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A Branch-and-price Approach to the ATM Switching Node Location Problem

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Abstract. We consider the ATM switching node location problem (ANLP). In this problem, there are two kinds of facilities, hub facilities and remote facilities, with different capacities and installation costs. We are given a set of customers with each demand requirements, a set of candidate installation sites of facilities, and connection costs between facilities. We need to determine the locations to place facilities, the number of facilities for each selected location, the set of customers who are connected to each installed hub via installed remote facilities with minimum cost, while satisfying demand requirements of each customer. We formulate this problem as a general integer programming problem and solve it to optimality. In this paper, we present a preprocessing procedure to tighten the formulation and develop a branch-and-price algorithm. In the algorithm, we consider the integer knapsack problem as the column generation problem. Computational experiments show that the algorithm gives optimal solutions in a reasonable time.

Keywords: Facility Location, Integer Programming, Column Generation, Branch-and-Price

1. INTRODUCTION

In the past few years, many experiments to implement broadband integrated services digital network (B-ISDN) have been conducted. To support these B-ISDN service requirements, ATM (Asynchronous Transfer Mode) has been proposed as the target technology. ATM is a packet-oriented transfer mode in which information is organized into a fixed-size entity known as a cell. ATM technology combines the flexibility of traditional packet-switching technology with the determinism of TDM (time division multiplexing) (Wu 1990).

To carry these flexible B-ISDN services, we should install ATM switching nodes. In this paper, we consider the ATM-MSS (ATM-MAN Switching System) Node Location Problem (ANLP) for the PVC (Permanent Virtual Connection) based leased line network. In this network, services are provided at a constant bit rate (CBR) or variable bit rate (VBR). In this paper, the QoS and/or statistical multiplexing are considered through the equivalent cell rate (ECR).

First, we consider the switching systems called the ATM-MSS switching nodes. We are given two kinds of

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facilities: Hub Switching Node (HSN) and Remote Switching Node (RSN). Each HSN accommodates several RSNs in the star topology. Each RSN accommodates user demands with various interfaces such as DS1E (2.048 Mbps), DS3 (44.736 Mbps) and STM-1 (155.520 Mbps), with the capacity of 284 DS1E. According to these functions, we may call the HSN the hub facility and the RSN the remote facility. For each candidate site of facilities, we may install more than one facility.

Then, the ANLP is defined as follows. We are given hub candidate sites (H), remote candidate sites(R) and users (U). Each user u should be connected to the remote facilities installed at one remote candidate site to satisfy demand requirement (r_u). The remote facilities connected to a user should be connected to the hub facilities installed at one hub candidate site. Each hub facility has a finite capacity (b_H) and a fixed cost (F_H) and each remote facility has a finite capacity (b_R) and a fixed cost (F_R). Connection cost d_{ur} arises when the unit demand (DS1E) of a user u is satisfied by the flow from the remote facility installed at a remote candidate site r. The connection cost d'_{rh} arises when a remote facility installed at a remote candidate site r is connected to the hub facility installed at a hub candidate site h.

Then, we determine the number of hub facilities and the number of remote facilities for each candidate site and the allocation of users to hub candidate sites via remote candidate sites with minimum total cost, the sum of facility installation cost and connection cost.

Facility location problems have received a great deal of attention for recent several decades. For the general capacitated plant location problem, Sa (1969), Davis and Ray (1969), Ellenwein and Gray (1971), Akinc and Khumawala (1977) have studied. But in most formulation of facility location problems, a single stage distribution system has been considered.

For the two stages distribution system where commodities are delivered from plants to customers via warehouses, Geoffrion and Graves (1974) considered a multicommodity two stage problem with the restriction that each customer should be served by only one warehouse. Tcha and Choi (1980) also studied of the single commodity two stages problem without aforementioned restriction. But since customers and plants are given, they determined warehouse locations only.

Kaufman *et al*. (1977) have studied of the problem of location simultaneously both plants and warehouses with no capacity restrictions. For the problem with single assignment restriction, Neebe and Rao (1983), Dee and Lieman (1972), Barcelo and Casanovas (1984), and Tang *et al*. (1978) are widely known.

Recently, the cutting plane method using polyhedral structure has become a standard technique. Survey on this method can be found in Nemhouser and Wolsey (1988). Crowder *et al*. (1982) and Johnson *et al*. (1985) reported the success of solving large scale 0-1 programming problems arising from planning models using this method. Especially Aardal *et al*. (1995) and Leung *et al*. (1989) have derived some valid inequalities for the capacitated facility location problem. Delayed column generation approach incorporated with the branch-and-bound procedure has also become a new technique for solving the combinatorial optimization problem. For example, Savelsbergh (1997) has reported the success of solving the generalized assignment problem.

In this paper, we use the delayed column generation and branch-and-price approach. We formulate this problem using tree variable. To solve the linear programming relaxation of the formulation, which has exponentially many variables, we solve the pricing subproblem, which is NP-hard. But we can solve the subproblem using the pseudo polynomial-time algorithm. Moreover, to tighten the bound of LP relaxation, a preprocessing procedure is devised by deriving some valid inequalities.

The remainder of the paper is organized as follows. In section 2, we state the notations for the problem description and formulate the problem using the concept of pattern generation. In section 3, the column generation subproblem is considered. In section 4, we present a preprocessing procedure to tighten the bound of LP relaxation of problem and present the augmented linear programming. In section 5, we present the branch-and-price algorithm and implementation details. In section 6, we show the computational results of our algorithm. Finally, we give concluding remarks.

2. FORMULATION OF THE PROBLEM

In this section, we present the formulation of ANLP. Since each user should be connected to a hub candidate site via a remote candidate site, we can allocate some users to be connected to a hub candidate site via a remote candidate site. For the fixed hub candidate site h and the fixed remote candidate site r, there are several kinds of allocation patterns. We call this allocation pattern as a tree. Then ANLP finds the set of trees and an assignment of facilities for each selected hub candidate site and the selected remote candidate site to minimize the cost while satisfying the demand requirements of the users.

First, we give some notation to be used in the formulation of the problem.

$NR(h)$: set of remote candidate sites that can be connected to hub candidate site h.

$NU(rh)$: set of users that can be connected to hub candidate site h via remote candidate site r.

$T(rh)$: set of feasible trees rooted at hub candidate site h

via remote candidate site r.

U_t^{rh} : set of users of a tree t, $t \in T(rh)$, $r \in R$, $h \in H$.

m_t^{rh} : the number of remote facilities of tree t, which are located on the remote candidate site r, and connected by hub candidate site h.

Using the above notation, ANLP can be formulated as follows.

(MP)

$$\min \sum_{h \in H} F_H u_h + \sum_{h \in H} \sum_{r \in NR(h)} \sum_{t \in T(rh)} c_t^{rh} x_t^{rh}$$

$$\text{s.t.} \sum_{h \in H} \sum_{r \in NR(h)} \sum_{\{t \in T(rh) | u \in U_t^{rh}\}} x_t^{rh} = 1 \quad \forall\, u \in U, \tag{1}$$

$$\sum_{h \in H} \sum_{r \in NR(h)} \sum_{t \in T(rh)} m_t^{rh} x_t^{rh} \le b_H u_h \ \forall\, h \in H, \tag{2}$$

$$\sum_{t \in T(rh)} x_t^{rh} \le 1 \qquad \forall\, r \in NR(h),\ h \in H \tag{3}$$

$$x_t^{rh} \in \{0,1\},\ u_h \in Z_+ \ \forall\, t \in T(rh),\ r \in NR(h),\ h \in H$$

where $c_t^{rh} = (F_{rh} + d'_{rh}) m_t^{rh} + \sum_{u \in U_t^{rh}} r_u d_{ur}$.

Note that c_t^{rh} is the cost of tree t for hub candidate site h and remote candidate site r. The decision variable x_t^{rh} =1 if a tree t for a hub candidate site h and a remote candidate site r is selected, otherwise it is 0. The decision variable u_h represents the number of hub facilities to be located on the hub candidate site h.

Constraints (1) imply that each user should be assigned to precisely one tree. Constraints (2) imply that if the hub candidate site h is selected as a hub site, then hub facilities should be located on the selected hub candidate site to accommodate the remote facilities. Constraints (3) imply that at most one feasible tree can be selected for the pair of the hub candidate site h and the remote candidate site r.

ANLP has exponentially many variables. But we can solve its LP relaxation by the (delayed) column generation method.

3. COLUMN GENERATION PROBLEM

In this section, we give an explanation of column generation problem and the algorithm to solve the problem.

Let (MPLP) be the linear programming relaxation of (MP). We need a feasible basis for (MPLP) to use the column generation method. If it is difficult to find an initial feasible solution, we can introduce artificial variables with big cost coefficient. We will mention how to find an initial feasible solution to (MPLP) in section 5.

Given a feasible basis to (MPLP), we need to generate columns to enter the basis. The column generation procedure to solve the (MPLP) is very similar to the one used for the generalized assignment problem (Savelsbergh 1997). Let α_u be the dual variable associated to the constraint in (1) for each user u. Let β_h be the dual variable associated to the constraint in (2) for each hub candidate site h. Let γ_{rh} be the dual variable associated to constraints in (3) for each remote candidate site r and hub candidate site h. Also let ($\bar{\alpha}, \bar{\beta}, \bar{\gamma}$) be the values of dual variables for a given solution to (MPLP). Then the solution is optimal to the restricted (MPLP) with current columns if

$$\max\{-c_t^{rh} + \bar{\alpha}_u + m_t^{rh} \bar{\beta}_h\} \le -\bar{\gamma}_{rh} .$$

Therefore, we may write the optimality condition for MPLP as follows.

$$\max\{(\bar{\beta}_h - F_{rh} - d'_{rh}) m_t^{rh} + \sum_{u \in U_t^{rh}} (\bar{\alpha}_u - r_u d_{ur})\} \le -\bar{\gamma}_{rh} .$$

Then the column generation problem associated to hub candidate site h and remote candidate site r can be formulated as follows.

(TGP(rh))

$$\max \quad (\bar{\beta}_h - F_{rh} - d'_{rh}) y_{rh} + \sum_{u \in NU(rh)} (\bar{\alpha}_u - r_u d_{ur}) w_u$$

$$\text{s.t.} \quad \sum_{u \in NU(rh)} r_u w_u \le b_R y_{rh}$$

$$w_u \in \{0,1\},\ y_{rh} \in Z_+ \qquad \forall\, u \in NU(rh),$$

The variable $w_u = 1$ if user u is connected to the hub candidate site h via the remote candidate site r, otherwise it is 0. The decision variable y_{rh} represents the number of remote facilities to be located on the remote candidate site r and to be connected to the hub candidate site h

Note that the (TGP(rh)) is a bounded variable knapsack problem. It can be seen easily by setting y_{rh} = $Upper\ bound - y'_{rh}$ and $y'_{rh} \le Upper\ bound$, $y'_{rh} \in Z_+$. If *Upper bound* is not given, we can set $Upper\ bound = \lceil \sum_{u \in U} r_u / b_R \rceil$. Therefore, there exists a pseudo polynomial-time algorithm for this problem. To solve the TGP((rh)), we may use the dynamic programming. For more results about this problem, refer to Nemhauser and Wolsey (1988). If the resulting value of the TGP is greater than $-\bar{\gamma}_{rh}$, then the generated column can be added to the current formulation. Otherwise, no column is generated with respect to the pair of remote candidate site r and hub candidate site h.

4. PREPROCESSING AND THE AUGMENTED LP

4.1 Preprocessing

In this section, we propose two valid inequalities as an efficient preprocessing procedure that is used to tighten the initial formulation. The procedure is based on the concept of the minimum demand requirements. Using the procedure, we can determine the minimum number remote facilities and the minimum number of hub facilities needed to satisfy the demands of users.

Proposition 1. *Following inequality is valid for ANLP.*

$$\sum_{h \in H} \sum_{r \in NR(h)} \sum_{t \in T(rh)} m_t^{rh} x_t^{rh} \geq \left\lceil \sum_{u \in U} r_u \Big/ b_R \right\rceil$$

Proof. Total demands of users, $\sum_{u \in U} r_u$ should be satisfied with remote facilities. Since the capacity of remote facility is b_R, to satisfy user demands, we should install at least $\lceil \sum_{u \in U} r_u / b_R \rceil$ number of remote facilities. Hence the above inequality should be satisfied. □

Proposition 2. Following inequality is valid for ANLP.

$$\sum_{h \in H} u_h \geq \left\lceil \left\lceil \sum_{u \in U} r_u \Big/ b_R \right\rceil \Big/ b_H \right\rceil$$

Proof. We know that each user should be connected to a hub candidate site via a remote candidate site. Proposition 1 also says that at least $\lceil \sum_{u \in U} r_u / b_R \rceil$ number of remote facilities should be installed. Since these facilities should be connected to hub facility and the capacity of hub facility is b_H, we should install at least $\lceil \lceil \sum_{u \in U} r_u / b_R \rceil / b_H \rceil$ number of hub facilities. So the above inequality should be satisfied. □

4.2 The Augmented LP

Now, consider the linear programming relaxation of the problem obtained by adding the valid inequalities to (MPLP). The augmented LP relaxation is as follows :

(AMP)

$$\min \quad \sum_{h \in H} F_H u_h + \sum_{h \in H} \sum_{r \in NR(h)} \sum_{t \in T(rh)} c_t^{rh} x_t^{rh}$$

s.t. (1), (2), (3)

$$\sum_{h \in H} \sum_{r \in NR(h)} \sum_{t \in T(rh)} m_t^{rh} x_t^{rh} \geq \left\lceil \sum_{u \in U} r_u \Big/ b_R \right\rceil \tag{4}$$

$$\sum_{h \in H} u_h \geq \left\lceil \left\lceil \sum_{u \in U} r_u \Big/ b_R \right\rceil \Big/ b_H \right\rceil \tag{5}$$

$$x_t^{rh} \geq 0 \quad \text{for all } t \in T(rh),\ r \in NR(h),\ h \in H$$

$$u_h \geq 0 \quad \text{for all } h \in H$$

Let μ be the dual variable associated with the constraint in (4). If we fix a hub candidate site h and remote candidate site r, then the column generation problem for (AMP) can be formulated as following column generation problem :

$$\max \quad (\overline{\beta}_h + \mu - F_{rh} - d'_{rh}) y_{rh} + \sum_{u \in NU(rh)} (\overline{\alpha}_u - r_u d_{ur}) w_u$$

$$\text{s.t.} \quad \sum_{u \in NU(rh)} r_u w_u \leq b_R y_{rh}$$

$$w_u \in \{0,1\},\ y_{rh} \in Z_+ \qquad \forall\, u \in NU(rh),$$

If the resulting value of the (TGP(rh)) is greater than $-\overline{\gamma}_{rh}$, then the tree can be added to the current formulation. Otherwise, no column is generated with respect to the pair of remote candidate site r and hub candidate site h.

5. BRANCH-AND-PRICE ALGORITM

5.1 Overview of the Algorithm

In this section, we give a brief and overall explanation of our algorithm. First, we construct the initial formulation of (AMP) using artificial variables and initial feasible solutions. After preprocessing and solving the initial (AMP), we decide if the present solution is dual feasible. If it is not, new columns are generated and added to (AMP). This process continues until no such columns are found. If the present solution is dual feasible, we have solved the problem at node 0 in the enumeration tree. Then we check if the solution obtained by solving the last LP is integral. If we have obtained an integral solution, we are done with an optimal solution of ANLP. Otherwise, we have to initiate the branch-and-bound procedure to find an optimal integer solution. Column generation is also allowed throughout the branch-and-bound tree.

For solving the column generation problem, we use the dynamic programming. The overall procedure of the algorithm is presented in Figure 1.

5.2 Initial feasible solution

To start the column generation procedure, we need to have an initial feasible solution to (AMP). We can obtain

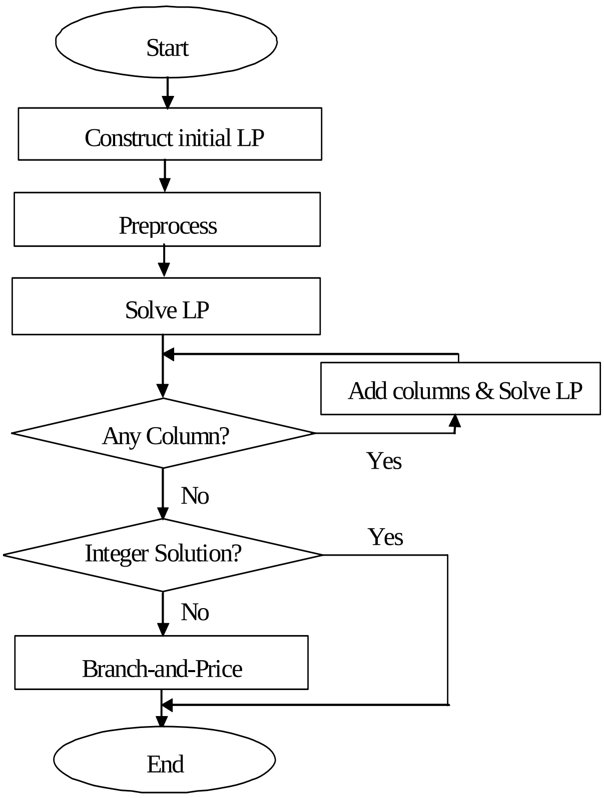

Figure 1. Overall procedure of the branch-and-price algorithm

an initial feasible solution by finding some feasible trees and use the trees as the initial columns in the formulation. Note that a feasible tree can also serve as an incumbent solution in the branch-and-price procedure. To find a feasible solution, we use a simple heuristic procedure.

We also introduce artificial variables with big cost coefficient into the formulation that guarantees the feasibility of (AMP). It may redundant to use artificial variables when the feasible solution exists. However, artificial variables are always needed when branching is done. Followings are the overall procedure for finding a feasible solution.

<u>*Greedy Heuristic Algorithm*</u>

Step 0 (*Start*)

Set $U(rh) = \emptyset$, for all $r \in NR(h)$, $h \in H$.

Step 1 (*Finding Minimum Cost Path*)

For each $u \in U$, find (r^*, h^*) where $(r^*, h^*) = \arg\min_{\{r\in R, h\in H | u \in NU(rh)\}} \{d_{ur} r_u + d'_{rh} \lceil r_u / b_R \rceil\}$, and set $U(r^* h^*) = U(r^* h^*) \cup \{u\}$.

Step 2 (*Compute the number of remote facilities*)

Set $y_{rh} = \lceil \sum_{u \in U(rh)} r_u / b_R \rceil$, for all $h \in H$, $r \in NR(h)$.

Step 3 (*Compute the number of hub facilities*)

Set $u_h = \lceil \sum_{r \in NR(h)} y_{rh} / b_H \rceil$, for all $h \in H$.

5.3 Branching rule

When the branch-and-price algorithm is used, the main difficulty arises in the column generation after some subset of variables is fixed to 0. To prevent the generation of columns that were set to 0, a careful branching rule should be used. In our case, we use the following branching rule. Let us define, $X_{urh} =$ 1, if user *u* is included in a tree whose hub candidate site is *h* and remote candidate site is *r*. 0, otherwise,

The variables are considered implicitly for branching purpose only. Note that $\sum_{h\in H} \sum_{r \in NR(h)} X_{urh} = 1$ should hold if a user *u* is included in a tree whose hub candidate site is *h* and remote candidate site is *r*. Based on the variables X_{urh}, Setting X_{urh} to 0 is equivalent to setting $\sum_{t \in T(u,rh)} x_t^{rh} = 0$, where $T(u, rh)$ is the set of trees having hub candidate site *h* and remote candidate site *r* and user *u*. In this case, we set $w_u = 0$ when solving TGP(*rh*) and set $x_t^{rh} = 0$ for all $t \in T(rh)$ such that $u \in U_t^{rh}$ when solving (MP). On the other hand, setting X_{urh} to 1 is equivalent to setting $\sum_{t \in \bar{T}(u,rh)} x_t^{rh} = 0$, where $\bar{T}(u, rh)$ is the set of trees containing user *u* and not having hub candidate site *h* and remote candidate site *r*. In this case, a similar method can also be used for column generation.

In branching, we also have to consider the variables u_h's which are given originally in the formulation. For these variables, the branching by variable dichotomy is applied, i.e., if $u_h = u_h^0 \notin Z$, then we divide current problem into two subproblems with $u_h \le \lfloor u_h^0 \rfloor$ and $u_h \ge \lfloor u_h^0 \rfloor + 1$, respectively.

When we select variables for branching, we select the variable u_h of which value is nearest to 0.5 first. If all of the variables are integral, we then select the connection (*u*, *r*, *h*) such that X_{urh} is nearest to 0.5 first in the similar way. We use the best bound rule when we select nodes to solve in the enumeration tree.

6. COMPUTATIONAL RESULTS

We have tested our algorithm on some randomly generated problem instances and two real problems. The randomly generated problems consist of several problem classes. For each problem class, we generated 15 problems. Candidate sites and users are randomly generated from a discrete uniform [0, 30] by [0, 30] plane.

Table 1. Computational results with distance limit

(|H|,|R|,|U|)		#LP	#COL	Gap0 (%)	Gap1 (%)	#Node		Time (sec.)	
						NOPRE	PRE	NOPRE	PRE
(2,5,10)	avg.	26.5	99.7	1.31	0.73	3.9	3.5	1.42	1.25
	max.	45.0	141.0	2.48	1.78	8.0	8.0	2.36	1.98
	min.	11.0	66.0	0.39	0.00	2.0	0.0	0.82	0.44
(2,5,15)	avg.	52.5	195.4	0.89	0.70	4.4	4.0	2.53	2.30
	max.	104.0	371.0	2.41	2.01	8.0	8.0	5.44	4.73
	min.	19.0	75.0	0.12	0.00	2.0	0.0	1.21	0.77
(3,8,20)	avg.	67.7	415.6	0.84	0.62	4.5	4.0	3.53	3.20
	max.	133.0	655.0	1.59	1.14	8.0	10.0	5.77	6.31
	min.	26.0	191.0	0.19	0.00	2.0	0.0	1.98	1.10
(3,8,25)	avg.	131.5	665.3	0.71	0.52	6.5	6.0	7.29	7.11
	max.	254.0	922.0	1.29	1.24	14.0	14.0	13.51	13.41
	min.	47.0	408.0	0.17	0.00	2.0	0.0	3.52	2.31
(2,9,28)	*	111.0	1122.0	0.53	0.44	4.0	4.0	6.81	6.98
(3,9,28)	*	273.0	1775.0	2.93	2.80	42.0	42.0	20.99	19.94

* : The real problem instance

The demands of users are randomly generated from a discrete uniform [1, $b_R \times 1.5$]. The connection cost between a candidate site and a user is set to be the Euclidean distance. The connection cost between a remote candidate site and a hub candidate site is set to be the product of the Euclidean distance and the maximum capacity of a remote facility.

In the test, we consider two cases. First, we consider distance limit as follows. If the distance between users and remote candidate sites is farther than 15, we set them not to be connected. We also apply it to the relations between remote candidate sites and hub candidate sites. In this case, the graph that consists of users and candidate sites can be sparse. Second, we do not consider distance limit. The distances are unlimited. In this case, the graph that consists of users and candidate sites can be dense. Then, we consider the facility parameters. We set hub facility cost to be 300 and remote facility cost to be 100. We also assume that b_H=5(remote facility/hub facility) and b_R =284 DS1E, referring to the hub facility capacity and remote facility capacity of ATM-MSS. The test problems were solved on Pentium PC(333MHz) and we used CPLEX 4.0 callable mixed integer library as a LP solver.

Table 1 and Table 2 summarize the computational results of our algorithm. The computational results with distance limit on test problem are shown in Table 1 and those without distance limit are shown in Table 2. In these tables, the column #LP refers to the total number of LP's solved in ANLP. The column #COL refers to the number of columns generated. The columns Gap0, Gap1 refer to the relative ratio between the integral optimum and the objective values obtained by solving the final LP without preprocessing, the final LP with preprocessing at node 0 respectively. The column #Node refers to the number of nodes in the enumeration tree of the branch-and-price algorithm. The column Time refers to the execution time in seconds needed to solve the problem. The columns NOPRE refers to the columns generated when no preprocessing is applied. The columns PRE refers to the columns generated when preprocessing is applied.

In Table 1, the average CPU time needed to solve the problem to optimality does not exceed 5 seconds when $|U|$=15, 7 seconds when $|U|$=20, and 15 seconds when $|U|$=25. Real problem instances are also solved in a reasonable time. But, the preprocessing procedure is not effective much in the test on the spare graph.

Table 2 shows that the preprocessing procedure yields better results in terms of gaps and generated nodes and execution time compared to the case where, no such procedure is applied. The number of nodes of the enumeration tree decreases when preprocessing procedure is applied.

The computational results show that the algorithm preformed well and the preprocessing procedure reduced the gap considerably, especially in Table 2. The number of columns generated increased as the size of problem instance increases.

7. CONCLUSIONS

In this paper, we proposed a branch-and-price algorithm for the ATM switching node location problem. We showed that the LP relaxation of the problem, which has exponentially many variables, can be solved by

Table 2. Computational results without distance limit

(|*H*|,|*R*|,|*U*|)		#LP	#COL	Gap0 (%)	Gap1 (%)	#Node		Time (sec.)	
						NOPRE	PRE	NOPRE	PRE
(2,8,20)	avg.	174.9	685.7	0.85	0.25	949.0	42.0	280.08	11.24
	max.	1208.0	1813.0	2.46	0.61	9324.0	532.0	(1)2526.68	93.76
	min.	43.0	486.0	0.27	0.00	2.0	0.0	3.57	1.98
(2,8,30)	avg.	151.6	1053.9	0.35	0.06	88.6	5.2	314.21	14.33
	max.	299.0	1307.0	0.69	0.29	488.0	34.0	(5)2564.52	30.32
	min.	90.0	829.0	0.00	0.00	0.0	0.0	20.71	7.80
(2,8,40)	avg.	509.9	2242.5	0.34	0.14	98.4	65.1	166.11	119.46
	max.	2873.0	5768.0	0.77	0.39	800.0	774.0	(4)634.88	(1)620.71
	min.	190.0	1576.0	0.05	0.00	2.0	0.0	54.43	27.51
(3,10,20)	avg.	140.5	1033.0	0.81	0.52	7.7	6.4	11.09	8.85
	max.	242.0	1184.0	1.64	1.21	14.0	14.0	23.67	15.99
	min.	40.0	787.0	0.24	0.00	2.0	0.0	6.54	2.09
(3,10,30)	avg.	256.6	1827.3	0.51	0.35	20.6	14.9	45.84	28.64
	max.	522.0	2283.0	0.78	0.78	58.0	38.0	(1)126.72	96.89
	min.	107.0	1448.0	0.08	0.00	2.0	2.0	11.37	10.16
(3,10,40)	avg.	446.7	2979.3	0.36	0.20	15.9	13.6	240.34	159.09
	max.	995.0	3793.0	0.67	0.42	56.0	48.0	2006.48	719.09
	min.	240.0	2389.0	0.05	0.00	4.0	2.0	50.86	43.61
(5, 15,20)	avg.	119.5	2214.3	3.14	2.91	10.9	11.2	9.29	9.92
	max.	175.0	2583.0	6.99	6.99	26.0	24.0	14.28	14.78
	min.	80.0	1952.0	1.79	1.79	6.0	6.0	6.64	6.43
(5, 15,30)	avg.	247.6	3735.5	1.25	1.10	12.9	12.9	34.71	35.32
	max.	397.0	4597.0	1.88	1.88	36.0	36.0	82.89	58.16
	min.	157.0	3003.0	0.35	0.35	6.0	6.0	19.83	22.47
(5, 15,40)	avg.	376.6	6363.0	0.93	0.78	14.1	14.7	154.74	151.43
	max.	517.0	7437.0	1.45	1.24	28.0	22.0	240.57	214.81
	min.	286.0	5009.0	0.57	0.47	8.0	12.0	67.39	79.81

(number) : the number of unsolved problem within 1 hour

column generation. To solve the column generation problem, we used the dynamic programming algorithm. We also devised two valid inequalities as the preprocessing procedure.

Computational results show that the algorithm can solve practically-sized problems to optimality within reasonable time. This problem can be extended to the case where the number of hub facilities and the number of remote facilities are constrained. It can also be extended to the problem where more than one type of hub facility or remote facility should be considered.

Solution approach using the tree variables can be also applied for the problems with the two stage distribution system or facility location problems with hierarchical structure.

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