IEMS Vol. 3, No. 1, pp. 38-51, April 2004.

Purchasing and Inventory Policy in a Supply Chain under the Periodic Review: A Single Manufacturer and Multiple Retailers' Case

K. Prasertwattana · N. Chiadamrong†
Industrial Engineering Program, Sirindhorn International Institute of Technology,
Thammasat University, Pathum-thani 12121, Thailand
Tel: +662-986-9009 Ext. 2104, E-mail: navee@siit.tu.ac.th

Abstract. Over the years, most or many companies have focused their attention to the effectiveness and efficiency of their business units. As a new way of doing business, these companies have begun to realize the strategic importance of planning, controlling, and designing their own supply chain system. This paper analyzes the coordination issues in supply chains that consist of one manufacturer and multiple retailers operating under uncertain end customer demand and delivery lead-time. We use the Genetic Algorithm (GA) to determine the appropriate ordering and inventory level at which the manufacturer and multiple retailers can maximize the profit of the chain. This is performed under three controlling policies: the traditionally centralized controlling policy under the manufacturer's perspective, the entire chain's perspective, and lastly the coordinating controlling policy with an incentive scheme. The outcome from the study reveals that the coordinating controlling policy with an incentive scheme can outperform the traditional centralized controlling policies by creating a win-win situation in which all members of the chain benefit from higher profit, thus resulting in more willingness from all members to join the chain.

Keywords: supply chain, inventory management, multi-retailers, genetic algorithm, incentive scheme

1. INTRODUCTION

The globalization of businesses has been accelerating in the last two decades due to the rapid development of technology in manufacturing and information, increased cost pressure and more aggressive demand from customers. Individual companies are no longer to compete as independent entities, but rather as integral part of supply chain links. As such, the ultimate success of a company will depend on its managerial ability to integrate and coordinate the intricate network of business relationships among supply chain members (Lambert and Cooper, 2000). The nature of manufacturer-retailer relationships has been a popular topic in supply management and strategic research due to its importance in promoting desired economic behaviors from suppliers and end customers. These relationships also play an important role in managing the uncertainties of the commercial exchange process that can increase the cost of conducting business activities. Most supply chain studies focus on two-stage problems in which one seller supplies the products to one buyer. For instance, Banerjee (1986), Goyal (1988), Hill (1997) and Ertek and Griffin (2002) studied a single supplier and a single retailer relationship but varying their models under different circumstances.

However, in practice, the seller (or the manufacture in our case) usually has a chance to sell its product to more than one buyer (or retailers in our case). In some situations, the assumption of only one buyer may not be so realistic, especially when the manufacturer has a higher bargaining power and usually has the ability to supply its product to more than one retailer. The main reason for this study is to add the issue of the distributing strategy to the problem. Under the multi-echelon inventory model with multi-buyers for the make to stock environment, if any shortage exists either from the erroneous demand forecasting or supply shortages or long lead times, the manufacturer must make a distributing decision to spread out a portion of available units to certain retailers. This strategy depends on each company's policy where the priority of each retailer must be set.

In this paper, we focus on a specific case of the supply chain under one manufacturer who produces the

† : Corresponding Author

products and distributes them to multiple retailers, who in turn sell the products to the end customers. The model takes into account such factors as demand and delivery lead time uncertainty. The paper is organized as follows. In the next sections, we give a review of literature on multi-echelon inventory models. In section 3, we describe the supply chain models under three controlling policies. In section 4, we introduce the use of Genetic Algorithm (GA) to generate the best ordering policy and inventory level to all members in the chain. Section 5, 6 and 7 contain a numerical example, the results and the conclusions respectively.

2. RELATED LITERATURE

The literature on supply chains is vast, allowing for the reviews of only those most relevant to the study. Recently, supply chain inventory problems are focusing on the multi-stage and multi-member system. Barnes-Schuster and Bassok (1997) developed the overall order/delivery strategy for a single-depot and multi-retailer inventory system. They suggested that when the truck size can be chosen, a direct shipping policy generated a very good result. Dekker *et al*. (1998) analyzed the effect of the break quantity rule on the inventory cost in a 1-warehouse, *N*-retailers distribution system. The break quantity rule is to deliver large orders from the warehouse and small orders from the nearest retailer where a so-called break quantity determines whether an order is small or large. They suggested the rule can lead to a significant cost reduction.

Parija and Sarker (1999) developed an ordering policy for raw materials and determined an economic batch size for a product at manufacturing center, which supplies finished products to multiple customers, with a fixed-quantity at a fixed time-interval to each of the customers. They suggested that both the manufacturing company and customers need to operate in harmonic logistics, and in order to keep the production-inventory system operating at minimal cost, the supply chain logistics of raw materials and finished products should be efficiently balanced. Ganeshan (1999) presented a near-optimal (s, Q) type of inventory-logistics cost minimizing model for a production/distribution network with multiple suppliers supplying a distribution center, which in turn distributes to a large number of identical retailers. The decisions in the model were made through a comprehensive distribution-based cost framework that includes the inventory, transportation, and transit components of the supply chain.

Chen *et al.* (2001) studied the coordination mechanisms for a distribution system with one supplier and multiple retailers in which the demand at each retailer is known and constant. They characterized an optimal strategy, maximizing total system wide profit in a centralized system. Moreover, they have also shown that the same optimum level of channel wide profits can be achieved in a decentralized system when the coordination is achieved via periodically charged, fixed fees, and a non-traditional discount pricing scheme under which the discount given to a retailer is the sum of three discount components based on the retailer's annual sales volume, order quantity, and order frequency. Boyaci and Gallego (2002) analyzed the problem of coordinating pricing and inventory replenishment policies with a supply chain consisting of one wholesaler, and one or more geographically dispersed retailer under deterministic, price-sensitive customer demand. Klastorin *et al*. (2002) analyzed the coordination of a firm supplying a product to multiple retailers facing a static demand in a decentralized multi-echelon inventory/distribution system. They proposed a policy in which the manufacturer out-sources production to an original equipment manufacturer and offers a price discount to retailers to coordinate the timing of their orders with its own.

3. SUPPLY CHAIN SYSTEM IN THE STUDY

The chain in the study consists of one supplier, one manufacturer, *N*-retailers and end customers. Each of these facilities is a representative of different supply chain echelons. However, this paper only focuses on a relationship formed between a manufacturer and *N*-retailers (supplier and end customers are considered as external members in the system when considering the profit of the chain) as shown in Figure 1. Inventories among members in the chain are controlled by a periodic review under the make-to-stock environment, in which the uncertain demand and lead-times are considered. Assumptions and descriptions of the models are as follows:

3.1 Model Assumption

1. The manufacturer uses the periodic review and lot sizing policy to control its inventory.
2. The retailer uses the periodic review with target stock level (T, S) to control its inventory.
3. End customer's demand and delivery lead time are randomly generated based on the normal distribution.
4. For both manufacturer and retailers, only one order is allowed to be outstanding at any period.
5. Production rate of the manufacturer is assumed to be fixed and higher than the mean demands from all retailers at each period.
6. Only a single product is considered. Without loss of the generality, the manufacturer uses one unit of raw material to produce one unit of the finished product.
7. Unfulfilled demand is considered to be shortage.
8. Supplier has unlimited capacity.
9. Since the supplier is not of interest in this dyadic

relationship, the shortage cost paid by the supplier to the manufacturer is not included in the profit function of the chain.

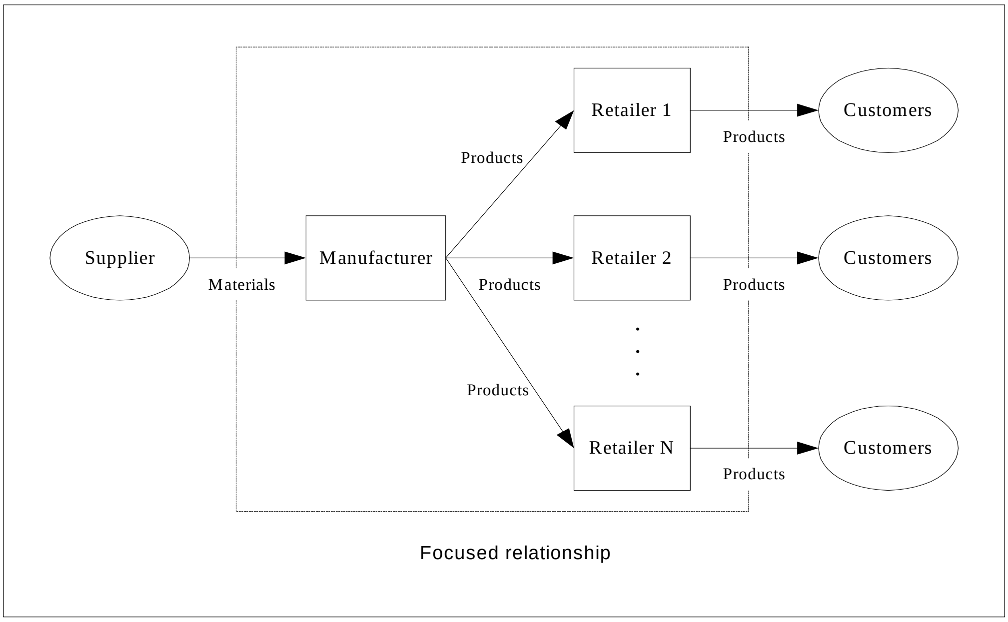

Figure 1. Members in the supply chain system

3.2 Model Description and Decision Variables

3.2.1 Manufacturer

The manufacturer has to decide its optimal Lot Sizing policy (*DLS*) and the safety stock level (*Optss*) to control its inventory. Under *DLS*, the manufacturer must decide in its ordering plan, whether to make an order following the lot for lot, or to combine its orders and make a bigger purchasing batch size. With 5 periods' planning horizon set in the study, there are $2^{5-1} = 16$ possible ordering policies. For instance, the first policy may follow the lot for lot or make an order in every period, which can save the holding cost but incur a high ordering cost. The second possible policy is to make an order in the first period and combine orders from period 2-5, which can save on the ordering cost but incur a high holding cost. Having received raw materials from the supplier, the manufacturer then transforms them to the finished products and distributes them to *N*-retailers. Each retailer penalizes any unfulfilled demand from the manufacturer as the shortage cost. Therefore, the manufacturer holds some safety stock to cover an effect of uncertainty in customer demand and delivery lead-time. The safety stock will be used only when a normal inventory level cannot satisfy the customer demand and it must be filled as soon as possible after having used.

3.2.2 Retailers

Each retailer aims to determine their optimal target stock level ($OptS_r$) to control their inventory. The target stock level of the retailer is not only to cover the end customer's demand but also to cover the effect of end customer demand's fluctuation as well as the late delivery and unfulfilled quantity from the manufacturer. Each retailer reviews its inventory and makes an order at every interval time *Tp*.

Even though the models in this study aims to maximize the profit, the customer satisfaction seems to be a major obligation. Therefore, the required customer service level constraint is added to the model so that the target stock level of the retailer satisfies the required customer service level. The customer service level considered here is called "*the fill rate*" which is set at 90%.

3.3 Model Development

For comparison purposes, the models in this study are constructed and separable according to each perspective, which are traditionally centralized controlling policy under manufacturer's perspective, the entire chain's perspective, and lastly coordinating controlling policy with an incentive scheme. The following notations will be used in all models.

r = 1,2,…,N represent retailer index.
t = 1,2,…,T represent period index.
k = 1,2,…,k_{max} represent generation index.
s = 1,2,…,S represent chromosome index.
Πs = Total profit of the supply chain ($)
Πm = Total profit of the manufacturer ($)
Πrs = Total profit of all retailers in the chain ($)
$\Pi joint$ = Total joint profit of the manufacturer and the retailers ($)
Π_m^C = Total profit of the manufacturer in the coordinated controlling system ($)
Π_m^C = Total profit of the retailer in the coordinated controlling system ($)
D_{rt} = End customer demand at retailer r at period t (units/day)
FD_t = Forecasted demand at the manufacturer at period t (units/day)
PR = Production rate per day at the manufacturer (units/day)
Tp = Duration of each period (days)
LTC_sm = Delivery lead time contract form between the supplier and the manufacturer (days)
LT_sm = Delivery lead time from the supplier to the manufacturer (days)
LTC_mr = Delivery lead time contract form between the manufacturer and the retailers (days)
LT_mr = Delivery lead time from the manufacturer to the retailers (day)
ESR_{rt} = Ending stock on hand level of finished products at the retailer r at period t (units)
Ess_t = Ending safety stock level of finished products at the manufacturer at period t (units)
Es_t = Ending stock on hand level of raw materials at the manufacturer at period t (units)
EI_t = Ending stock on hand level of finished products at the manufacturer at period t (units)
Qor_{rt} = Ordering quantity of the retailer r at period t (units)
Qm_t = Ordering quantity of the manufacturer at period t (units)
$Qpro_t$ = Production quantity of the manufacturer at period t (units)
$QsellR_{rt}$ = Sales volume per period at the retailer r at period t (units)
$QsellM_{rt}$ = Sales volume per period at the manufacturer to retailer r at period t (units)
$ShortQm_{rt}$ = Shortage quantity at the manufacturer at period t (units)
$RShortQ_{rt}$ = Shortage quantity at the retailer r at period t (units)
$Latem_{rt}$ = Late time delivery from the manufacturer to the retailer r at period t (days)
FR_{rt} = Fill rate of the retailer r at period t (%)
FRm_{rt} = Fill rate of the manufacturer to the retailer r at period t (%)
β = Desired customer service level or desired fill rate (%)
Ch_m = Unit holding cost of finished product at the manufacturer ($/unit/period)
Ch_rmtl = Unit holding cost of raw material at the manufacturer ($/unit/period)
Cs_m_r = Unit shortage cost at the manufacturer that is paid to the retailer r ($/unit)
Cpm = Unit purchasing cost at the manufacturer ($/unit)
Cpr_m = Unit production cost at the manufacturer ($/unit)
$Corder\_m$ = Ordering cost of the manufacturer ($/period)
$Corder\_m_t$ = Ordering cost of the manufacturer at each period t ($/period)
Cac_m = Activated cost per period at the manufacturer ($/unit)
$Cact\_m_{rt}$ = Activated cost at the manufacturer that is paid to push an on time delivery to the retailer r at period t ($)
Cp_r = Sales price per unit of the product at the manufacturer to the retailer r ($)
Ch_r_r = Unit holding cost of finished product at the retailer r ($/unit/period)
Cs_r_r = Unit shortage cost at the retailer r ($/unit)
Ca_r_r = Administration cost at the retailer r ($/unit)
$Corder\_r_r$ = Ordering cost of the retailer r ($/period)
$Bonus$ = Bonus cost per period at each retailer ($/period)
$Bonus_{rt}$ = Bonus cost of the retailer r that is paid to the manufacturer at period t ($)
$Sell_r$ = Sales price per unit of finished product at the retailer ($/unit)
ER_r = Pre-determined quantity discount level for the retailer r (units)
Pc = Probability of crossover (%)
Pm = Probability of mutation (%)

There are six decision variables to decide in the study. The notations can be shown as:

DLS = Discrete lot sizing at the manufacturer (ordering policy)
$Optss$ = Safety stock level at the manufacturer (units)
$OptS_r$ = Target stock level at the retailer r (units)
ACB_r = Decision to accept or reject bonus offered by the retailer r of the manufacturer
EQ_r = Percentage of exceeding units' quantity discount offered by the manufacturer to the retailer r (%)
DS_r = Percentage of distributing the amount of products on hand at the manufacturer to the retailer r (%)

3.3.1 Centralized Controlling Policy under the Manufacturer' Perspective

Under this policy, the chain is created using full information sharing, demanding that information of the end customers is available to all parties so that minimum error in setting the manufacturer's production level can be achieved. This is to avoid multiple demand forecast updates and make demand data downstream, available to the upstream site. However, the manufacturer's perspective aims at maximizing the profit of the manufacturer and represents the situation where the manufacturer can dominate the chain in which the retailers may not willing to join the chain.

Objective of Centralized controlling policy under Manufacturer's perspective

Maximize profit of the manufacturer (Πm)

Where:

$$\begin{aligned}\Pi m \;=\; & \sum_{r=1}^{N}\sum_{t=1}^{T} Cp_r \times QsellM_{rt} \\ & - \sum_{t=1}^{T} Ch\_rmtl \times Es_t \\ & - \sum_{t=1}^{T} Ch\_m \times (EI_t + Ess_t) \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cs\_m_r \times ShortQm_{rt} \\ & - \sum_{t=1}^{T} Corder\_m_t \\ & - \sum_{t=1}^{T} Cpm \times Qm_t \\ & - \sum_{t=1}^{T} Cpr\_m \times Qpro_t \end{aligned} \tag{1}$$

Subject to:

$$FRm_{rt} = 1 - (ShortQm_{rt} / Qor_{rt}) \tag{2}$$

$$FRm_{rt} \geq \beta \tag{3}$$

For $r = 1, 2\ldots,N$ and $t = 1,2\ldots,T$.

3.2.2 Centralized Controlling Policy under the Entire Chain's perspective

Centralized controlling policy under the entire chain's perspective is the system operating under one controller. This controlling policy is easier to implement if all members in the chain belong to only one owner, since it is quite difficult in practice for companies with different interests to be joined by a single entity and attain centralization. Thus, the objective of this policy is to maximize the profit of the entire chain rather than one member in particular.

Objective of the Centralized Controlling Policy under the entire chain's perspective

Maximize profit of the supply chain *(Πs)*

= Total profit of the manufacturer *(Πm)*

+ Total profit of the retailer (*Πrs)*

Where:

Πm is similar to the manufacturer perspective (Equation 1).

$$\begin{aligned}\Pi rs \;=\; & \Big(\sum_{r=1}^{N}\sum_{t=1}^{T} Sell_r \times QsellR_{rt} \\ & + \sum_{r=1}^{N}\sum_{t=1}^{T} Cs\_m_r \times ShortQm_{rt}\Big) \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Ch\_r_r \times ESR_{rt} \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cs\_r_r \times RshortQ_{rt} \\ & - \sum_{r=1}^{N} Corder\_r_r \times T \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cp_r \times QsellM_{rt} \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Ca\_r_r \times QsellR_{rt} \end{aligned} \tag{4}$$

Subject to:

$$FR_{rt} = 1 - (RshortQ_{rt} / D_{rt}) \tag{5}$$

$$FR_{rt} >= \beta \tag{6}$$

For $r = 1, 2\ldots,N$ and $t = 1,2\ldots,T$.

3.2.3 Coordinating the Controlling Policy with an Incentive Scheme

Coordinating the controlling policy with an incentive scheme occurs when a strong relationship is formed between the manufacturer and the retailers. Each member in the chain intends to share the information at the point of sales and exchange, giving incentive to strengthen their relationship. In practice, such information sharing cannot be achieved without the incentive and full cooperation of all parties in the chain. Therefore, each member should contribute some form of incentive to their partner as a means of strengthening their relationship and improving the performance of the supply chain, as well as their own companies. As a result, the objective of this policy is not only to maximize the total profit of the supply chain system but also to strengthen a relationship which can lead to an improvement of the overall channel profitability. However each member in the chain still has the authority to accept or reject the incentives offered by its partners.

Under this controlling policy, we focus on a situation where the manufacturer offers the discount to the retailer for each unit purchased beyond the pre-determined quantity discount level (ER), which is called "*the exceeding units' quantity discount policy (EUQD)*". At the same time, the retailer offers a bonus to the manufacturer as an exchange in order to speed up its delivery. As a consequence, the bonus is paid to the manufacturer only when the products are delivered to the retailer at right quantity and on time. In order to deliver the product on time, the manufacturer may need to pay an extra cost which is called "*the activated cost*" to gain extra effort for such activities.

Model of coordinating controlling policy with an incentive scheme

Maximize joint profit of the chain (Π_{joint})

= *Total profit of the manufacturer* (Π_m^C)

+ *Total profit of the retailer* (Π_r^C)

Where:

$$\begin{aligned}\Pi_m^C = & \sum_{r=1}^{N}\sum_{t=1}^{T} Cp_r \times QsellM_{rt} \\ & + \sum_{r=1}^{N}\sum_{t=1}^{T} Bonus_{rt} \\ & - \sum_{t=1}^{T} Ch\_rmtl \times Es_t \\ & - \sum_{t=1}^{T} Ch\_m \times (EI_t + Ess_t) \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cs\_m_r \times ShortQm_{rt} \\ & - \sum_{t=1}^{T} Corder\_m_t \\ & - \sum_{t=1}^{T} Cpm \times Qm_t \\ & - \sum_{t=1}^{T} Cpr\_m \times Qpro_t \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cact\_m_{rt}\end{aligned} \quad (7)$$

$$\begin{aligned}\Pi_r^C = & \Big(\sum_{r=1}^{N}\sum_{t=1}^{T} Sell_r \times QsellR_{rt} \\ & + \sum_{r=1}^{N}\sum_{t=1}^{T} Cs\_m_r \times ShortQm_{rt}\Big) \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Ch\_r_r \times ESR_{rt} \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cs\_r_r \times RshortQ_{rt} \\ & - \sum_{r=1}^{N} Corder\_r_r \times T \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Cpr_r \times QsellM_{rt} \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Ca\_r_r \times QsellR_{rt} \\ & - \sum_{r=1}^{N}\sum_{t=1}^{T} Bonus_{rt}\end{aligned} \quad (8)$$

Subject to:

$$Cp_r = \begin{cases} Cp_r & \text{if } Qor_{rt} \le ER_r \\ ((Cp_r \times ER_r) + (Cp_r)(Qor_{rt} - ER_r)(1-(EQ_r/100)))/Qor_{rt} & \text{otherwise} \end{cases} \quad (9)$$

$$Bonus_{rt} = \begin{cases} bonus & \text{if } ACB_r = 1 \text{ and } Latem_{rt} \le 0 \text{ and } shortQm_{rt} = 0 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

$$Cact\_m_{rt} = \begin{cases} Cac\_m & \text{if } ACB_r = 1 \text{ and } Latem_{rt} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

$$FR_{rt} >= \beta \quad (12)$$

For $r = 1, 2\ldots,N$ and $t = 1,2\ldots,T$.

4. SOLUTION TECHNIQUES

This paper studies a supply chain system where a manufacturer distributes a single product to *N*-retailers, who in turn sell the product to end customers. Since the end customer demand and delivery lead-time are uncertain. The optimal settings for decision variables are determined using the Genetic Algorithm (GA). The flow chart of Genetic algorithm is shown in Figure 2.

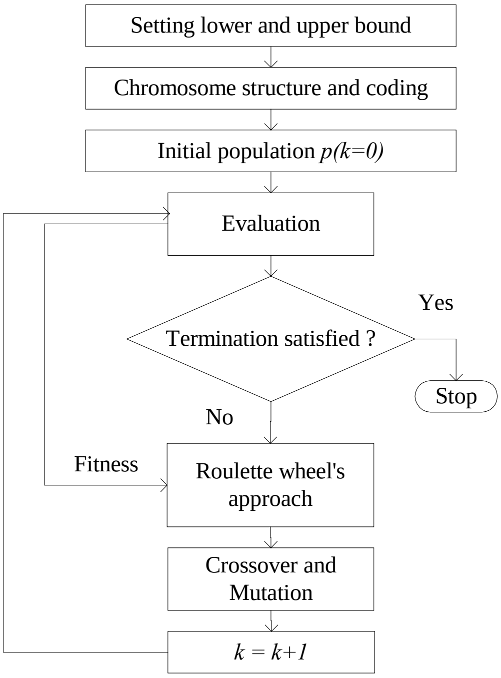

Figure 2. Genetic Algorithm's Procedures

4.1 Genetic Algorithm

The algorithm of the GA as shown in Figure 2 can be explained as follows:

4.1.1 First step: Setting the Lower Bound and Upper Bound for the Solution

This boundary is set to limit the computational time from the GA algorithm. However, the searching boundary for each decision variable must be large enough to ensure that the optimal solution will fall inside the boundary. The following steps are used to set lower bound (l_i) and upper bounds (u_i) on all six decision variables (i.e., *DLS*, *Optss*, $OptS_r$, ACB_r EQ_r and DS_r where $r = 1,2\ldots,N$).

Discrete Lot Sizing (DLS)

- Lower bound of Discrete Lot Sizing (*DLS*) is 1 or making an order in every period.
- Upper bound of Discrete Lot Sizing (*DLS*) is equal to the number of discrete lot sizing pattern, which is equal to $2^{T\text{-}1}$. Since the planning horizon (n) is equal to 5, the upper bound of *DLS* is equal to $2^{5\text{-}1} = 16$.

Safety Stock Level (Optss)

- Lower bound of the safety stock level at the manufacturer (*Optss*) is set at 0 or no inventory is kept at the manufacturer.
- During preliminary experiments, the safety stock level that was generated from GA has never exceeded 30% of the average demand in each period, the upper bound of the safety stock level at the manufacturer (*Optss*) is then limited to holding no more than 30% from the average demand in each period, which is equal to 2,025 units.

Target stock level (OptS)

- Lower bound of the target stock level at the retailer ($OptS_r$) should be at least equal to the expected demand during the review period plus the delivery lead time contract from the manufacturer, which is equal to

$$\overline{D_r} \times (Tp + LTC\_mr) \tag{13}$$

Where $\overline{D_r}$ is the mean end customer demand occurs at each retailer, LTC_mr is the lead-time contract from the manufacturer. Therefore, the lower bound of the target stock level for each retailer can be calculated as follows:

Lower bound of $OptS_1$ = 100 × (30+7) = 3,700 units

Lower bound of $OptS_2$ = 50 × (30+7) = 1,850 units

Lower bound of $OptS_3$ = 75 × (30+7) = 2,775 units

Upper bound of the target stock level at each retailer ($OptS_r$) under the traditionally coordinating policy without an incentive scheme is set to achieve 99.85% service level, which should be high enough to cover the optimal solution. The equation can be shown as:

$$\overline{D_r} \times (Tp + LTC\_mr) + z\sqrt{(\overline{lt\_mr} \times \sigma^2_{D_r}) + (\overline{D_r}^2 \times \sigma^2_{lt\_mr})} \tag{14}$$

(see Tersine (1996) for further information)

Where $\overline{D_r}$ and σ_{Dr} are the mean and standard deviation for the end customer demand occurring at each retailer, ($r = 1,2\ldots N$), $\overline{lt\_mr}$ and σ_{lt_mr} are the mean and standard deviation of the delivery lead-time of finished products from the manufacturer to the retailers. Therefore, the upper bound of the target stock level at the retailers can be calculated as follows:

The upper bound of $OptS_1$

$$= 100(30+7) + 3\sqrt{(7 \times 25^2) + (100^2 \times 2^2)}$$

= 4,331units

The upper bound of $OptS_2$

$$= 50(30+7)+3\sqrt{(7\times 12^2)+(50^2\times 2^2)}$$
$$= 2{,}164\text{units}$$

The upper bound of $OptS_3$

$$= 75(30+7)+3\sqrt{(7\times 15^2)+(75^2\times 2^2)}$$
$$= 3{,}240 \text{ units}$$

However, when the manufacturer offers quantity discount incentive to each retailer, it appeared that the retailers are willing to take such an advantage by purchasing more products. This results in the upper bound of the target stock level at each retailer ($OptS_r$) under the coordinating policy with an incentive scheme being set at 20% higher than the upper bound of the traditionally coordinating policy, which are equal to 5,197 units, 2,596 units and 3,888 units at the retailer 1, 2 and 3, respectively.

Accept or Reject the bonus (ACB)

- A decision to accept or reject a bonus (ACB_r) is represented by a binary number (0 and 1). One means accepting the bonus while zero means rejecting the bonus. As a result, this decision variable requires only one bit in each chromosome.

Percentage of Quantity Discount (EQ_r)

- The lower bound of the percentage of exceeding units' quantity discount (EQ_r) is set at 0% or does not offer any discount.
- The upper bound of the percentage of exceeding units' quantity discount (EQ_r) is limited at a 20% discount from a unit purchasing cost for each unit purchased beyond the predetermined quantity discount level (4,100 units at the retailer 1, 2,050 units at the retailer 2, and 3,100 units at the retailer 3, which are set just above at the target stock level recommended by the Genetic Algorithm during the centralized controlling policy under the entire chain's perspective without incentive scheme).

Distribution Strategy of finished product (DS_r)

- The lower bound of the percentage of distributing the amount of products on hand at the manufacturer to each retailer (DS_r) is set at 0% or does not supply any products to that retailer.
- The upper bound for the percentage of distribution of the amount of products on hand at the manufacturer to each retailer (DS_r) is set at 100% or supplies all products to only that retailer.

4.1.2 Second Step: Chromosome Structure and Coding

The binary coding is selected to represent the solution for this problem. Therefore, all parameters should be converted to binary strings. The required bits (denoted with m_i) for each decision variable. They are calculated as follows:

$$2^{m_i - 1} < (u_i - l_i) \le 2^{m_i} - 1 \qquad (15)$$

Where i is a decision variable index.

For example, under the centralized controlling policy with both manufacturer and the entire chain's perspectives, the combined multi-parameter chromosome has the following form: $<DLS><Optss><OptS_1><OptS_2><OptS_3><DS_1><DS_2>$. Under the coordinating controlling policy with an incentive scheme, the chromosome has the following form: $<DLS><Optss><OptS_1><OptS_2><OptS_3><DS_1><DS_2><ACB_1><ACB_2><ACB_3><EQ_1><EQ_2><EQ_3>$. The length of the chromosome can be calculated as follows:

For DLS: $2^3 <(16\text{-}1)<= 2^4\text{-}1$, so the required bit for $DLS = 4$.
For $Optss$: $2^{10} < (2{,}025\text{-}0) <= 2^{11}\text{-}1$, so the required bit for $Optss = 11$.
For $OptS_1$: $2^9 < (4{,}331\text{-}3{,}700) <= 2^{10}\text{-}1$, so the required bit for $OptS_1 = 10$.
For $OptS_2$: $2^8 < (2{,}164\text{-}1{,}850) <= 2^9\text{-}1$, so the required bit for $OptS_2 = 9$.
For $OptS_3$: $2^8 < (3{,}240\text{-}2{,}775) <= 2^9\text{-}1$, so the required bit for $OptS_3 = 9$.
For DS_r: $2^{13} < (10000\text{-}0) <= 2^{14}\text{-}1$, so the required bit for $DS_r = 14$. (For $r = 1, 2$)
For ACB_r: $2^0 < (1\text{-}0) <= 2^1\text{-}1$, so the required bit for $ACB_r = 1$. (For $r = 1, 2, 3$)
For EQr: $2^4 < (20\text{-}0) <= 2^5\text{-}1$, so the required bit for $EQr = 5$. (For $r = 1, 2, 3$)

Therefore, the length of the chromosomes in the traditionally centralized controlling policies under the manufacturer's and the entire chain's perspective are equal to 71 bits and the length of the chromosome in the coordinating controlling policy with incentive scheme is equal to 89 bits.

4.1.3 Third step: Initializing Population

Genetic Algorithm starts with an initial set random solution called population. The population is set to contain 10 chromosomes. We use a random number generator to generate the initial population $P(k{=}0)$ in forms of binary numbers, where k is a generation index.

4.1.4 Fourth step: Evaluation

Each chromosome in the population represents a potential solution to the problem. The evaluation function is responsible for rating these potential solutions by substituting a real number back to the objective function as a measure of it's fitness. This is carried out in two steps as follows:

Step 1: Mapping binary number to real number

・Convert the binary string from base 2 to base 10

$$\left(\left\langle b_{m_i-1} b_{m_i-2} \cdots b_0 \right\rangle\right)_2 = \left(\sum_{a=0}^{m_i-1} b_a 2^a \right) = decimal(chromosome_i) \tag{16}$$

Where a is the position of each bit and b_{m_i} is the binary coding of chromosomes.

・Find a corresponding real number (x_i)

$$x_i = l_i + \left(decimal(chromosome_i) \cdot \frac{u_i - l_i}{2^{m_i} - 1} \right) \tag{17}$$

Step 2: Setting the evaluation function

Due to the fact that this problem is a constraint optimization, the penalty function is selected to handle all the constraints. Penalty strategy transforms the constrained problem into an unconstrained problem by penalizing infeasible solutions, in which a penalty term is added to the objective function for any violation of the constraints. Under a centralized controlling policy with the manufacturer's perspective, the solution will be penalized when the fill rate of the manufacturer is less than the required service level (β). However, under the centralized controlling policy with the entire chain's perspective and coordinating controlling policies, the solutions will be penalized when the fill rate at any retailers is less than the required service level (β).

The evaluation function ($Eval_t$) of the centralized controlling policy under the manufacturer's perspective is presented as follows:

$$Eval_t = \Pi m - \begin{cases} 0 & \text{if } FRm_{rt} > \beta \\ (\beta - FRm_{rt}) \times D_{rt} \times k & \text{otherwise} \end{cases} \tag{18}$$

The evaluation function ($Eval_t$) of the centralized controlling policy under the entire chain's perspective is presented as follows:

$$Eval_t = \Pi s - \begin{cases} 0 & \text{if } FR_{rt} > \beta \\ (\beta - FR_{rt}) \times D_{rt} \times k & \text{otherwise} \end{cases} \tag{19}$$

The evaluation function ($Eval_t$) of the coordinating controlling policy with an incentive scheme is presented as follows:

$$Eval_t = \Pi jo\,int - \begin{cases} 0 & \text{if } FR_{rt} > \beta \\ (\beta - FR_{rt}) \times D_{rt} \times k & \end{cases} \tag{20}$$

Note that the measures of infeasibility from equation 18 to 20 are multiplied by the number of generations, so as to increase the pressure on infeasible individuals. Other penalty functions such as multiplying infeasible results with a large constant numbers have also been tested but they gave inferior results (i.e. lower the chain profit).

4.1.5 Fifth Step: Selection and Genetic Operators

The roulette wheel approach is chosen as the method to select the chromosomes and create a mating pool for their reproduction. The roulette wheel approach belongs to the fitness-proportion selection and can select a new population with respect to the probability distribution based on fitness values.

Having selected the operation, a mating pool is formed. The next step is to do a crossover operation. The crossover operation used in the study is a random one cut-point, which exchanges the right parts of two parents to generate an offspring. Then, the mutation operator flips a bit in a chromosome by a random method. After the first generation has been completed, the new population will be collected. Then, the process repeats itself until 10,000 generations.

4.2 Number of Replications and Replication Length

Due to the stochastic nature of the problem in this study, 10 replications with 5 periods (150 days) in each replication have been experimented with. This number of replications and its length have been analyzed to given sufficient data and allow the half width of 95% confident intervals of the interested observation (the total profit of the chain) within 5% of its mean in all perspectives.

5. NUMERICAL EXAMPLE

An example is built to demonstrate the proposed algorithms and to compare the results under three controlling policies: the centralized controlling policy under the manufacturer's perspective, the entire chain's perspective, and the coordinating controlling policy with an incentive scheme. Table 1 presents the parameters used in the study.

Under make to stock environment, the manufacturer has to produce the products and supply them to the retailers based on its forecasted demand (FD_t). On the other hand, end customer demands (D_{rt}) are generated at the retailers. The mean and standard deviation of the forecasted demand at the manufacturer are based on the summation of end customer's demand at each retailer.

This is due to the fact that the demand information can be passed to all members in the chain. The end customer's demand at each retailer (D_{rt}) are randomly generated under the normal distribution, in which approximately 25% of their mean value is considered as its standard deviation.

With one manufacturer and three retailers ($N=3$) in this study, the mean and variance of the end customer's demand at the retailer 1 is the highest, followed by those of retailer 3 and the retailer 2. We try to distinguish each retailer by their different characteristics as they mark up the profit. As a result, the sales price of the manufacturer to each retailer is different, and so is the sales price of each retailer to their customers. In general, the manufacturer gains the highest profit margin to retailer 1, whereas the retailer 3 gains the highest profit margin to its customers. Due to problems of uncertainty with the demand and lead-time, the products available on hand at the manufacturer at any period may be insufficient to supply all retailers as requested and needs to be proportionally distributed to each retailer according to the objective of each perspective. As each retailer has different characteristics, the percentage of distribution of the amount of products on hand to each retailer is also one of the decision variables optimized by using the Genetic Algorithm's approach.

Table 1. Data Input

Input parameters	**Set values**
End customer demand per day at retailer 1 (D_{1t}) End customer demand per day at retailer 2 (D_{2t}) End customer demand per day at retailer 3 (D_{3t})	Normal (100, 25^2) units Normal (50, 12^2) units Normal (75, 15^2) units
Forecasted end customer demand per period (FD_t)	Normal (225, 32^2) × *Tp* units
Delivery lead time from the supplier (*LT_sm*)	Normal (7, 2^2) days
Delivery lead time from the manufacturer (*LT_mr*	Normal (7, 2^2) days
Delivery lead time contract from the supplier (*LTCsm*)	7 days
Delivery lead time contract from the manufacturer (*LTC_mr*)	7 days
Production rate per day (*PR*)	300 units
Planning horizon (*T*)	5 periods
Number of days in each period (*Tp*)	30 days
Required service level (β)	90 %
Cost parameters	**Set values**
Sales price per unit of raw material from the supplier (*Cpm*)	\$350 per unit
Unit holding cost of raw material at the manufacturer per period (*Ch_rmtl*)	10% of raw material value
Sales price per unit of the product at the manufacturer to retailer 1 (Cp_1) Sales price per unit of the product at the manufacturer to retailer 2 (Cp_2) Sales price per unit of the product at the manufacturer to retailer 3 (Cp_3)	\$615 per unit \$610 per unit \$600 per unit
Unit production cost at the manufacturer (*Cpr_m*)	\$150 per unit
Unit holding cost of product at the manufacturer and the retailers per period (*Ch_m,*and Ch_r_r)	10% of product value
Ordering cost at the manufacturer and the retailers (*Corder_m,*and $Corder\_r_r$)	\$500 per order
Unit shortage cost paid by the manufacturer to the retailer 1 (Cs_m_1) Unit shortage cost paid by the manufacturer to the retailer 2 (Cs_m_2) Unit shortage cost paid by the manufacturer to the retailer 3 (Cs_m_3)	\$35 per units \$30 per units \$25 per units
Unit opportunity shortage cost at the retailer 1 (Cs_r_1) Unit opportunity shortage cost at the retailer 2 (Cs_r_2) Unit opportunity shortage cost at the retailer 3 (Cs_r_3)	\$50 per units \$45 per units \$55 per units
Unit administration cost at the retailer 1 (Ca_r_1) Unit administration cost at the retailer 2 (Ca_r_2) Unit administration cost at the retailer 3 (Ca_r_3)	\$50 per unit \$60 per unit \$50 per unit
Sales price per unit of finished product at the retailer 1 ($Sell_1$) Sales price per unit of finished product at the retailer 2 ($Sell_2$) Sales price per unit of finished product at the retailer 3 ($Sell_3$)	\$750 per unit \$745 per unit \$755 per unit
Bonus cost per period offered by the retailer to the manufacturer (*bonus*)	\$ 10,000 per period
Activated cost per period at the manufacturer (*Cac_m*)	\$ 2,000 per period
GA parameters	**Set values**
Number of chromosomes in the population (*S*)	10 chromosomes
Probability of crossover (*Pc*)	50%
Probability of mutation (*Pm*)	20%
Stopping generation (k_{max})	10,000 generations

6. RESULTS

The results are divided into 3 sections according to each comparative perspective. Table 2 presents the profit and the fill rate from each member in the chain as well as the total chain's profit. The numbers shown in the table are the average value from 10 replications over 5 periods (150 days) per replication. In addition, Table 3 reveals the best settings for each decision variable as generated by the Genetic Algorithm. Lastly Table 4 presents the relevant costs from each member in the chain.

6.1 Centralized Controlling Policy under Manufacturer's Perspective

In this instance, the manufacturer tries to optimize its own profit by increasing its revenue and reducing its costs. As shown in Table 3, the operations have forced retailers 1 and 2 to set relatively high levels of their target stock at 4,178 units and 2,084 units respectively. At the same time the manufacturer has also decided to hold more stock (highest safety stock level in relative to all perspectives) and to supply more products to retailers 1 and 2 (as shown by 100% fill rate for both retailers) but only to supply a 90.95% fill rate to retailer 3. This is due to the fact that the profit margin of selling one unit of the product to retailer 1 (≈ 19.7%) and retailer 2 (≈ 18.8%) are higher than the profit margin per unit of selling to retailer 3 (≈ 17.1%). Moreover, retailer 1 and retailer 2 have also charged the manufacturer a higher shortage cost. Having done that, the manufacturer can increase its revenue and avoid paying a high shortage cost when only looking at its own perspective.

As its objective, even though all members gain profit, the manufacturer gains the highest profit when there exist some level of unfair treatment to the retailers in the chain, especially in the case of the retailer 3, who seems to have the lowest bargaining power and as a result, has been given lower priority in supplying parts from the manufacturer.

6.2 Centralized Controlling Policy under the Entire Chain's Perspective

This policy is intended to optimize the profit of the whole chain, rather than optimizing the sole profit of one particular member. The results from Table 2 show that the profit of the entire chain has increased by 17.30% from the manufacturer's perspective. As expected, the profit of the manufacturer has reduced while the profit of the retailers has increased. Due to the entire chain's perspective, rather than solely looking at the manufacturer's benefit, each member operates by aiming at the common benefit of the entire chain. As a result, the manufacturer tends to supply more products to retailer 3, as retailer 3 is the main profit contributor to the chain, and who sells the product with the highest profit margin to the end customers (≈ 16.2%). On the other hand, the profit margin at the retailer 1 and retailer 2 to their customers are only set at approximately 12.8% and 11.5% respectively. By setting a high target stock level for retailer 3 (3,071 units as show in Table 3), retailer 3 can supply more products to the end customers.

This has been shown by the fill rate from the manufacturer to the retailer 3, which is as high as 97.97% (others are dropped to around 83-86%) and the fill rate from retailer 3 to the end customers, which is at the highest with 99.59% (retailer 1 is at 98.20% and retailer 2 is only at 95.38%).

6.3 Coordinating Controlling Policy with an Incentive Scheme (Exceeding Units Quantity Discount and Bonus)

It is obvious, from Table 2 as well as Figure 3 that by implementing an incentive scheme between the manufacturer and all retailers, the profit of the entire chain is at the highest when compared to the profits obtained from all comparative perspectives. In addition, individual profits from all members in the chain are also at their highest. By giving the discount at the rate of 2% to retailer 3 and 3% to the retailers 1 and 2, all retailers have been shown to take advantage of buying the discounted price (also leading to a reduction their holding cost per unit) and set a higher target stock level. As a result, each retailer spends higher holding cost for more stock holding. However, this not only increases an ability of the retailers to supply to more end customer's demand but also brings higher revenues and saves the shortage cost at the retailers as illustrated in Table 4.

In order to get bonus from each retailer, the manufacturer has to achieve two constraints (both on time delivery and correct amount). The manufacturer can speed up its delivery by paying the activated cost and prevent any shortage by holding and producing sufficient amount of stock. Since the objective of this controlling policy is to optimize the profit of the chain (not individual). The results from GA suggest the manufacturer not to accept all retailers' bonus simultaneously. This is to avoid holding too much safety stock and incurring too high holding cost. Due to the fact that only a 2% discount is given to retailer 3 and that the highest profit margin can be made from this retailer, the manufacturer normally accepts the bonus from the retailer 3 (accepting 7 from 10 replications), but rarely accepts the bonus from the retailer 1 and the retailer 2 (accepting 2 and 3 from 10 replications, respectively).

Table 2. Comparison of the performances among different perspectives (5 periods' planning horizon)

Performance of the supply chain system	Centralized control		Coordinating control with EUQD & bonus
	Manufacturer's perspective	Entire chain's perspective	
Profit of the chain ($)	5,447,711	6,390,114	6,898,525
Profit of the manufacturer ($)	3,435,081	3,005,882	3,447,515
Profit of the retailer 1 ($)	866,235	1,729,542	1,732,814
Profit of the retailer 2 ($)	371,712	627,293	631,471
Profit of the retailer 3 ($)	774,684	1,027,397	1,086,726
Fill rate of the manufacturer to retailer 1 (%)	100.00	85.71	91.83
Fill rate of the manufacturer to retailer 2 (%)	100.00	82.90	89.91
Fill rate of the manufacturer to retailer 3 (%)	90.95	97.97	95.12
Fill rate of retailer 1 to end customers (%)	99.88	98.20	99.78
Fill rate of retailer 2 to end customers (%)	99.58	95.38	98.66
Fill rate of retailer 3 to end customers (%)	89.59	99.59	99.95

Table 3. Comparison of decision variables among different perspectives (5 periods' planning horizon)

Decision variable	Centralized control		Coordinating control with EUQD & bonus
	Manufacturer's perspective	Entire chain's perspective	
Discrete lot sizing at the manufacturer (ordering policy)	Lot for lot	Lot for lot	Lot for lot
Safety stock level at the manufacturer (units)	1,289	121	452
Target stock level at the retailer 1 (units)	4,178	4,061	4,280
Target stock level at the retailer 2 (units)	2,084	2,010	2,215
Target stock level at the retailer 3 (units)	2,835	3,071	3,267
Accept or reject the bonus offered from the retailer 1	-	-	Accept 2 from 10 replications
Accept or reject the bonus offered from the retailer 2	-	-	Accept 3 from 10 replications
Accept or reject the bonus offered from the retailer 3	-	-	Accept 7 from 10 replications
Percent discount offering to retailer 1 (%)	-	-	3
Percent discount offering to retailer 2 (%)	-	-	3
Percent discount offering to retailer 3 (%)	-	-	2
Percentage of parts distributing to the retailer 1 (%)	45.20	40.99	42.30
Percentage of parts distributing to the retailer 2 (%)	29.95	22.76	25.26
Percentage of parts distributing to the retailer 3 (%)	24.85	36.35	32.44

Table 4. Comparison of cost parameters among different perspectives (5 periods' planning horizon)

Cost parameter and sales volume	Centralized control Manufacturer's perspective	Centralized control Entire chain's perspective	Coordinating control with EUQD & bonus
Holding cost of raw materials at the manufacturer ($)	124,311	103,145	113,255
Holding cost of products at the manufacturer ($)	206,371	104,322	202,250
Shortage cost of the manufacturer give to retailer 1 ($)	0	96,096	44,604
Shortage cost of the manufacturer give to retailer 2 ($)	0	50,482	27,309
Shortage cost of the manufacturer give to retailer 3 ($)	21,823	6,511	13,254
Holding cost of product at the retailer 1 ($)	346,532	198,449	292,358
Holding cost of product at the retailer 2 ($)	198,294	114,131	149,369
Holding cost of product at the retailer 3 ($)	164,017	228,474	261,637
Opportunity shortage cost of the retailer 1 ($)	3,030	14,547	3,483
Opportunity shortage cost of the retailer 2 ($)	2,708	16,191	5,503
Opportunity shortage cost of the retailer 3 ($)	42,378	2,843	1,441

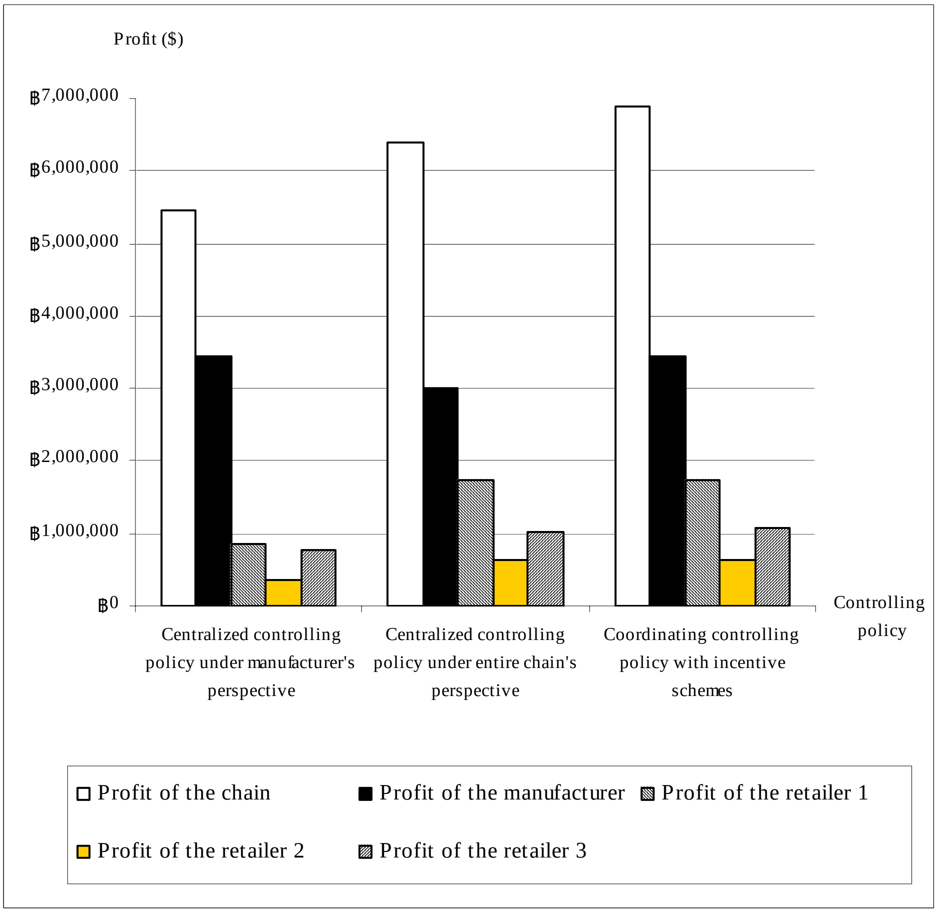

Figure 3. Comparison of the profit under the different controlling policies

7. CONCLUSIONS

As companies foster longer-term and co-operative relationships in the supply chain, buying and supplying companies must better understand how to manage these relationships. This study presented the purchasing and inventory control policies of a manufacturer and multiple retailer case. In these types of systems, the manufacturer tends to dominate in the chain, (since the retailers have to rely on them to supply the products) and has a higher bargaining power over them. As in the case of the centralized controlling policy under the manufacturer's perspective, (which operates under a chain by aiming to maximize one member's profit or solely looking at its own benefit in particular) there might not be willingness among some of other members to cooperate since they may be concerned about the confidentiality of the information and may not see a long-term benefit in sharing information.

When (as in the case of the centralized controlling policy under the entire chain's perspective) the aim is to maximize the profit of the entire chain, it is found that the profit of the whole chain can be increased within the centralized model. However, the slight drop of the manufacturer's profit in this perspective may hinder the manufacturer to move ahead with the plan. Without any compensation, the manufacturer would not be totally happy to join the chain as the chain is pretty much dominated by the manufacturer. However, exchanging incentives among them, a win/win game for all parties can be achieved. Since all transacting parties have been linked with further improved benefits, without feeling being taken advantage of, this is the first step in forming the long-term strategic partnership.

Buyer-supplier relationships are characterized by having high levels of interdependence and an ongoing commitment. They rarely use any formal contractual or legal enforcement means. The objective of these relationships is to maximize total relationship outcomes, which satisfy both individual and group needs. The objectives for long-term cooperation and loyalty from all parties to the relationship are expected and necessary for strong relationship continuity. One important challenge in this kind of the coordination is termed "*flexibility*" and is a very important part in this complex business world. The results obtained from the optimization-searching algorithm of GA in this study can provide flexibility and accommodate independency among members in the chain. In these situation all parties aim at maximize the profit of the chain and adjust their own operating policies to accommodate such objectives, for instance, how much to order, how to keep their stock and how much to distribute to each retailer outlet.

Another purpose of this paper is to study the problem of why and how to establish an incentive scheme. The aim is not only to maximize the total profit of the supply chain system as well as the overall channel profitability beyond the traditional chain, but also to enhance the relationship among members, which allow the chain to achieve its best performance. Through the above numerical example, the research has fulfilled its purpose, which is to demonstrate the proposed algorithm using the Genetic Algorithm, and to suggest the optimal setting of parameters for each member in the chain, as well as highlighting the benefits of the functionally coordinating policy with an incentive over the traditional chain.

ACKNOWLEDGEMENT

This research is supported by Thaikhadi Research Institute, Thammasat University. This support is acknowledged gratefully.

REFERENCES

Banerjee, A. (1986) A joint economic-lot-size model for purchaser and vendor. *Decision Science*, **17**(3), 292-311.

Barnes-Schuster, D. and Bassok, Y. (1997) Direct shipping and the dynamic single-depot/multi-retailer inventory system, *European Journal of Operational Research*, **101**, 509-518.

Boyaci, T. and Gallego, G. (2002) Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailer. *Int. J. Production Economics*, **77**, 95-111.

Chen, F., Federgruen, A. and Zheng, Y.S. (2001), Coordination mechanisms for a distribution system with one supplier and multiple retailers, *Management Science*, **47**(5), pp.693-708.

Dekker, R., Kleijn and M.K., Kok, A.G. (1998) The break quantity rule's effect on inventory costs in a 1-warehouse, N-retailers distribution system, *Int. J. Production Economics*, **56-57**, 61-68.

Ertek, G. and Griffin, P.M. (2002) Supplier-and buyer-driven channels in a two-stage supply chain. *IIE Transaction*, **34**(8), 691-700.

Ganeshan, R. (1999) Managing supply chain inventories: a multiple retailer, one warehouse, multiple supplier model, *Int. J. Production Economics*, **59**, 341-354.

Goyal, L.K. (1988) A joint economic-lot-size model for purchaser and vendor: A comment. *Decision Sciences*, **19**(1), 236 -241.

Hill, R.M. (1997) The single vendor single-buyer integrated production-inventory model with a generalised policy. *European Journal of Operational Research*, **97**(3), 493-499.

Klastorin, T.D., Moinzadeh, K. and Son, J. (2002) Coordinating orders in supply chains through price discounts. *IIE Transactions*, **34**(8), 679-689.

Lambert, D.M. and Cooper, M.C. (2000) Issues in supply chain management, *Industrial Marketing Management*, **29**, 65-83.

Minner, S. (2003) Multiple-supplier inventory models in supply chain management: A review, *Int. J. Production Economics*, **81-82**, 265-279.

Parija, G.R. and Sarker, B.R. (1999) Operations planning in a supply chain system with fixed-interval deliveries of finished goods to multiple customers. *IIE Transactions*, **31**, 1075-1082.

Tersine, R.J. (1996) *Principle of inventory and material management*, Prentice Hall, Englewood Cliffs, New Jersey.