# Analysis and Compare for Control Charts Under the Changed Alarm Rule

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## Abstract

This paper mainly studies to build control charts under different alarm rule. For different alarm rule, the control limit parameters of a control chart should be changed, then some kinds of control schemes under different alarm rule were compared and the methods of calculating ARL for different control schemes were given.

**Key Words:** alarm rule, Average Run Length, Markov chain

## 1. Introduction

Control chart is a kind of chart with control limit, which is used to analyze and judge whether process is in control. A chart is often employed to show quality variations in production process at any time and refer to manager to adopt corresponding measure in time to eliminate the influence of systematic factors and maintain the steady state of process. It is a basic method of control and quality management in enterprises, and playing the most important role for quality control of product. The control chart used to detect production process is proposed by Shewhart in 1924. In this kind of chart, we think the key characteristic or the process being detected is in out of control state when a point falls out $3\sigma$ control limit. This kind of control chart is simple and easy to use discovering larger shifts of the process quickly. So the control chart was being used extensively. However, for relative less shifts, the chart becomes not very sensitive.

In order to monitoring better for less shifts, people have used many methods to improve the conventional control chart. Some persons adopted complex control chart schemes, such as CUSUM chart and EWMA chart. Although those charts are sensitive to detect less shifts, complex calculation is needed to build those schemes. So those schemes are hard to get extensive application. On the other hand some persons seek to adjust the alarm rules of control chart. They increased some other alarm rules on the original rule of one point beyond 3σ control limit. Those added rules are called as western electrical rule. However people discovered that increasing alarm rules could enhance the sensitivity of control chart for less shifts, but at the same time increased false alarm rate. Therefore a kind of simple and effective method was put forward in this paper, it can improve the sensitivity of conventional control chart and at the same time keep the false alarm rate unaltered. The design method of control charts under different alarm rules was first introduced in this paper, such as two successive points out of limits, two of three successive points out of limits, three successive points out of limits etc. Then we compare those kinds of charts and Shewhart chart and elicit a conclusion that the chart under the alarm rule of two of three successive points out of limits has better effect in monitoring less shifts of process mean and shifts of process variations. Finally the calculation method of ARL of charts under changed alarm rules is given.

## 2. Computation Of Control Limit Parameters

Now we discuss the problem of how to get those control limit parameters under different alarm rules. The precondition is to make sure the false alarm rate unaltered, that means $\alpha = 0.0027$ for any control chart,.

First, when alarm rule is two successive points out of limits, the control limit parameter is defined as $k_{22}$, then the probability of one point out of limits is $\rho = 2\phi(-k_{22})$; the probability of two successive points out of limits is $\rho' = \rho^2 = [2\varphi(-k_{22})]^2$. From $\alpha = 0.0027$, we can get a formula:

$$[2\phi(-k_{22})]^2 = \alpha = 0.0027; \tag{1}$$

And we can get $k_{22}$=1.94345.

When alarm rule is two of three successive points out of limits, the control limit parameter is defined as $k_{32}$, then the probability of one point out of limits is $\rho = 2\phi(-k_{32})$; the probability of two successive points out of limits is: $\rho' = (1-\rho)\cdot\rho^2 + \rho\cdot(1-\rho)\cdot\rho + \rho^2 = 3\rho^2 - 2\rho^3$;

From $\alpha = 0.0027$, then :

$$3[2\phi(-k_{32})]^2 - 2[2\phi(-k_{32})]^3 = 0.0027; \tag{2}$$

and we can get $k_{32}$=2.17009

When alarm rule is three successive points out of limits, the control limit parameter is defined as $k_{33}$, then the probability of one point out of limits is $\rho = 2\phi(-k_{33})$; the probability of two successive points out of limits is $\rho' = \rho^3$. From $\alpha = 0.0027$, we can get:

$$\left[2\phi(-k_{33})\right]^3 = 0.0027; \tag{3}$$

and we can get $k_{33}$=1.47579.

## 3. Comparing Those Charts By The Shifted Mean

In last section, we have determined control limit parameters of control charts under different alarm rules. Then we evaluate all of those charts and Shewhart control chart by comparing their accepting rate. And the sample size of all kinds control schemes adopts the value Shewhart control chart usually used, $n$=5.

Firstly, under the condition of process mean shifted, we use the shift value of process mean $d$( $d=\frac{|d_1-d_0|}{\sigma}$, $d_1$, the process mean after a shift occurred; $d_0$, the original process mean) as transverse axis, use accepting rate of a point after a shift occurred as vertical axis, and then we can get the OC curve as Figure 1. From Figure 1 we can get a conclusion as follow:

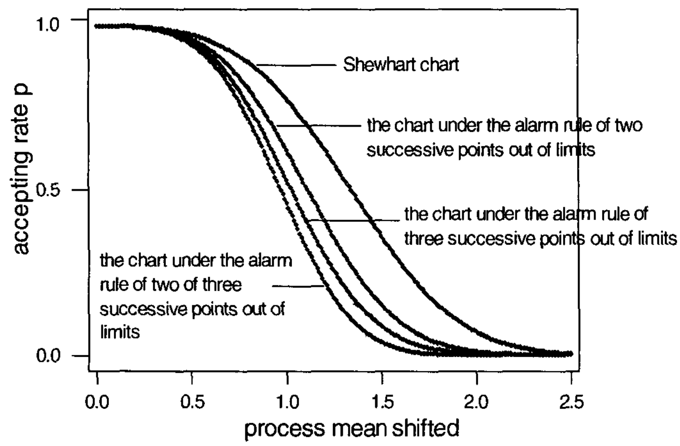


**Figure 1.** The OC curves of all kinds of charts about shifts of process mean

When the process mean is unchanged, because the false alarm rates of all kinds of charts are same, accepting rates of all charts are equal: $p$=1-0.0027=0.9973; When the shift value of process mean is $0<d<2.5$, the order of accepting rate of all kinds control schemes is: Shewhart control chart > the chart under the alarm rule of successive two points out of limits > the chart under the alarm rule of successive three points out of limits > the chart under the alarm rule of two of successive three points out of limits. Under the condition of a same false alarm rate, the less accepting rate, the less the probability of fail to alarm, the more sensitive this kind of control chart to shifts of process mean. So, when the shift value of process mean is $0<d<2.5$, the chart under the alarm rule of two of three successive points out of limits is the best monitoring scheme of process mean in those kinds of charts and Shewhart chart is the worst one; When the shift value of process mean is bigger than 2.5, the accepting rates of all kinds of charts become same again. This means they have same monitoring effect.

## 4. Comparing Those Charts For Process Variation

Now, we discuss those schemes under the condition that the variance of process changed. Usually we only need to prevent the increasing of process variance, so here we only consider the change of accepting rate of all kinds of charts when process variance increased. We use increasing rate of process variance $h$( $h=\frac{\sigma_1}{\sigma_0}$, $\sigma_1$ express increased variance, $\sigma_0$ express process original variance) as transverse axis, use accepting rate of a point after an increase of variation occurred as vertical axis, and then we can get the OC curve as Figure 2. From Figure 2 we can get a conclusion as follow:

When the process variation is unchanged, because the false alarm rates of all kinds of charts are same, accepting rates of all charts are equal: $p$=1-0.0027=0.9973; When the process variation is increased, the order of accepting rate of all kinds control schemes is: the chart under the alarm rule of successive three points out of limits great than the chart under the alarm rule of successive two points out of limits great than Shewhart control chart great than the chart under the alarm rule of two of successive three points out of limits.

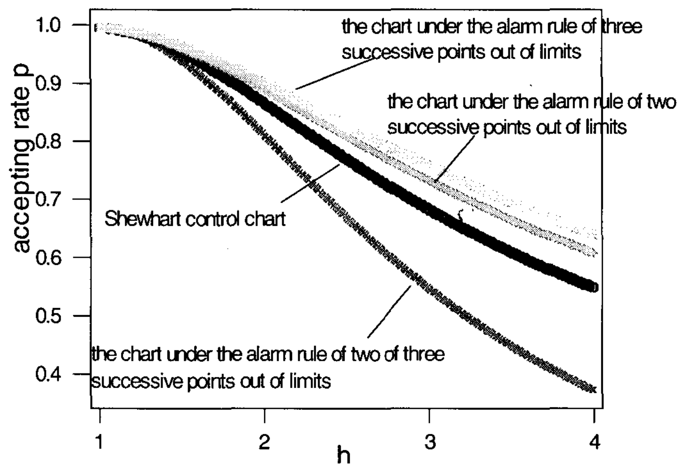


**Figure 2.** The OC curves of all kinds of charts about increase of process variation

## 5. The Calculation Method Of ARL

Now we discuss the calculation methods of ARL of the control charts under changed alarm rules. Because of the changes of alarm rule, run length of a control chart do not obey a geometry distribution. Therefore the calculation methods of ARL also need to have corresponding change. Klein( 2000) calculates average operation length with Markov transfer matrix. We introduce the following method.

A control chart can be viewed as consisting of three regions: above upper control limit, between two control limits, below the low control limit. The probabilities of a point on control chart falling in this three regions can be expressed by $p_u$, $p$, and $p_l$ respectively. The values of these three probabilities can determine parameters of the control limits. The relationship of this the three probabilities can be showed as:

$$p_u = p_l$$
$$p + p_u + p_l = 1 \tag{4}$$

For a control chart under the alarm rule of successive two points out of limits, if we regard dotting process as a Markov chain, it have four possible states at a time: state 1 representing no points beyond either control limits, state 2 representing a point beyond the upper control limit, state 3 representing a point beyond the lower control limit, state 4 representing two successive points are beyond just one of the two limits. The former three states means that the process is still in control, but state 4 express the process is out of control. And state 4 is an absorbing point. If the chart under state 4, it can't be changed to others state again. So we can get the transition probabilities matrix of this Markov chain as Table 1:

**Table 1.** Transition probabilities Matrix of Markov process

| | | States at time t+1 | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 |
| States at time t | 1 | P | $P_u$ | $P_L$ | 0 |
| | 2 | P | 0 | $P_L$ | $P_u$ |
| | 3 | P | $P_u$ | 0 | $P_L$ |
| | 4 | 0 | 0 | 0 | 1 |

The expected value of the first passage step from starting state 1 to state 4 is the ARL, it can be computed by equations below:

$$R_{14} = 1 + p \cdot R_{14} + p_u \cdot R_{24} + p_l \cdot R_{34}$$
$$R_{24} = 1 + p \cdot R_{14} + p_l \cdot R_{34}$$
$$R_{34} = 1 + p \cdot R_{14} + p_u \cdot R_{24} \tag{5}$$

Here, $R_{i4}$ express the expected value first passage step from state to state 4 and $R_{14}$ is the ARL. The calculation method of ARL is easy to get from formula (5):

$$ARL = R_{14} = \frac{1}{1 - p - \frac{p_u}{1+p_u} - \frac{p_l}{1+p_l}} \tag{6}$$

According to same method, we may also get ARL calculation methods of other two kinds of charts under changed alarm rules. Because run length obey different distributions under different alarm rule, in-control ARL of all kinds of charts are different ever though those charts have a same false alarm rate. For example, under the precondition of $\alpha = 0.0027$, the in-control ARL of chart under the alarm rule of two successive points out of limits is 760, and the in-control ARL of Shewhart control chart is 370. Under the precondition of fixing $ARL_0$=370, Klein (2000) had put forward a kind of design scheme of control charts under the alarm rule of two successive out of limits and under the alarm rule of two of three successive points out of limits. However the precondition of this kind of design increased greatly the false alarm rate.

In Figue.1, we had discovered that control chart under the alarm rule of two successive points out of limits is better than Shewhart control chart in monitoring shifts of process mean. Now this conclusion can be proved further by comparing ARL values of the two design schemes.

In Table 2, when process is in control statement, the ARL value of control chart under the alarm rule of two successive points out of limits is much bigger than the ARL value of Shewhart control chart. The bigger ARL can make control chart under the alarm rule of two successive points out of limits work without false alarm for longer time and reduce lose bringing by false shutdown to overhaul. At the same time, when process is out of control, the ARL value of control chart under the alarm rule of two successive points out of limits is decreased quickly, and even is smaller than the ARL value of Shewhart control in the section of $0.8 \leq d \leq 2.4$ . In this way control chart under the alarm rule of two successive points out of limits can detect shifts of process mean quicker than Shewhart control chart.

**Table 2.** The ARL values of two kinds of charts

| d | ARL value of Shewhart control chart | ARL value of control chart under the alarm rule of two successive points out of limits |
|---|---|---|
| 0 | 370 | 760 |
| 0.2 | 308.4 | 543.7 |
| 0.4 | 200.1 | 274.9 |
| 0.6 | 119.7 | 135.3 |
| 0.8 | 71.6 | 70.4 |
| 1.0 | 43.9 | 39.3 |
| 1.2 | 27.8 | 23.5 |
| 1.4 | 18.2 | 15.0 |
| 1.6 | 12.4 | 10.2 |
| 1.8 | 8.7 | 7.4 |
| 2.0 | 6.3 | 5.6 |
| 2.2 | 4.7 | 4.4 |
| 2.4 | 3.6 | 3.6 |
| 2.6 | 2.9 | 3.1 |
| 2.8 | 2.4 | 2.8 |
| 3 | 2.0 | 2.6 |
| 4 | 1.2 | 2.0 |

## Acknowledgment

This research was jointly supported by National Science Funds for Distinguished Young Scholars 70125004, and NSFC Projects 9870100, 70072029.

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