Optimized Algebra LDPC Codes for Bandwidth Efficient Modulation

GiYean Hwang · Yu Yi · MoonHo Lee

Abstract

In this paper, we implement an efficient MLC/PDL system for AWGN channels. In terms of the tradeoff between the hardware implementation and system performance, proposed algebra LDPC codes are optimized by the Gaussian approximation(GA) according to the rate of each level assigned by the capacity rule and chosen as the component code. System performance with Ungerboeck Partitioning(UP), Mixed Partitioning(MP) and Gray Mapping(GM) of 8PSK are evaluated, respectively. Many results are presented in this paper; they can indicate that the proposed MLC/PDL system using optimized algebra LDPC codes with different code rate, capacity rule and Gray mapping(GM) can achieve the best performance.

Key words : LDPC Code, Multilevel Coding, Parallel Decoding on Levels, Gaussian, Ungerboeck.

Ⅰ. Introduction

Multilevel coding(MLC) proposed by Imai in 1977[1] is a coded modulation technique used for the bandwidth efficient transmission. The idea of *MLC* is to protect the individual bits at each level of signal points by separating binary codes[2]. Usually *Multistage Decoding* (MSD) is employed at the receiver to decode the signals starting from the lowest level and take into account decisions of prior decoding stages. However, MSD is not suitable for the real time communications because of the large time delay and error propagation from low level to high level. Parallel Decoding on Levels(PDL) is an efficient decoding strategy for MLC transmissions. In contrast to the MSD approach, all decoders in PDL are working in parallel. This can avoid error propagation and low decoding delay. Based on these advantages, PDL is adopted in our MLC system.

As described in [2], performance of MLC/PDL system mainly depends on three points such as component codes design, rate rules for each level and labeling strategies. Many error-correction codes have been applied to the MLC system such as BCH codes, GAC and Turbo codes[3]. Recently, Low-density parity-check(LDPC) code is a newly developed as a channel coding technique. LDPC codes with low decoding complexity were demonstrated to outperform the Turbo codes and achieve more reliable transmission for a SNR extremely close to the Shannon limit[4]. However, the main disadvantage of LDPC codes is high encoding complexity. In terms of the hardware implementation and system performance, Novel algebra LDPC codes are proposed in this paper because of their simply parity-check matrix and the better performance[5]. According to the rate of individual level, proposed algebra LDPC codes are optimized by the Gaussian approximation[6] to get the best degree distribution and chosen as the component codes. For the rate design rules, capacity rule is used to assign the rate to the each level because it has the similar performance with other rate rules and is a good choice for practical codes[2]. Several mapping rules of 8PSK are investigated respectively in this paper such as Ungerboeck Partitioning(UP), Mixed Partitioning(MP) and Gray Mapping(GM).

In this paper, we implement an efficient MLC/PDL system for AWGN channels. In terms of the tradeoff between hardware implementation and system performance, proposed algebra LDPC code is optimized by Gaussian approximation(GA) according to the rate of each level assigned by the capacity rule and chosen as the component code. System performance with UP, MP and GM of 8PSK are evaluated, respectively. Many results are presented in this paper; they indicate that the proposed MLC/PDL system based on the optimized algebra LDPC codes, capacity rule and GM can obtain the very good performance.

This paper is organized as follows; the system model

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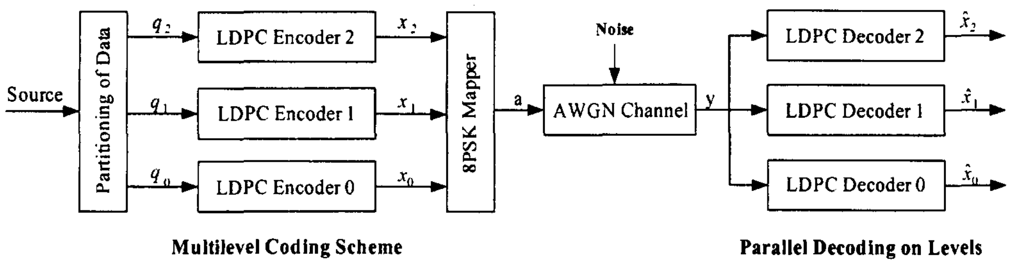

Fig. 1. Block diagram of the MLC/PDL system model.

is introduced in section Ⅱ including multilevel coding, capacity rule, parallel decoding on levels and labeling strategies. We proposed the algebraic LDPC codes and optimized them for our MLC/PDL system in section Ⅲ. Section Ⅳ shows the simulation result and draws a conclusion.

Ⅱ. System Model

The block diagram of the proposed system model used in this paper is shown in Fig. 1.

Fig. 1 shows the source information is separated into the different LDPC levels according to the each level code rate. Especially, 8PSK is considered in the proposed MLC/PDL system. Each bit from the different level is mapped by 8PSK mapping rule and transmitted over the AWGN channels. In the receiver, LDPC decoder can detect and decode the received symbol. In the following, we introduce the system model in details including the multilevel coding, capacity rule, parallel decoding on levels and labeling strategies.

2-1 MLC(Multilevel Coding)

For the multilevel coding scheme, a block of K binary source data $q=(q_1,\dots,q_K), q_l \in \{0,1\}$, is partitioned into l blocks.

$$q^i=(q_1^i,\dots,q_{K_i}^i),\ i=0,\dots,l-1 \qquad (1)$$

of length K_i with $\sum_{i=0}^{l-1} K_i = K$. Each data block q^i is fed into an individual binary encoder generating words $x^i=(x_1^i,\dots,x_N^i), x_r^i \in \{0,1\}$, of the component code C_i. The codeword symbols x_r^i, $r=1,\dots,N$, of the codewords $x^i, i=0,\dots l-1$, at one time instant r, form the binary label $x_r=(x_r^0,\dots,x_r^{l-1})$, which is mapped to the signal point a_r. The code rate R of the scheme is equal to the sum of the individual code rates $R^i = K_i/N$, namely,

$$R=\sum_{i=0}^{l-1} R_i = \frac{1}{N}\sum_{i=0}^{l-1} K_i = \frac{K}{N}. \qquad (2)$$

2-2 Capacity Rule

Since the mapping in MLC is bijective and hence loss less in sense of the information theory, mutual information $I(Y;A)$ between the transmitted signal point $a \in A$ and received signal $y \in Y$ equals $I(Y;X^0,X^1,\dots,X^{l-1})$ between address vector $X \in \{0,1\}^l$ and received signal point:

$$I(Y;A)=I(Y;X^0,X^1,\dots,X^{l-1}) \qquad (3)$$

Applying the chain rule[2] to get the expression,

$$\begin{aligned} I(Y;A) &= I(Y;X^0,X^1,\dots,X^{l-1}) \\ &= I(Y;X^0)+I(Y;X^1 \mid X^0)+\cdots \\ &\quad +I(Y;X^{l-1} \mid X^0,X^1,\dots,X^{l-2}). \end{aligned} \qquad (4)$$

From the chain rule the mutual information

$$I(Y;X^i \mid X^0,\dots,X^{i-1}) \qquad (5)$$

of the equivalent channel i can be easily calculated by

$$\begin{aligned} I(Y;X^i \mid X^0,\dots,X^{i-1}) &= I(Y;X^i \dots X^{l-1} \mid X^0 \dots X^{i-1}) \\ &\quad - I(Y;X^{i+1} \dots X^{l-1} \mid X^0 \dots X^i) \end{aligned} \qquad (6)$$

The capacity C of the equivalent channel i is given by the respective mutual information $I(Y;X^i \mid X^0,\dots,X^{i-1})$ for these specific channel input probabilities.

$$C^i = I(Y;X^i \mid X^0,\dots,X^{i-1}) \quad i=0,1,\dots,l-1 \qquad (7)$$

For a 2^l-ary digital modulation scheme the rate R^i at the individual coding level i of a multilevel coding scheme should be chosen equal to the capacity C^i of the equivalent channel i, $i=0,\dots,l-1$,

$$R^i = C^i \qquad (8)$$

(1) Ungerboeck partitioning of 8PSK

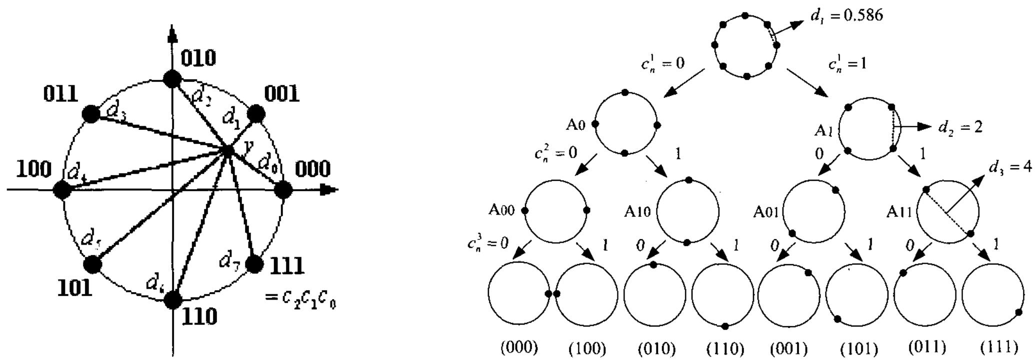

Fig. 2. Ungerboeck partitioning of 8PSK.

(2) Mixed partitioning of 8PSK

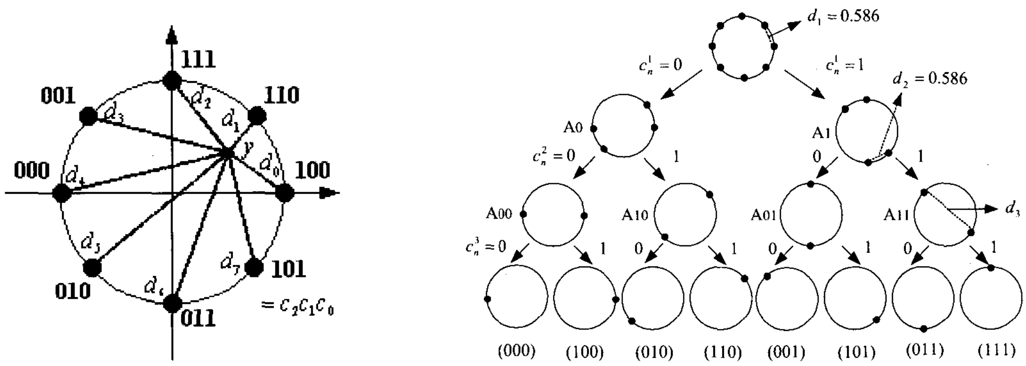

Fig. 3. Mixed partitioning of 8PSK.

(3) Gray mapping of 8PSK

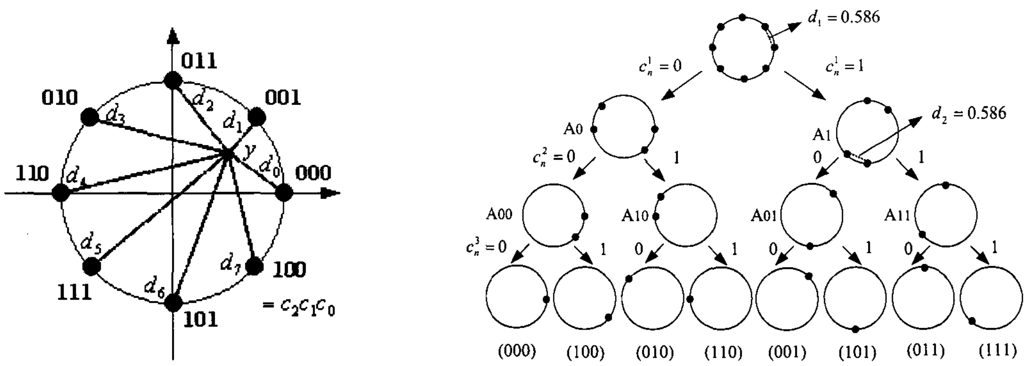

Fig. 4. Gray mapping of 8PSK.

2-3 Labeling Strategies

Several labeling strategies are analyzed respectively such as UP, MP and GM in this section.

According to capacity rule, we show these rate distributions used in this paper in Table 1[2].

Ⅲ. Proposed Algebra LDPC Code Design

In this section, we propose an efficient approach to design the algebra LDPC codes[7] for the MLC/PDL system as component codes in the following,

□ Let $B_{jk}(i)$ be an I identity matrix with t×t located

Table 1. Rate distributions for different partition methods.

Partition	Individual Code Rates	MLC Rate
UP	$R^0/R^1/R^2=0.200/0.810/0.990$	2 bits/s/Hz
MP	$R^0/R^1/R^2=0.505/0.505/0.990$	2 bits/s/Hz
GM	$R^0/R^1/R^2=0.510/0.745/0.745$	2 bits/s/Hz

at the *j*th block row and *k*th block column of parity check matrix having its rows shifted to right *i* mod *t* positions for

$$i\in S=\{0,1,\ 2,\dots,\ t-1\}; \tag{9}$$

□ Exist a q such that $q^k\equiv 1(\bmod t)$, S can be divided into several sets C and one set containing integer s is the set

$$\{s,sq,sq^2,\dots,sq^{m_s-1}\}; \tag{10}$$

where *ms* is the smallest positive integer satisfying

$$sq^{m_s}\equiv s(\bmod t) \tag{11}$$

□ The location of 1's in H^d can be determined using the set $C_1, \cdots, C_j$ and the parameter t.

We can propose the recursive algorithm to construct B_{jk}(i) with different parameters i, j and k,

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Bjk(i) (Recursive algorithm):
    i=p (shift number)
        For j1=1 to t (t is block size of Bjk(i))
            For j2=1 to t
                B[j1][j2]=0 (initialization)
                For j1=1 to t
                    j2= (j1+p) mod t
                    B[j1][j2]=1 (shifted procession)
                End
            End
    End
```

We can show an example with parameters j=3, k=5 and t=31 in Fig. 5, where N = $k\cdot t$ and M = $j\cdot t$.

$$Coderate=\frac{K}{N}=\frac{N-M}{N}=\frac{155-93}{155}=0.4 \tag{12}$$

According to the second step of the construction, we can show C_i, $i\in[0,13]$ distributions in Table 2. The location of 1's in H matrix can be decided by C_1, C_2, $C_{j=3}$.

According to the construction method and Table 1, we can design parameters of LDPC codes for each level and show them in Table 3,

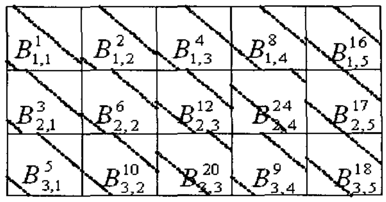

Fig. 5. An example for algebra LDPC Code with j=3, k=5 and t=31.

Table 2. Permutation number distributions.

s/m_s	$sq^{m_s}\equiv s(\bmod t)$	$C=\{s,sq,sq^2,\dots,sq^{m_s-1}\}$
$s=0/m_s=1$	$0\cdot 2^{m_s}=0(\bmod 31)$	{0}
$s=1/m_s=5$	$1\cdot 2^{m_s}=1(\bmod 31)$	{1,2,4,8,16}
$s=3/m_s=5$	$3\cdot 2^{m_s}=3(\bmod 31)$	{3,6,12,24,17}
$s=5/m_s=5$	$5\cdot 2^{m_s}=5(\bmod 31)$	{5,10,20,9,18}
$s=6/m_s=5$	$6\cdot 2^{m_s}=6(\bmod 31)$	{6,12,24,17,3}
$s=7/m_s=5$	$7\cdot 2^{m_s}=7(\bmod 31)$	{7,14,28,25,19}
$s=9/m_s=5$	$9\cdot 2^{m_s}=9(\bmod 31)$	{9,18,5,10,20}
$s=10/m_s=5$	$10\cdot 2^{m_s}=10(\bmod 31)$	{10,20,9,18,5}
$s=11/m_s=5$	$11\cdot 2^{m_s}=11(\bmod 31)$	{11,22,13,26,21}
$s=12/m_s=5$	$12\cdot 2^{m_s}=12(\bmod 31)$	{12,24,17,3,6}
$s=13/m_s=5$	$13\cdot 2^{m_s}=13(\bmod 31)$	{13,26,21,11,22}
$s=14/m_s=5$	$14\cdot 2^{m_s}=14(\bmod 31)$	{14,28,25,19,7}
$s=15/m_s=5$	$15\cdot 2^{m_s}=15(\bmod 31)$	{15,30,29,27,23}

Table 3. Code parameters for different partition methods.

Partition	LDPC Codes (N, M, p) for $R^0/R^1/R^2$
UP	(305,244,61)/(1220,244,61)/(6100,61,61)
MP	(366,183,61)/(366,183,61)/(6100,61,61)
GM	(366,183,61)/(732,183,61)/(732,183,61)

We can optimize the proposed LDPC code according to GA and choose degree distribution. For example, computation results for Mixed Partitioning are shown in the Table 4.

According to Table 4, p=61 is selected to design LDPC code for the MP again. We can get optimized LDPC code parameters: (366,183,61)/(366,183,61)/

Table 4. Degree distribution for different level.

Level 0			Level 1	Level 2		
j, k	Rate	GA	GA	j, k	Rate	GA
3, 6	**0.50**	**3.7**	**3.7**	**3,300**	**0.99**	**3.8**
4, 8	0.50	4.1	4.1			
5, 10	0.50	4.5	4.5			

(18300,183,61) for each level. Also we can use this rule to optimize LDPC codes for other mapping rules. Sum product algorithm is used for proposed codes to decode received bits.

Fig. 6 compares the performance of proposed LDPC code with random LDPC codes over the AWGN channels. Obviously, we can see the algebra LDPC codes are better than random LDPC codes. For example, when Bit Error Rate(BER) is 1e-5, we can get the 0.25 dB gain from the random codes for different code rate such as 0.510 or 0.745.

Ⅳ. Simulation Results

We can compare the performance of the MLC/PDL system based on optimized LDPC codes with different mapping rules in Fig. 7. For example, the bandwidth efficient is 2 bits/s/Hz. From the Fig. 7, we can draw a conclusion that the proposed MLC/PDL system based on optimized LDPC code, capacity rule and Gray mapping has the best performance. When BER is 10^{-4}, we get **4.2 dB** gain from uncoded QPSK modulation[3].

Ⅴ. Conclusions

In this paper, we implement an efficient MLC/PDL system for AWGN channels. In terms of the tradeoff

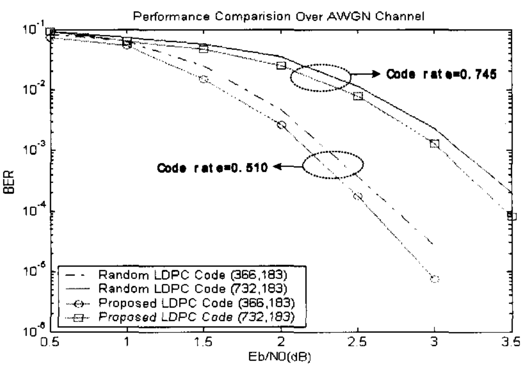

Fig. 6. Performance over AWGN channel.

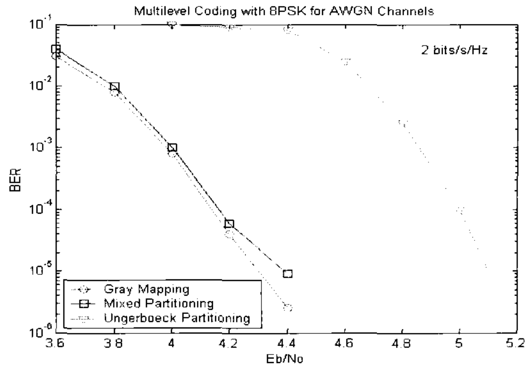

Fig. 7. Performance of MLC/PDL over AWGN channel.

between hardware implementation and system performance, many results are presented in this paper; they indicate that the proposed MLC/PDL system using the optimized algebra LDPC code, capacity rule and Gray mapping(GM) can achieve the best performance.

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