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The Evaluation of Groundwater Pumping Capacity at a Catchment Area with Interrelated Wells in Volcanic Island: I. Without Consideration of Water Quality

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상관우물들이 분포하는 화산섬 집수역에 대한 지하수 양수능의 평가 I. 水質을 고려하지 않은 경우

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요 약

본 연구는 數値解析을 이용하여 상관우물들의 분포를 갖는 집수역에서 수질을 고려하지 않은 상태의 지하수양수능을 평가하는 것을 목적으로 한다. 연구지역은 日本의 미야께지마(三宅島)이며, 미야께지마는 최근에 이르기까지 火山噴火가 계속되고 있는 화산섬으로 水文地質學的으로 매우 복잡한 구조를 갖고 있다. 각각의 우물들에 대한 양수능은 個別양수에 의해서 구해진 IMY(i,t)로써 추정되었으며, 全 연구지역의 양수능은 同時양수에 의해서 구해진SSMY(i,t)로써 추정되었다. 이러한 결과들은 미야께지마와 같은 화산섬에서 用水공급을 위한 계획의 수립에 있어서 확실한 공급원의 확보에 이용될 수 있다.

동시양수의 경우, 우물 5와 6에서의 양수는 타이로이께(大路池)부근에 존재하던 지하수가 연구지역의 내부에 까지 침투하는 것에 대한 障害우물로써 작용했다. 그러므로, 본 연구는 質的, 量的 측면에서 용수공급을 위한 지하수의 最適 관리방법으로써 동시양수를 제안한다.

주요어 : groundwater pumping capacity, interrelated wells, individual withdrawal, simultaneous withdrawal, and barrier well

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Abstract

The purpose of this paper is to evaluate the groundwater pumping capacity at a catchment area containing interrelated wells without considering their qualities by using numerical simulation in Miyake Island, young volcanic island with very complicated hydro-geological formations. The groundwater pumping capacities of each well and over entire study area were estimated as the IMY(i,t) by individual withdrawals and the SSMY(t) by simultaneous withdrawals. These results can be used to secure a sure source for taking a plan for supplying water use in young volcanic island as Miyake Island.

In simultaneous withdrawals, the withdrawals from well no. 5 and 6 should have the roles as the barrier wells against the intrusion of the groundwater of the part adjacent to Tairo Pond into the inner part of study area. Therefore, it can be suggested to adopt the simultaneous withdrawals as the optimal approach of groundwater management for supplying water use with respect to quantity and quality.

Key words : groundwater pumping capacity, interrelated wells, individual withdrawal, simultaneous withdrawal, and barrier well

I. Introduction

To secure the stable sources of water use for human activities is an important problem in Miyake Island, Japan. Most of young volcanic islands generally have little permanent surface runoffs regardless of having abundant precipitation. The major causes might be in the very steep slopes from its center to coastlines and the thick volcanic surface deposits with high permeability. While the former brings about very rapid discharge of a part of precipitation through intermittent surface runoff into sea, the latter causes the infiltration of the other part of precipitation into underground. Even in young volcanic islands, the existence of groundwater could be found.

It is very difficult to find out the distribution and catchment area of groundwater with a high quality and abundant quantity in a young volcanic island, because not only the topographical features of the thick volcanic surface deposits often have a very poor agreement with their underlying layers but also they have remarkable local differences with respect to hydro-geological properties. In the use of groundwater as the source of water use, also, the withdrawal of groundwater from wells is necessary and this should results in; the interference of pumping yields by interrelation between wells and the spread of contaminants from the wells or sources with low quality. In order to solve these problems, it is required to evaluate the pumping yields without and with consideration of water quality using numerical simulation. Pinder and Bredehoeft (1968) used the digital computer for aquifer evaluation. Pinder and Frind (1972) described the application of Galerkin's procedure to aquifer analysis. Pricket (1975) and Bachmat et al. (1978) used numerical models for evaluating the quantity of regional groundwater. Orlob and

Woods (1967) considered the management of water quality in irrigation systems. However, it was never mentioned to evaluate the pumping yields of interrelated wells without and with consideration of water quality using numerical simulation in order to secure a sure source for taking a plan for supplying water use in the young volcanic island as Miyake Island.

The purpose of this paper is to evaluate the groundwater pumping capacity at a catchment area containing interrelated wells without consideration of water quality using numerical simulation in the young volcanic island with very complicated hydro-geological formations. This result can be used to secure a sure source for taking a plan for supplying water use in the young volcanic island. The evaluation of groundwater pumping capacity with consideration of water quality will be subsequently discussed.

II. Study Area

This study was accomplished at the deposits with a light slope and the area of about 0.2 km^2 between pond and steep rock slope within a small basin (Figure 1). The pond is called 'Tairo Pond' and is located at the center of a catchment area with a distinct basin type. The basin is the most important source for water use in Miyake Island. Also, it was reported to be the crater formed by the volcanic eruption 2500 years ago (Tsukui and Suzuki, 1998). Miyake Island is about 180 km south of Tokyo, Japan and its geological formation is mainly composed of the complicate alternations of basalt and volcanic deposits by several volcanic activities with the interval of about 20 years.

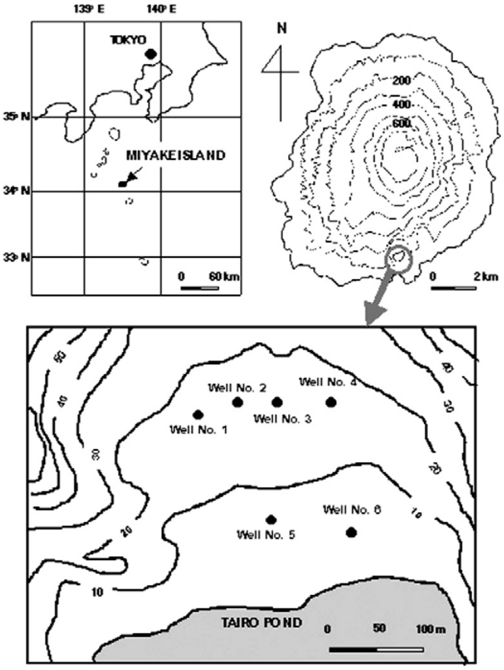

Figure 1. Study area.

Table 1. The groundwater and pumping limited heads at wells and the hydraulic conductivities around wells.

	Groundwater head$^{\#\&}$ (unit: m)	Pumping limited head$^{\#\$}$ (unit: m)	Hydraulic conductivity (unit: m/sec)
well no. 1	3.092	-0.869	1.663 x 10^{-3}
well no. 2	3.081	-1.334	7.490 x 10^{-4}
well no. 3	3.070	-1.185	1.662 x 10^{-3}
well no. 4	3.070	-0.790	5.696 x 10^{-4}
well no. 5	2.997	0.583	2.320 x 10^{-2}
well no. 6	3.002	-2.908	1.035 x 10^{-2}

The references of groundwater and pumping limited head are at sea level.
& Groundwater heads were measured during February to April 2002.
$ Pumping limited head means the minimum level in withdrawal, which is determined by the structure of installed pump equipment.

The hydraulic conductivities around wells were obtained by pumping test, and the result is shown in Table 1.

III. Theory

The formulation of the equation describing groundwater flow in horizontal two dimensions can be written:

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial t}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)=S_s\frac{\partial h}{\partial t} \tag{1}$$

where h is hydraulic head in m, K_x and K_y are the components of saturated hydraulic conductivity in the x and y coordinate directions in m/sec, S_s is specific storage in m^{-1}, and t is time in sec. In order to apply Eq. (1) into the study area of Figure 1, the appropriate boundary and initial conditions are required.

The initial condition, $h(x, y, 0)$ was given by the measured groundwater and pond water heads during February to April 2002, $h_0(x, y)$, and the result is shown as the groundwater heads in Table 1 and the distribution of equipotential lines in Figure 2. Thus, the initial condition for Eq. (1) is:

$$h(x,y,0)=h_0(x,y) \text{ for } t=0 \text{ in } \Gamma \tag{2}$$

where Γ denotes the entire flow domain.

The boundary condition of this study area is separated into two parts. The one part, Ω_1 is the borderline between Tairo pond and study area and this was treated as Dirichlet boundary condition with a constant head, that is, the initial head of Tairo pond. The other part, Ω_2 is the borderline between steep rock slope and study area. This part is given as Neumann boundary condition with no flow. Thus, the boundary conditions for Eq. (1) are:

$$h(x,y,t)=h_0(x,y) \text{ for } t>0 \text{ in } \Omega_1 \tag{3a}$$

$$-\left(K_x\frac{\partial h}{\partial x}\bar{n}_x+K_y\frac{\partial h}{\partial y}\bar{n}_y\right)=q_n(t) \text{ for } t>0 \text{ in } \Omega_2 \tag{3b}$$

where $\bar{n}_x$ and $\bar{n}_y$ are unit vectors in the x and y directions and q_n is the flux normal to the boundary. Water flux at any point in the flow region is given by:

$$q_x=-K_x\frac{\partial h}{\partial x} \tag{4a}$$

$$q_y=-K_y\frac{\partial h}{\partial y} \tag{4b}$$

where q_x and q_y are the components of flux in the x and y directions.

This analysis used the linear quardrilateral elements as Figure 3. Its linear shape functions can be defined as:

$$N_i=\frac{1}{4}(1+\varepsilon_i\varepsilon)(1+\eta_i\eta) \tag{5}$$

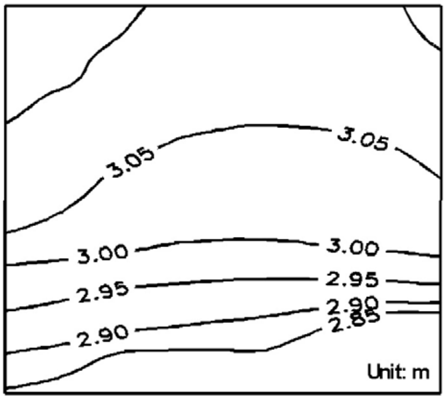

Figure 2. The distribution of equipotential lines in initial condition.

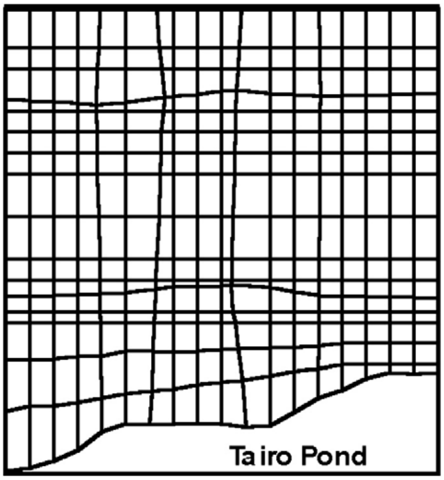

Figure 3. Placements of nodes and linear quardrilateral elements for applying Galerkin finite element method into study area.

$$\frac{\partial N_i}{\partial \varepsilon} = \frac{\varepsilon_i}{4}(1+\eta_i\eta) \qquad \frac{\partial N_i}{\partial \varepsilon}\Big|_{\varepsilon=0,\eta=0} = \frac{\varepsilon_i}{4} \tag{6a}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\eta_i}{4}(1+\varepsilon_i\varepsilon) \qquad \frac{\partial N_i}{\partial \eta}\Big|_{\varepsilon=0,\eta=0} = \frac{\eta_i}{4} \tag{6b}$$

where ε and η are local coordinates and the values of ε_i and η_i are given by Huyakorn and Pinder (1983).

Applying initial and boundary conditions, Eq. (1) was formulated into the local matrices of 4×4 size by the Galerkin finite element method with the linear quadrilateral elements of 4 nodes. Then, the local matrices were assembled into the global matrix as:

$$\begin{gathered}([C] + \omega\Delta t[K])\{h\}_{t+\Delta t} = ([C] - (1 - \omega)\Delta t[K]) \\ \{h\}t + \Delta t((1 - \omega)\{F\}t + \omega\{F\}_{t+\Delta t})\end{gathered} \tag{7}$$

where $[C]$, $[K]$, $\{h\}$, and $\{F\}$ called the global matrices of capacitance, conductance, head, and specific flow, respectively, ω is the relaxation factor, and Δt and $t + \Delta t$ are the time steps of present and next, respectively.

The simulator was coded by using the Microsoft Fortran language.

IV. Methods

The evaluation of pumping capacity was accomplished through the use of the pumping rates obtained by individually and simultaneously withdrawing from six wells within the study area. The withdrawals from wells were carried out by numerical simulation. In individual withdrawals, the pumping rate required for initial head to attain at pumping limited head during a pumping time was individually obtained at each well. This pumping rate will be referred to as '*IMR*' hereinafter. In simultaneous withdrawals, the pumping rates required for initial head to attain at pumping limited head during a pumping time were simultaneously obtained at all wells. They will be referred to as '*SMR*' hereinafter. The multiplications of *IMR* and *SMR* by pumping times, *t*, become the individual and simultaneous duration maximum pumping that will be referred to as '*IMY*' and '*SMY*' hereinafter, respectively. The pumping times adopted in both withdrawals were 6, 12, 24, 48, 72, 144, and 288 hours. Also, the changes of *IMY* and *SMY* with pumping time were summarized as the approximate expressions for *IMY* and *SMY* with pumping time, $IMY(i,t)$ and $SMY(i,t)$, in which i and t indicate well number and pumping time, respectively. Furthermore, the sum of $SMY(i,t)$ at all wells was given as $SSMY(t)$. Therefore, while *IMR* and *SMR* or $IMY(i,t)$ and $SMY(i,t)$ mean the duration maximum pumping rate or yield at each well without and with considering the interrelation between wells, $SSMT(t)$ indicates the duration maximum pumping yield over entire study area.

In withdrawing due to numerical simulations, the heads of nodes were obtained. Using these values, the distributions of equipotential lines on over-all study area were produced out. They gave the important information for describing the behavior of groundwater on individual and simultaneous withdrawals from wells and taking a plan for the optimal management in withdrawing groundwater.

V. Results and Discussion

1. Evaluation of pumping capacity by individual withdrawals

At each well, the *IMP* required for the initial head to attain at the pumping limited head during the pumping times of 6, 12, 24, 48, 72, 144, and 288 hours was obtained by numerical simulation with trial and error. Figure 4 shows the head changes occurred by applying the *IMP*'s corresponding to pumping durations, 6, 12, 24, 48, 72, 144, and 288 hours into well no. 6. Here, the curves of 72, 144, and 288 hours were one another superimposed. The similarity in these head changes can be acquired in the case that their pumping rates approach a constant value under steady state. This fact can be confirmed from very small differences between the values of the last three plots at each well in Figure 5, also.

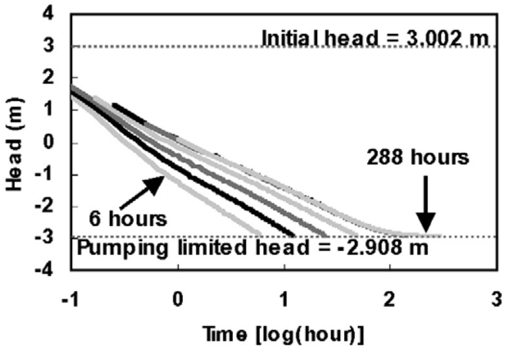

Figure 4. Head changes occurred by individually applying the *IMP*s of pumping times, 6, 12, 24, 48, 72, 144, and 288 hours at well no. 6. The curves of 72, 144, and 288 hours are one another superimposed.

Because the changes of *IMP*'s with pumping times relations in Figure 5 show very severe curvilinear trends, it is impossible to summarize them into a exact expression. This difficulty can be overcome by multiplying *IMP* by pumping time, namely obtaining *IMY*. The relations between *IMY*'s and pumping times at wells were found to be linear in Figure 6. Summarizing them, the approximate expression for *IMY*, *IMY*(*i*,*t*), can be obtained in general form as:

$$IMY(i,t) = a_i t + b_i \tag{8}$$

where a_i and b_i the slope and intercept in the approximate expression for *IMY* at well *i*, respectively. The values of a_i and b_i at each well are given in Table 2 and the coefficients of determination, R^2, in all cases were greater than 0.999. The application of Eq. (8) is allowable between 6 and 288 hours.

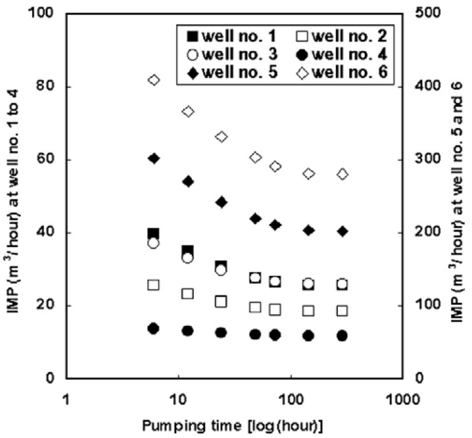

Figure 5. Changes of *IMP*s with pumping times at wells.

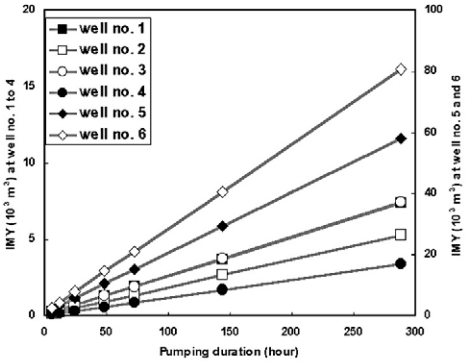

Figure 6. Relations between *IMY*s and pumping times at wells. The lines indicate approximate expressions, *IMY*(*i*,*t*), and the coefficients of determination, R^2, in all cases were more than 0.999.

Table 2. The values of the aw, bw, cw, and dw for the approximate equations obtained by individual and simultaneous withdrawals.

	Individual pumping		Simultaneos pumping	
	a_w	b_w	c_w	d_w
well no. 1	25.236	105.880	11.090	216.220
well no. 2	18.154	56.617	5.748	112.090
well no. 3	25.646	82.397	8.933	175.440
well no. 4	11.693	18.035	1.834	66.297
well no. 5	198.710	858.820	104.870	1519.300
well no. 6	276.590	1067.200	252.360	1194.800

It should be noted that the values of b_i in Table 2 are greater than zero. It might be natural that *IMP* or *IMY* becomes zero at t = 0. The cause is in adopting the Dirichlet boundary condition with the assumption of a constant head at the borderline between Tairo Pond and study area. This assumption is resulted from the fact that the withdrawals from wells have no effect on the head of Tairo Pond, since the area of Tairo Pond is very larger than that of study area. In the case having a remarkable head drop with withdrawal, it might be desirable for obtaining the exact solution to adopt the procedure of Lee (2002) that the loss due to head drop and the inflow through borderline are converged by iteration.

Because $IMY(i,t)$ indicates the maximum pumping capacity of respective wells as the function of pumping time without considering the interrelation between wells, it is useful in determining the details related with the pump installation as pump capacity, electric capacity, magnitude of reservoir or pipe, etc.

2. Evaluation of pumping yield by simultaneous withdrawals

The computations of *SMP* and *SMY*(*i*,*t*) were accomplished by the similar procedures such as the cases of *IMP* and *IMY*(*i*,*t*). Since in the former not only the complicated interrelations between wells but also the heads of the six wells with various hydraulic conductivities, initial heads, pumping limited heads, and distances to two different boundary conditions must be simultaneously attained at pumping limited heads with the smaller error than 10^{-5}, the estimation of *SMP* through numerical simulation requires an enormous computation time.

Figure 7 shows the head changes occurred by simultaneously applying the *SMP*'s of 288 hours into wells. All curves except for that of well no. 6 have S shapes. The main cause is considered that the horizontal spreading of depression cone due to withdrawal at well no. 6 obstructs them at the other wells. It should be noted that similarly to the cases of *IMP* (Figure 5) the *SMP*'s of well no. 6, also, were the largest (Figure 8) even though the hydraulic conductivity of well no. 5 is the largest (Table 1). This means that pumping limited head is an important parameter in determining pumping rate as well as hydraulic conductivity.

The relations between *SMY*'s and pumping times at wells were shown in Figure 9. The lines

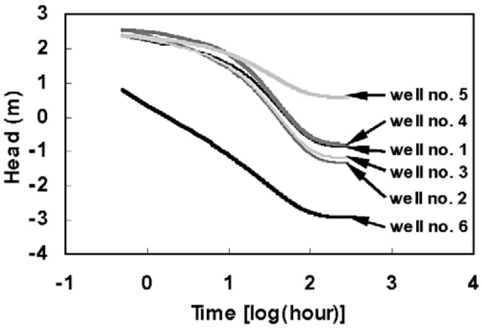

Figure 7. Head changes occurred by simultaneously applying the *SMP*s of 288 hours into all wells.

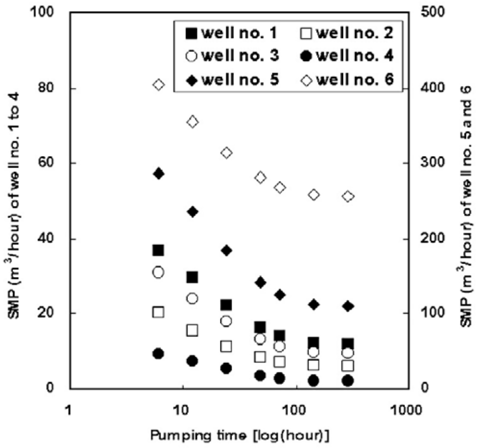

Figure 8. Changes of *SMP*s with pumping times at wells.

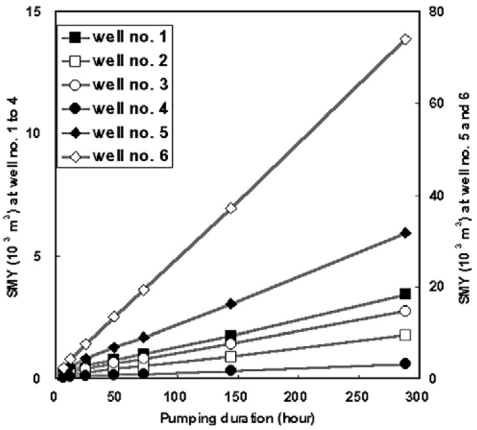

Figure 9. Relations between *SMY*s and pumping times at wells. The lines indicate approximate expressions, *SMY*(*i,t*) and the R^2 in all cases were more than 0.999.

indicate approximate expressions, *SMY*(*i,t*), and the R^2 in all cases were higher than 0.999. The *SMY*(*i,t*) can be expressed in the general form as:

$$SMY(i,t) = c_i t + d_i \tag{9}$$

where c_i and d_i are the slope and intercept in the approximate expression for *SMY* with the pumping duration *t* at well *i*, respectively. The values of ci and di in Eq. (9) were given in Table 2. Therefore, the sum of the *SMY*(*i,t*) in Eq. (9), *SSMY*(*t*), can be obtained by:

$$SSMY(t) = \sum_{i=1}^{6} SMY(i,t) = t\sum_{i=1}^{6} c_i + t\sum_{i=1}^{6} d_i \tag{10}$$

The lower and upper limits of pumping time, *t*, in applying Eq. (10) are 6 and 288 hours.

Because *SSMY*(*t*) means the duration maximum pumping yield over entire study area, the evaluation on pumping capacity of a catchment area with interrelated wells must be discussed by this parameter. This can be used to estimate the supply capacity of a catchment area as a source for water use.

3. Behavior of groundwater due to individual and simultaneous withdrawals

The withdrawal of groundwater at a point within catchment area has some effects on behavior of groundwater. The effects can be found by the changes of the distribution of equipotential lines occurred due to withdrawal. Figure 10 shows the distributions of equipotential lines obtained by individually applying the *IMY*'s of 288 hours into wells. In all cases, it clearly appears that the hydraulic gradients were formed with very steep slopes from Tairo Pond toward the inner part of study area as well as the respective pumping wells. This means that groundwater moves from Tairo Pond toward the inner part of study area at high rate. If the part adjacent to Tairo Pond were highly concentrated, the rapid spread of the contaminants of the part adjacent to Tairo Pond into the inner part of study area by individual withdrawals could not escape.

Figure 11 shows the distributions of equipoten-

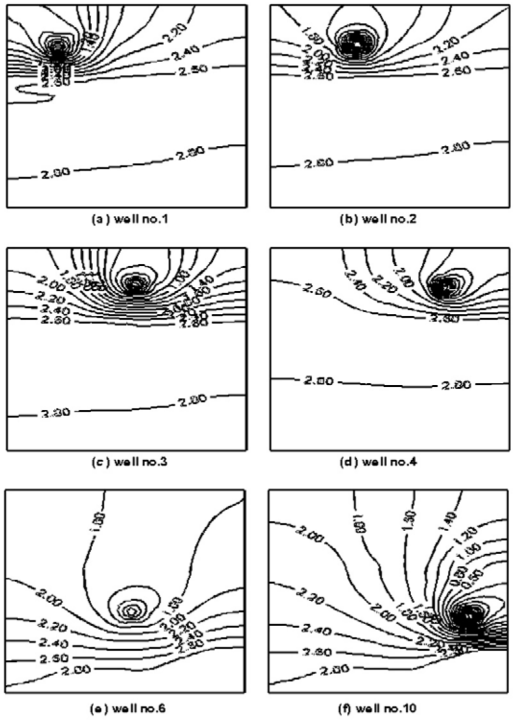
(a) well no.1 (b) well no.2 (c) well no.3 (d) well no.4 (e) well no.6 (f) well no.10

Figure 10. Distributions of equipotential lines obtained by individually applying the IMYs of 288 hours into wells.

tial lines obtained by simultaneously applying the *SMY*'s of 12, 24, 48, 72, 144, and 288 hours into all wells. The shadow parts in (a) to (f) indicate the ridge parts of equipotential lines between the alignment of well no. 1 to 4 and that of well no. 5 and 6. Since the heads in these ridge parts have higher than them in the alignments of well no. 1 to 4 and of well no. 5 and 6, they should have the role for preventing the spread of the groundwater of the part adjacent to Tairo Pond into the inner part of study area by simultaneous withdrawals.

Therefore, it is desirable to adopt the simultaneous withdrawals as the optimal management for supplying water use with respect to its quantity and quality. The more detailed considerations for controlling the quality of groundwater by simultaneous withdrawals will be discussed in the subsequent report.

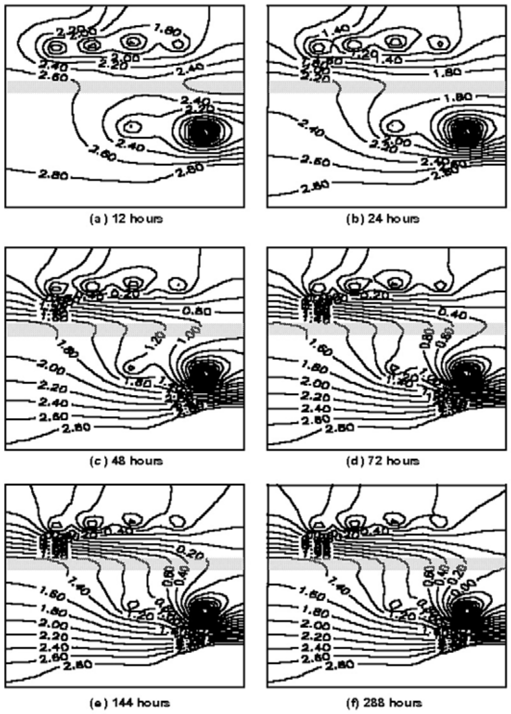
(a) 12 hours (b) 24 hours (c) 48 hours (d) 72 hours (e) 144 hours (f) 288 hours

Figure 11. Distributions of equipotential lines obtained by simultaneously applying the SMYs of 12, 24, 48, 72, 144, and 288 hours into all wells. The shadow parts in (a) to (f) indicate the ridge part of equipotential lines between the alignment of well no. 1 to 4 and that of well no. 5 and 6.

VI. Conclusion

We evaluated the groundwater pumping capacity at a catchment area containing interrelated wells without consideration of water quality in Miyake Island, a young volcanic island with very complicated hydro-geological formations. All parameters obtained by Galerkin horizontal 2-dimensional transient simulator. The withdrawals from wells were individually and simultaneously accomplished.

The groundwater pumping capacity of each well was estimated as the *IMY*(*i,t*) by individual withdrawal. The duration maximum pumping rate, *IMP*, of each well can be easily obtained by dividing *IMY*(*i,t*) by the pumping time. They are useful in determining the details related with the pump installation as pump capacity, electric capacity, magnitude of reservoir or pipe, etc.

The groundwater pumping capacity over the entire study area was estimated as the *SSMY*(*t*) by simultaneous withdrawals. The pumping rate of each well correspondent to *SSMY*(*t*) can be easily obtained by dividing *SMY*(*i,t*) by the pumping time, where the sum of *SMY*(*i,t*) becomes *SSMY*(*t*). Using *SSMY*(t), the exact estimation of the supply capacity of a catchment area as a source for water use should be possible.

The groundwater behavior resulted from individual and simultaneous withdrawals was given by the distributions of equipotential lines. While individual withdrawal brought about the rapid movement of the groundwater of the part adjacent to Tairo Pond into the inner part of study area, in simultaneous withdrawals the movement of the groundwater of the part adjacent to Tairo Pond was interrupted by the occurrence of the ridge parts of equipotential lines between the alignment of well no. 1 to 4 and that of well no. 5 and 6. In simultaneous withdrawals, the withdrawals from well no. 5 and 6 should have the roles as the barriers for the intrusion of the groundwater of the part adjacent to Tairo Pond into the inner part of study area. Therefore, it can be suggested to adopt the simultaneous withdrawals as an approach of the optimal management of groundwater for supplying water use with respect to quantity and quality.

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최종원고채택 03. 07. 08