Tension Based 7 DOFs Force Feedback Device: SPIDAR-G

Seahak Kim, Yasuharu Koike, and Makoto Sato

Abstract: In this paper, we intend to demonstrate a new intuitive force-feedback device for advanced VR applications. Force feedback for the device is tension based and is characterized by 7 degrees of freedom (DOF); 3 DOF for translation, 3 DOF for rotation, and 1 DOF for grasp). The SPIDAR-G (Space Interface Device for Artificial Reality with Grip) will allow users to interact with virtual objects naturally by manipulating two hemispherical grips located in the center of the device frame. We will show how to connect the strings between each vertex of grip and each extremity of the frame in order to achieve force feedback. In addition, methodologies will be discussed for calculating translation, orientation and grasp using the length of 8 strings connected to the motors and encoders on the frame. The SPIDAR-G exhibits smooth force feedback, minimized inertia, no backlash, scalability and safety. Such features are attributed to strategic string arrangement and control that results in stable haptic rendering. The design and control ofthe SPIDAR-G will be described in detail and the Space Graphic User Interface system based on the proposed SPIDAR-G system will be demonstrated. Experimental results validate the feasibility of the proposed device and reveal its application to virtual reality.

Keywords: haptic interface, human interface device, space graphic user interface, grasp manipulation

I. Introduction

The development of computer technology is enabling users to interact with various virtual environments. When users want to interact with virtual objects in a manner similar to those in the real world, an intuitive force-feedback device with multiple degrees of freedom (DOF) becomes a necessity.

In general, the physical act of gripping (or grasping) allows human beings to perform several important functions including using instruments to hit, translate, rotate, puncture and cut objects. Before doing the above-mentioned tasks, we select the necessary instruments by grasping it. Depending on the size and shape of the object, we can generally grasp an object using our thumb and our other fingers.

So far force-feedback devices have presented users with simple ways of representing this grasping function, such as pushing a button on a mouse or keyboard. We believe that an effective force-feedback device should not be limited to feedback on only the differential sense of width. Force-feedback should also be coupled with grasping. Such an "intuitive" force-feedback device has not yet been reported in the literature so much. The purpose of this paper is to realize such a tension-based 7 DOF force-feedback device that can allow users to not only grasp an object, but to also sense the width of an object as in real life object manipulation.

Up to the present, many force feedback devices has been developed. We can divide the force-feedback devices that have been developed into two categories: ground-based type and body-based type. Examples of body-based force-feedback devices are the LRP data glove[1], the Cybergrasp force feedback glove by Virtual Technologies Inc[2], and the Rutgers Masters (RM-II)[3] developed at Rutgers University. Fundamentally, body-based force feedback devices have the advantage of allowing the user to grasp an object, but also present the disadvantage of not being able to represent the weight of an object. Examples of ground-based devices include the PHANToM [4](Serial like type), the Haptic Master [5] (Parallel link type), the CyverForce[6] (Link type + Glove), the Freedom6S [7](Link type + string), CMU's magnetic force feedback device[8], and the SPIDAR [9][10][11] (String type). Ground-based force-feedback devices can generally be classified as link type, magnetic levitation type, and tension based type. To date, there is few intuitive force feedback device to display 7 DOFs force to the user.

Manuscript received: Aug. 23, 2001, Accepted: Nov. 16, 2001.
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II. Force displaying using tension

2.1 Vector closure

One characteristic of using strings to display forces is that they can only be used to represent tension. In other words, the strings can be used to pull and not push. We can determine the number of strings needed by applying vector closure to the indispensable condition of displaying n-DOF reflective forces using strings. When generating an n-dimensional force vector $q \in R^n$, using m-strings, the force vector q added to the target object from m-strings can be shown as in equation(1).

$$q = [w_1, w_2, \cdots, w_m]\tau \qquad (1)$$
$$w_i \in R^n \quad (i = 1, \cdots, m)$$
$$\tau = (\tau_1, \tau_2, \cdots, \tau_m)^T$$

Where w_i represents a force vector, when unit tension is added to the i-th string and τ is the tension vector. The following theories (1 and 2) outline the conditions realizing a positive solution for any τ in equation (1) [12][13].

[Theory 1]

If $A = [w_1, w_2, \cdots, w_m]$, the indispensable condition to have a positive solution in equation (1) is as follows:

m>n

[Theory 2]

If $A = [w_1, w_2, \cdots, w_{n+1}]$, the indispensable condition to have positive solution in equation (1) is as follows:

$$rank(A) = n$$

2. Using the remain row vector, any $w_i (i = 1, \cdots, n+1)$ can be represented as

$$w_i = -\sum_{j=1(j \neq i)}^{n+1} \alpha_j w_j \quad (\alpha_j > 0) \qquad (2)$$

However, n is the dimension of the work coordinate. Therefore, we can conclude that for the user to move an object in any direction in n-dimensional space, n+1 strings are need. Furthermore, the connection of the strings has to satisfy theory 2. In our case, SPIDAR-G needs at least 8 strings to display 7 DOF.

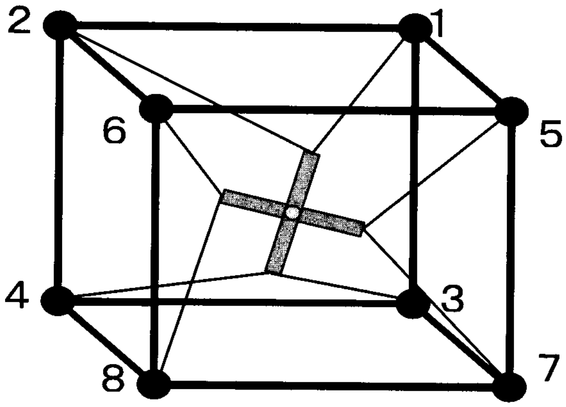

Fig. 1. Basic structure of SPIDAR-G.

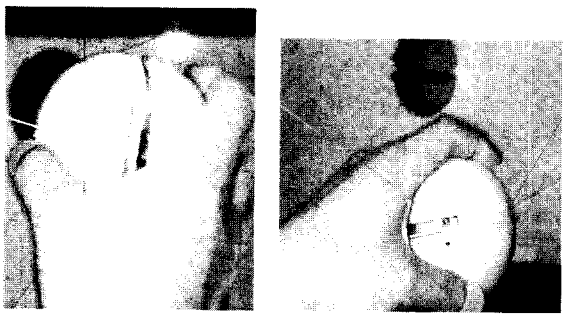

Fig. 2. State of grip before grasping (left) and after grasping (right).

III. Structure of SPIDAR-G

3.1 Basic Structure

Although we deduced that it was sufficient for us to use only 8 strings and that the connection has to comply with theory 2 (from vector closure), we still need to choose the best possible configuration for the connection of strings. This is because the magnitude, direction and area of force depend on the types of connections between the frame and grip. In general, we assume the users of our device would work in the central region of the frame. We choose the simplest way to display 7 DOF force in the central areas of the frame by using low torque. In other words if the position vector of grip was set to $A(\in R^{7\times 8})$, the larger the result of $\det|A^T A|$, the better it is for our purpose. Using this type of an analysis we could take the best connection between a vertex of a grip and a corner of a frame, as in figure 1. At each corner of the frame, an encoder and a motor are attached. The 8 strings are connected to each of the corners of the frame. On the opposite side, the other 8 strings are connected to each of the 2 strings on the vertex of the grip as well. The encoder calculates the length of string and the motors produce tension by pulling on the string.

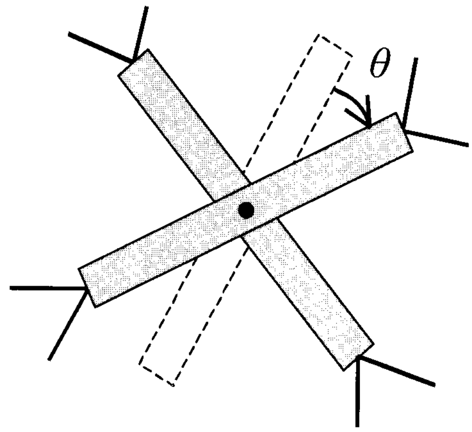

Fig. 3. Grasp manipulation.

3.2 Structure to grasp

Human beings are naturally skilled at grasping objects using their thumb and fingers. To display feedback force on the individual fingers, we initially tried attaching strings to the tips of each finger. This approach was not successful as it provided to be difficult to display translational, rotational, and grasping forces using only 8 strings.

In this paper we suggest a new mechanism for the grip. The new grip allows its users to manipulate with 7 DOF by grasping the grip between the thumb and other fingers. In order to incorporate the "grasping" functionality of the grip, it is best to consider a spherical shape. In figure 2, the proposed mechanism is broken into 2 hemispherical structures. If the user grasps the grip using thumb and fingers, the 2 poles rotate depending on the magnitude of the grasp force. Hence, it is possible to control the grasp functionality of the grip. The basic structure of the cross type grip is shown in figure 3.

The crossing degree θ changes with the magnitude of the grasping force and is used to quantify the action of grasping.

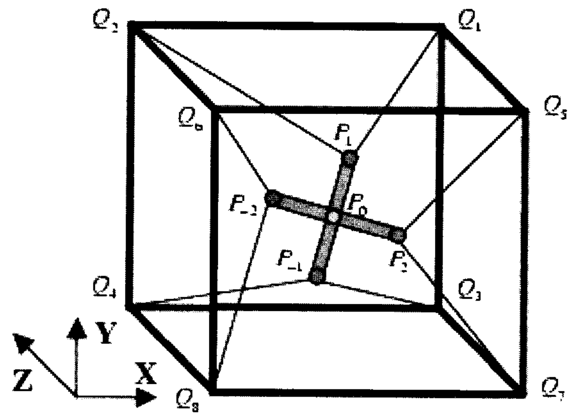

Fig. 4. Definition of symbols.

IV. Calculation of position

Position (translation, rotation, and grasp) is calculated from the length of 8 strings. The frame is rectangular parallel piped with the side lengths(along the X, Y and Z axis) represented as $2a, 2b, 2c$. The center of frame is taken to be the origin $(0,0,0)$. Each position vector Q_i $(\in R^3)$ in i-th extremity of frame is as follows.

$$Q_1 = (\ a,\ b,\ c) \qquad Q_2 = (\ -a,\ b,\ c)$$
$$Q_3 = (\ a,\ -b,\ c) \qquad Q_4 = (\ -a,\ -b,\ c)$$
$$Q_5 = (\ a,\ b,\ -c) \qquad Q_6 = (\ -a,\ b,\ -c)$$
$$Q_7 = (\ a,\ -b,\ -c) \qquad Q_8 = (\ -a,\ -b,\ -c)$$

Position vectors of the grip (P_0) and the 4 extremities $(P_1, P_{-1}, P_2, P_{-2})$, $P_i \ (\in R^3)$ are defined below.

$$P_0 = (x, y, z)$$
$$P_1 = (x + x_1, y + y_1, z + z_1)$$
$$P_{-1} = (x - x_1, y - y_1, z - z_1)$$
$$P_2 = (x + x_2, y + y_2, z + z_2)$$
$$P_{-2} = (x - x_2, y - y_2, z - z_2)$$

If we set the length of each pole to $2d$, the following equation is the result.

$$x_1^2 + y_1^2 + z_1^2 = x_2^2 + y_2^2 + z_2^2 = d^2$$

The grip extremities(P_i) are connected to the i-th frame corners (Q_i). The connectivity is described in Fig. 4. The length of the i-th string, l_i can be represented with following :

$$l_i = \left\| Q_i - P_{(i)} \right\| \quad (i = 1, \cdots, 8) \tag{3}$$

To calculate translation, rotation, and grasp, we have to solve (x, y, z), (x_1, y_1, z_1), and (x_2, y_2, z_2) from the length of 8 strings. The above equation can be converted into the following equations.

$$(x + x_1 - a)^2 + (y + y_1 - b)^2 + (z + z_1 - c)^2 = l_1^2 \tag{4}$$

$$(x + x_1 + a)^2 + (y + y_1 - b)^2 + (z + z_1 - c)^2 = l_2^2 \tag{5}$$

$$(x - x_1 - a)^2 + (y - y_1 + b)^2 + (z - z_1 - c)^2 = l_3^2 \tag{6}$$

$$(x - x_1 + a)^2 + (y - y_1 + b)^2 + (z - z_1 - c)^2 = l_4^2 \tag{7}$$

$$(x + x_2 - a)^2 + (y + y_2 - b)^2 + (z + z_2 + c)^2 = l_5^2 \tag{8}$$

$$(x - x_2 + a)^2 + (y - y_2 - b)^2 + (z - z_2 + c)^2 = l_6^2 \tag{9}$$

$$(x + x_2 - a)^2 + (y + y_2 + b)^2 + (z + z_2 + c)^2 = l_7^2 \tag{10}$$

$$(x - x_2 + a)^2 + (y - y_2 + b)^2 + (z - z_2 + c)^2 = l_8^2 \tag{11}$$

Using the above equations, we can solve (x, y, z), (x_1, y_1, z_1), and (x_2, y_2, z_2). We can solve the above variables using 4 arithmetical operations because of the redundancy of strings. We show the detailed algorithm in index 1.

V. Display of force

In this section, we explain how to determine the tension of the 8 strings that will result in 7 DOF force display at the "cross-type" grip. The force vector $q (\in R^7)$ can be represented as

$$q = (f_x f_y f_z m_x m_y m_z, g)^T$$

Where f_x, f_y, f_z represent translation forces, m_x, m_y, m_z rotation forces, and g is the grasp force.

We define the tension of string $\tau_{(i)}$ $(i = 1, \cdots, 8)$, and the tension vector τ as follows

$$\tau = (\tau_1, \tau_2, \cdots, \tau_8)^T \quad (\in R^8)$$

We take w_i to be the force vector generated in the grip. As the unit tension is added to i-th string. w_i is defined as

$$w_i = \begin{bmatrix} c_i \\ r_{(i)} \times c_i \\ \delta_i \cdot n \cdot r_{(i)} \times c_i \end{bmatrix}$$

However,

$$c_i = \frac{Q_i - P_{(i)}}{\left\| Q_i - P_{(i)} \right\|} \quad (i = 1, 2, \cdots, 8)$$

$$r_{(i)} = P_{(i)} - P_0$$

$$\delta_i : \begin{Bmatrix} 1 & i = 1,2,3,4 \\ -1 & i = 5,6,7,8 \end{Bmatrix}$$

$$n = \frac{r_1 \times r_2}{\left\| r_1 \times r_2 \right\|}$$

If we set $A \in R^{7 \times 8}$ into $A = (w_1, w_2, \cdots, w_8)$, given tension vector τ, the force vector q, can be represented as

$$q = A\tau$$

To display force vector q to cross type grip, we have to solve the tension vector τ which satisfies the above equation. However, the tension vector must be a positive value vector $(\tau_i \geq 0, i = 1, \cdots, 8)$. If we solve the quadratic programming problem, we can obtain the tension vector.

$$\left\| q - A\tau \right\|^2 \rightarrow \min$$
$$s.t. \ \tau_{\max} > \tau \geq 0$$

Because SPIDAR-G uses the tension of strings to display force, there are certain location within the frame where SPIDAR-G can not display appropriate forces. However, near the center of frame, SPIDAR-G can display 7 DOF force appropriately.

VI. SPIDAR-G system

We show manufactured SPIDAR-G in figure 5. The length

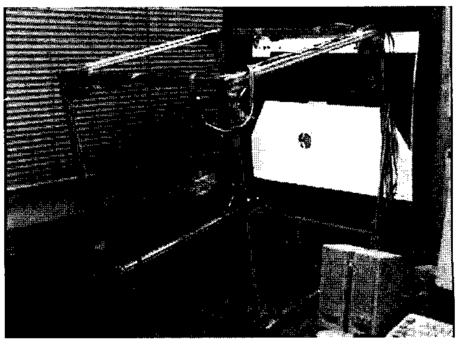

Fig. 5. Manufactured SPIDAR-G.

of frame is 48cm, and the radius of grip is 4cm. The computer which was used with the SPIDAR-G is based on a Pentium 400 MHz processor. The encoder is a HEDS-5540 made by HP. The DC motor is a product of Maxon[14].

Table 1. Error of Measured string

Real length (cm.)	Method 1 Measured length	Error of M1 (cm.)	Method 2 Measured length	Error of M2 (cm.)
10	9.15	0.85	9.85	0.15
20	18.68	1.32	19.79	0.21
30	28.20	1.80	29.74	0.26
40	37.89	2.11	39.70	0.30
50	47.59	2.41	49.70	0.30
60	57.08	2.92	59.72	0.28
70	66.54	3.46	69.77	0.23

6.1 Error of each string

Our proposed force feedback device calculates the position of a grip based on the length of the strings. We used encoders that can detect 2000 pulses per 1 rotation and pulleys that each have a radius of 8 *mm*. The length per pulse is 0.0251327*mm*. A length per pulse multiplied by the counter value is the length of string [method 1(M1)]. Because this method of calculating the string length can result in considerable measured errors, we used least squared error criterion to reduce the measured error [method 2(M2)]. The equation used to determine the length of each string is as follows.

$$\text{Each length(cm)} = 0.50744 + 0.0026282 * \text{counter pulse}$$

Using this equation, we measured the error of various string lengths over five trials with the results shown in table 1. The measured error proved to be below 0.3cm for strings less than or equal to 70cm in length. Even though we used a 2000 P/R high resolution encoder, some large errors were observed. Possible explanations for these errors follow.

1. Tension of string: The string can be stretched depend on the magnitude of tension. If a large amount of tension is continuously added, the string will be excessively stretched and cannot be restored to its original length.

2. Changing pulley radius: If the string does not evenly wind around the pulley, the string will begin to wrap on top of itself. This changes the effective width of the pulley. To solve this problem, grooves should be added to the pulley so that a string will wind evenly.

[Consideration]

In the case of explanation 1, the magnitude of stretch depends on the length of the string. Errors are more pronounced when the grip is manipulated away from the center of the workspace toward the edge of the frame. In these regions, the lengths of some strings are maximized. To correct this problem, software must restrict the workspace to the center region of the frame. In the case of explanation 2, string length is irrelevant and users can manipulate anywhere within the frame. Error are constant regardless of position, but the entire workspace within the frame can be used.

6.2 Workspace of SPIDAR-G

The connection of the strings between each extremity of the grip and each vertex of the frame is similar in length about the X and Y axis. Therefore, the Y dimension will be ignored in the following experiment.We define capacity to display force as the displayed force divided by the target force.

The SPIDAR-G displays force $q\,(\pm 5,0,0,0,0,0,0)$ to the grip without rotation. The translation forces obtained about the X-axis were measured in the center of the workspace deviating no more than 20cm from the center.

The displayed translation force about the X-axis is as follows.

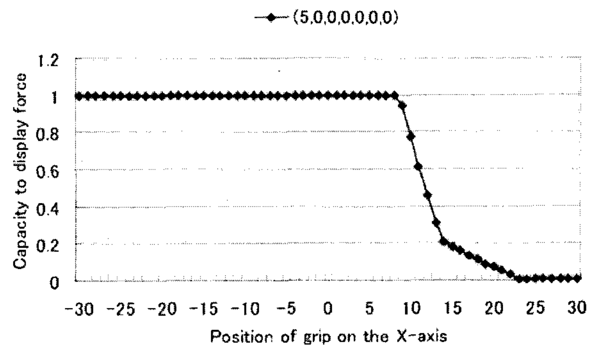

Fig. 6. X-axis workspace about translation 1.

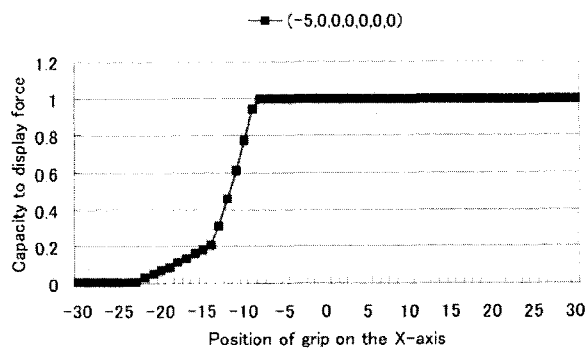

Fig. 7. X-axis workspace about translation 2.

Like translations about the X-axis, the translation forces obtained about the Z-axis were measured in the center of the workspace. The resulting translation force about the Z-axis is shown below.

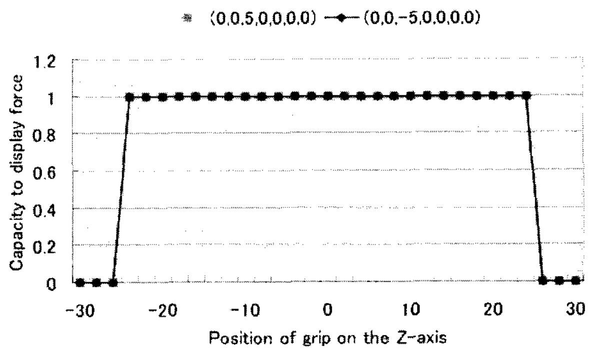

Fig. 8. Z-axis workspace about translation.

The rotation force about the X-axis was measured between -90degree and 90degree. The displayed rotation force about X-axis is as follows.

Table. 2 Average grip positions and the differences with respect to the marker (Unit in cm.)

	-20	-15	-10	-5	0	5	10	15	20
X	-19.7	-14.7	-9.9	-4.9	0	4.9	9.9	14.7	19.6
Y	-20.4	-15.4	-9.7	-4.9	0	4.8	9.6	14.7	20.4

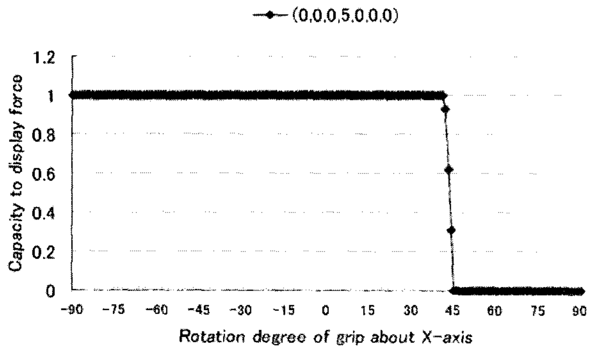

Fig. 9. X-axis workspace about rotation 1.

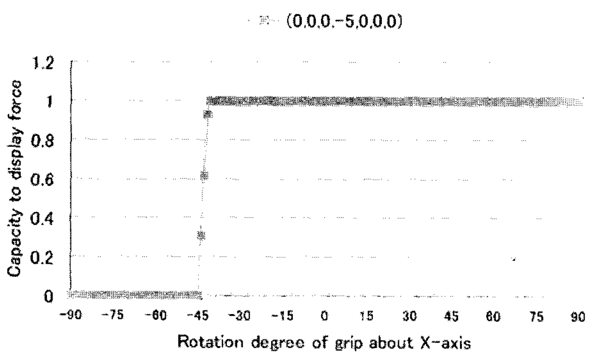

Fig. 10. X-axis workspace about rotation 2.

The rotation force about Z-axis were obtained similarly to those about the X-axis. The displayed rotation force about Z-axis is shown in figure 11.

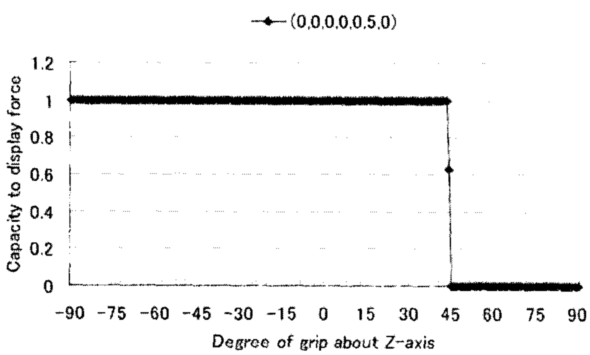

Fig. 11. Z-axis workspace about rotation 1.

The grasp force between -45degree and 45degree was measured. The SPIDAR-G easily displayed grasp force within this work space.

6.3 Accuracy of position measurement (experiment1)

We positioned the grip of the SPIDAR-G on markers located every 5cm on the X-axis and Z-axis (without applying rotation) and determined the average grip position with the method defined in section 4. A start point of (0,0,0) was taken in the center of frame. The results are shown in table 2.

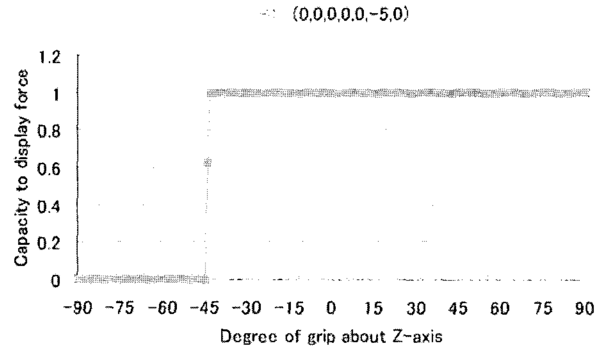

Fig. 12. Z-axis work space about rotation 2.

The input error of the SPIDAR-G proved to be below ± 4 mm when measured within a 40-centimeter cube region positioned in the center of the frame. In addition, a 45 degree rotation of the grip was applied about the Y-axis and the grip was translated along the X and Z axis. While maintaining a 45 degree rotation, the angle was determined with the method described in section 4. The results are shown in table 3. The error resulting for measuring grip orientation is most likely due to inaccurate string measurement. The rotation errors are larger in magnitude than the translation errors, indicating greater sensitivity to inaccuracies in string length measurement. If the angle of a cross typed grip is between 20degree and 50 degree, we assumed that the grip is in grasp status. Otherwise, release is assumed. Given these criteria, every point within a 40-centimeter cubical region positioned in the center of the frame could be grasped and released.

VII. SGUI

We created a 3D grip device for the space Graphic User Interface(SGUI) to be used with the SPIDAR-G. The grip device is shown in figure 13.

1) Hardware:

CPU: Pentium 400MHz (corresponding), 128MB

OpenGL graphic accelerator board: FireGL 4000 (corresponding), USB interface

2) Software

OS: Windows98/2000

Graphic library: OpenGL

www browser : Microsoft Internet

7.1 Experiment environment

To validate the proposed environment, a subject group of 12 was chosen that included a range of people who were inexperienced to very experienced with personal computers. Ideally, a range of experience with haptic devices would have been preferred, but almost all the subjects interviewed had no experience with force feedback devices. The participants were either office workers or students. All worked in field related to CG, software development, e-commerce and VR.

Table 3. Step valuations.

Step	feeling
Valuation 5	Best
Valuation 4	Good
Valuation 3	Average
Valuation 2	Little bad
Valuation 1	Bad

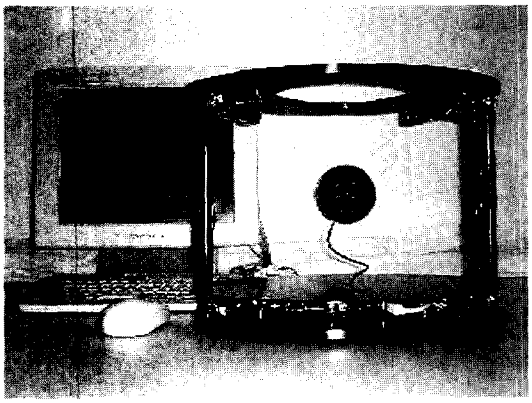

Fig. 13. SGUI system.

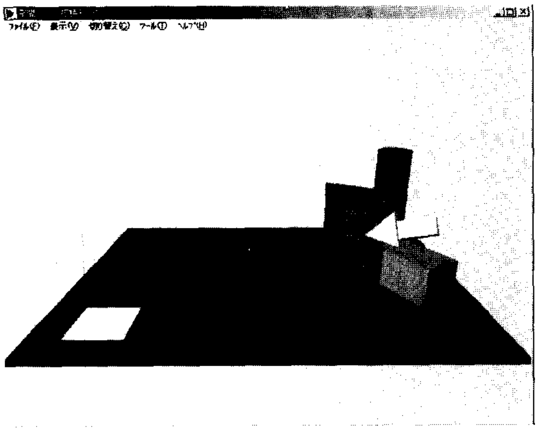

Fig. 14. Pick and Place task (Experiment 2).

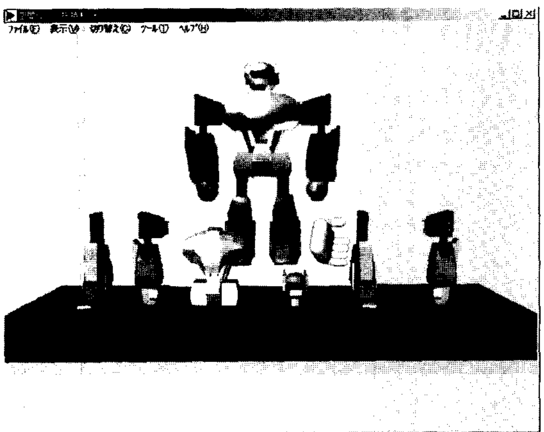

Fig 15. Link part together (experiment 3).

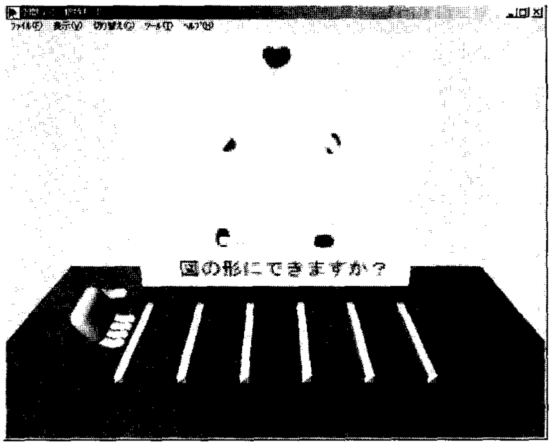

Fig. 16. Link matches together (experiment 4).

7.2 Evaluation of the 3D grip

Three tasks that required 7 dofs manipulation were presented to the users. The first experiment required the user to pick up objects and place them on corresponding targets (figure 14). The second experiment required the user to make a robot by linking parts together (figure 15). The third experiment demanded that a user link matches together in order to create a predefined target shape (figure 16).

We measured the task time to pick up objects and move them to their respective targets. The subjects were also asked to subjectively assess the SPIDAR-G interface for each experiment on a five-point scale. These tasks were also presented through a mouse and the CosmoPlayer, however, auto animation had to be incorporated so that this 2-D interface could be used to perform 3D tasks. The objective was to compare a typical 2D interface to the 3D interface of the SPIDAR-G.. Ideally, we would have developed experiments that compared the SPIDAR-G to other haptic devices. Unfortunately, no commercial haptic device exists that provides 7 DOFs. For this reason, only simple subjective experiments were designed that tested the users satisfaction with the interface. An attempt was made to device tasks that required the use of all 7 DOFs. Grip was a critical element in each experiment.

The perceived effectiveness using the 3D grip decreased in experiment 4 as compared to the results obtained in experiment 2 through 3 (60-70%). The reason for this is probably due to the increased difficulty of the task. Thin objects, such as matches, ended up being harder to pick up. However, when evaluating overall manipulation, subjects were satisfied (80%) with the interface for all the tasks.

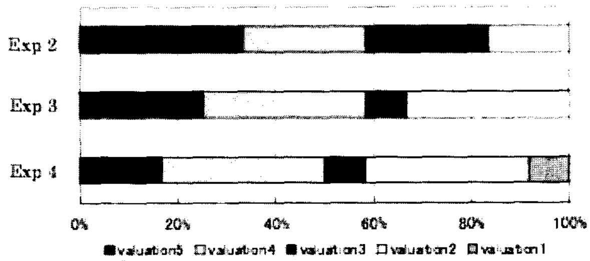

Fig. 17. Grasp virtual objects.

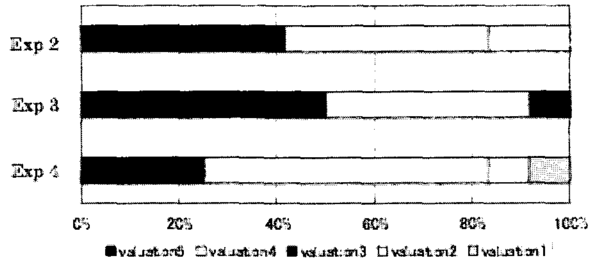

Fig. 18. Translation and rotation.

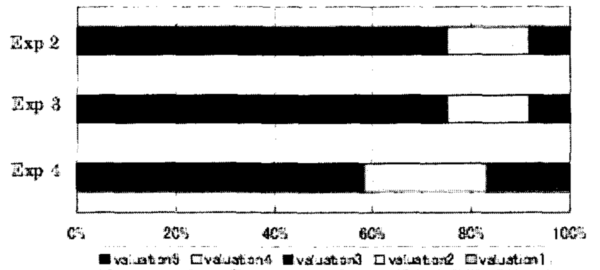

Fig. 19. Comparison with a mouse.

VIII. Conclusion

In this paper, we described a tension based force feedback device with 7 DOFs. We demonstrated how to obtain precise solutions for the position of the grip using the redundancy of strings and the unique geometric characteristic of this system. We have also shown a new way to calculate position with 7 DOFs and to display force over 7 DOFs using a grip. Through examples, we have demonstrated the effectiveness of our proposed SPIDAR-G when performing 3D tasks. The examples prove that our contrived SPIDAR-G provides users with not only effective translation and rotational manipulation, but also grasp, which is accurate and efficient. Given only simple verbal instructions, subjects could easily complete the tasks presented in experiment 2,3 and 4 and all subjects expressed a high level of satisfaction with the interface. Future work will be devoted to developing more robust algorithms that more accurately measure string length.

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From equation (4)~(11), we can get each x, y, x_1, y_2 as follows.

About x : -eq(4) + eq(5) - eq(6) + eq(7)
About y : -eq(8) - eq(9) + eq(10) + eq(11)
About x_1 : -eq(4) + eq(5) + eq(6) - eq(7)
About y_2 : -eq(8) + eq(9) + eq(10) - eq(11)
Therefore,

$$x = \frac{1}{8a}\left(-l_1^2 + l_2^2 - l_3^2 + l_4^2\right)$$

$$y = \frac{1}{8a}\left(-l_5^2 - l_6^2 + l_7^2 + l_8^2\right)$$

$$x_1 = \frac{1}{8a}\left(-l_1^2 + l_2^2 + l_3^2 - l_4^2\right)$$

$$y_2 = \frac{1}{8a}\left(-l_5^2 + l_6^2 + l_7^2 - l_8^2\right)$$

We substitute the above solutions into equation (12)~(15).

eq(4) + eq(5) + eq(6) + eq(7) (12)

eq(4) + eq(5) - eq(6) - eq(7) (13)

$$\text{eq}(8) + \text{eq}(9) + \text{eq}(10) + \text{eq}(11) \tag{14}$$

$$\text{eq}(8) - \text{eq}(9) + \text{eq}(10) - \text{eq}(11) \tag{15}$$

We can get 4 equations about Z. These 4 equations can be changed into 2 six degrees equations. General case, we use numeric method to solve six degrees equations. But this method requires suitable conditions, substantial time due to the iterative nature of the technique, and the results are only approximations. It is necessary to reduce the amount of calculations to maintain the haptic servo loop and we need precise results rather than approximations to earn high resolution. Numerical methodologies are therefore unsuitable. Fortunately, in the case of using strings, we can know that the above 2 six degree equations contain the same result about z due to the redundancy of the strings. By either adding or subtracting 2 equations, we are able to get a six degree equation and a five degree equation which have a common solution about z. By dividing the high degree equation by the low degree equation, we get the result of Z. Using this, we can solve other variables (y_1, z_1, x_2, y_2, z_2).

$$y_1 = \frac{1}{2b}\{k_1 + (z-c)^2\}$$

$$x_2 = \frac{1}{2a}\{k_3 + (z+c)^2\}$$

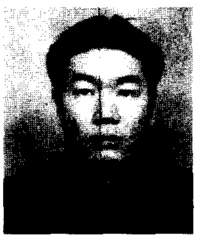
$$y_2 = \frac{1}{8b}\left(-l_5^2 + l_6^2 + l_7^2 - l_8^2\right)$$

$$z_2 = \{\frac{1}{8}(l_5^2 - l_6^2 + l_7^2 - l_8^2) - yy_2 - ax - xx_2\}/(z+c)$$

However,

$$k_1 = x^2 + y^2 + d^2 + a^2 + b^2 - \frac{1}{4}\sum_{i=1}^{4} l_i^2$$

$$k_2 = x^2 + y^2 + d^2 + a^2 + b^2 - \frac{1}{4}\sum_{i=5}^{8} l_i^2$$



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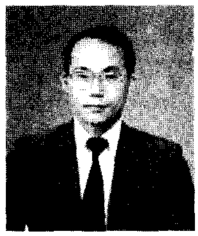

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