Positive Real Control for Uncertain 2-D Singular Roesser Models

Huiling Xu, Lihua Xie, Shenyuan Xu*, and Yun Zou

Abstract: This paper discusses the problem of positive real control for uncertain 2-D linear discrete time singular Roesser models (2-D SRM) with time-invariant norm-bounded parameter uncertainty. The purpose of this study is to design a state feedback controller such that the resulting closed-loop system is acceptable, jump modes free and stable, and achieves the extended strictly positive realness for all admissible uncertainties. A version of positive real lemma for the 2-D SRM is given in terms of linear matrix inequalities (LMIs). Based on the lemma, a sufficient condition for the solvability of the positive real control problem is derived in terms of bilinear matrix inequalities (BMIs) and an iterative procedure for solving the BMIs is proposed.

Keywords: 2-D singular systems, positive real control, positive realness, LMIs, state feedback.

1. INTRODUCTION

The concept of positive realness has played an important role in control and system theory [14-16]. In the past years, the problem of positive real control has received much attention. The objective is to design controllers such that the resulting closed-loop system is stable and the closed-loop transfer function is positive real [17]. The motivation for studying the positive real control problem stems from robust and nonlinear control, in which a well-known fact is that the positive realness of a certain loop transfer function will guarantee the overall stability of feedback systems if uncertainty or nonlinearity can be characterized by a positive real system [16]. Now, it is known that a solution to such a problem for a known linear time-invariant system involves solving a pair of Riccati inequalities [18]. When parameter uncertainty appears, the results in [18] were extended by [19,20], where observer-based controllers were designed and an LMI design method was developed. The corresponding results for discrete-time systems can be found in [21]. Recently, the positive real control problem has been extended to 1-D discrete-time singular systems [22].

On the other hand, 2-D singular systems have received much interest due to their extensive applications in many practical areas [1-11]. A great number of fundamental results on 1-D singular systems have been extended to 2-D singular systems [3-11]. Using the z-transformation approach, Kaczorek [1,2] studied the general response formula and minimum energy control problem for 2-D general descriptor models in both shift-invariant and varying coefficient cases. Karamanciogle *et al.* [3] extended the geometric method to the 2-D singular case. The admissibility of input of 2-D singular systems was investigated in [4] and some results were obtained. Zou and Campbell [5] proposed an asymptotic stability theory based on the concept of jump modes, which was further improved in [6]. It reveals that the existence of jump modes in such systems is one of the characterizations that regular systems do not have. Moreover, some results of structural stability of 1-D singular systems were also extended to 2-D singular systems in [7] and [8] by different approaches. For the problem of state observer design, Zou and Wang [9] extended the notion of detectability to 2-D singular systems, and a singular observer design approach was developed for a detectable singular system. The problem of regular state observers for 2-D singular systems was studied in [10]. Reference [12] discussed the problem of robust H_∞ control for linear discrete time 2-D singular Roesser models with time- invariant norm-bounded parameter uncertainty. However, the problem of positive real control for 2-D discrete-time singular systems has not been investigated in literature

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In this paper, we consider the problem of positive real control for uncertain 2-D singular Roesser models (2-D SRM). The parameter uncertainties are assumed to be time-invariant and norm-bounded. Attention is focused on the design of state feedback controllers such that, for all admissible uncertainties, the resulting closed-loop system is acceptable, jump modes free, and stable while the closed-loop transfer function matrix from the disturbance to the controlled output is *extended strictly positive real* (ESPR). A sufficient condition for 2-D SRM to achieve acceptability, causality, stability and ESPR property is proposed. Based on this, a sufficient condition on the existence of desired state feedback controllers is derived in terms of BMIs, which can be solved by an iterative procedure.

The paper is organized as follows: In Section 2, the problem formulation and some necessary preliminaries are presented. Section 3 presents a version of positive realness for 2-D SRM. We briefly summarize the paper in Section 4.

Notation: Throughout the paper, the superscripts 'T' and '*' stand for the transpose and complex conjugate transpose, respectively. The notation of $X \geq Y$ ($X \geq Y$), where X and Y are real symmetric matrices, represents that X-Y is a positive semi-definite (positive definite) matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a 2-D SRM of the following form.

$$E\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = (A+\Delta A)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bu(i,j) \quad (1)$$
$$+Ld(i,j),$$

$$z(i,j) = H\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Gd(i,j) \quad (2)$$

with the so-called standard quarter plane boundary conditions[25]:

$$x^h(0,j) = x_j^h,\ x^v(i,0) = x_i^v,\ i,j = 0,\ 1\ ,\ 2,\ldots \quad (3)$$

where $x^h(i,j) \in R^{n_1}$, $x^v(i,j) \in R^{n_2}$ are respectively the horizontal and vertical states; $u(i,j) \in R^m$, $z(i,j) \in R^l$ are the input and output vectors, respectively; and $d(i,j) \in R^p$ is the disturbance vector; A, B, G, L, H are real matrices of appropriate dimensions. ΔA is a time-invariant parameter uncertainty. E is possibly singular, satisfying the 2-D regular pencil condition, i.e., for some finite pair (z,w)

$$\det[EI(z,w) - A] = \sum_{k=0}^{\bar{n}_1}\sum_{l=0}^{\bar{n}_2} a_{kl} z^k w^l \neq 0,$$

where $I(z,w) = \operatorname{diag}\{zI_{n_1}, wI_{n_2}\}$. I_n is the identity of dimension $n \times n$. $0 \leq \bar{n}_k \leq n_k$. Such a condition guarantees that the 2-D SRM is uniquely solvable. For simplicity, we call system (1)-(2) with $\Delta A = 0$ the nominal system of (1)-(2). When $a_{\bar{n}_1,\bar{n}_2} \neq 0$, system (1)-(2) is called acceptable [1-5]. It is revealed in [5] that in the 2-D singular case, only acceptable systems can be considered as well posed in some sense. Hence from now on we only discuss the acceptable systems. In this paper, we assume that the parameter uncertainty ΔA is of the form

$$\Delta A = MFN, \quad (4)$$

where M and N are known real matrices, and $F \in R^{i \times j}$ is an unknown real matrix satisfying

$$FF^T \leq I. \quad (5)$$

The parameter uncertainty ΔA is said to be admissible if both (4) and (5) hold. Consider the following nominal unforced 2-D SRM of (1)-(2)

$$E\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Ld(i,j), \quad (6)$$

$$z(i,j) = H\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Gd(i,j). \quad (7)$$

When system (6)-(7) is regular, the transfer function of this system is as follows:

$$\Im(z,w) = H[EI(z,w) - A]^{-1} L + G. \quad (8)$$

Throughout this paper, we shall use the following concept of positive realness for the 2-D SRM, which is extended from the 1-D case [18].

Definition1:

(1) The 2-D SRM (6)-(7) is said to be positive real (PR) if its transfer function matrix $\Im(z,w)$ is analytic in $|z| > 1$, $|w| > 1$ and satisfies $\Im(z,w) + \Im^*(z,w) \geq 0$ for $|z| > 1$, $|w| > 1$.

(2) The 2-D SRM (6)-(7) is said to be strictly positive real (SPR) if its transfer function matrix $\Im(z,w)$ is analytic in $|z| \geq 1$, $|w| \geq 1$ and satisfies $\Im(e^{j\omega_1}, e^{j\omega_2}) + \Im^*(e^{j\omega_1}, e^{j\omega_2}) > 0$ for $\omega_1, \omega_2 \in [0,\ 2\pi)$.

(3) The 2-D SRM (6)-(7) is said to be extended strictly positive real (ESPR) if it is SPR and $\Im(\infty,\infty) + \Im(\infty,\infty)^T > 0$.

Now, the **positive real control problem** for 2-D

SRM (1)-(2) to be addressed in this paper can be formulated as follows: for a given uncertain 2-D SRM (1)-(2), find a state feedback

$$u(i,j)=Kx(i,j),\quad K\in R^{m\times n},\quad n=n_1+n_2 \tag{9}$$

such that, for all admissible uncertainties (4)-(5), the closed-loop system

$$E\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}=A_c\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}+Ld(i,j), \tag{10}$$

$$z(i,j)=H\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}+Gd(i,j) \tag{11}$$

is acceptable, internally stable, jump modes free and ESPR. Here $A_c=A+\Delta A+BK$ and $\Im(z,w)=H[EI(z,w)-A_c]^{-1}L+G$ is the transfer function matrix from the exogenous input $d(i,j)$ to the controlled output $z(i,j)$.

Remark 1: This jump mode-free requirement of system (6)-(7) can be defined equivalently by the nonexistence of nonzero positive power items ($z^i w^j$, $i>0$ or $j>0$) in the Laurent expansion of the matrix $(EI(z,w)-A)^{-1}$, $\infty>|z|\geq 1$, $\infty>|w|\geq 1$ [5]. We require that the closed loop system to be free of jump modes because the jump modes may amplify the disturbances dramatically [5]. It is also equivalent to requiring the closed loop system be a causal one.

We conclude this section by introducing several lemmas, which will be used in the proof of our main results.

Lemma 1 [5,6]**:** The 2-D SRM (6)-(7) is acceptable and internally stable if and only if

$$p(z,w)\neq 0,\ 0<|z|\leq 1,\ 0<|w|\leq 1, \tag{12}$$

where $p(z,w)=\det[E-AI(z,w)]$.

Lemma 2 [12]**:** The 2-D SRM (6)-(7) is acceptable, internally stable, and jump modes free if there exists a symmetric block-diagonal matrix $P=\text{diag}\{P_h,P_v\}\in R^{n\times n}$ with $P_h\in R^{n_1\times n_1}$ and $P_v\in R^{n_2\times n_2}$ such that

$$EPE^T\geq 0, \tag{13}$$

$$APA^T-EPE^T<0. \tag{14}$$

Moreover, if (14) holds, then P is nonsingular.

Lemma 3 [13]**:** Let T be a nonsingular symmetric matrix and F satisfy (5). If M,N are constant matrices of appropriate dimensions and there exists a constant $\varepsilon>0$ such that $\varepsilon\ I-M^TTM>0$, then

$$(A+MFN)T(A+MFN)^T$$
$$\leq ATA^T+ATN^T(\varepsilon I-NTN^T)^{-1}NTA^T+\varepsilon\ MM^T.$$

3. POSITIVE REALNESS OF 2-D SINGULAR SYSTEM

The following theorem gives a sufficient condition for the 2-D SRM (6)-(7) to be acceptable, internally stable, jump modes free and ESPR. This result will play a key role in solving the positive real control problem for the 2-D SRM in the next section.

Theorem 1: The 2-D SRM (6)-(7) is acceptable, internally stable, jump modes free, and ESPR, if there exists a symmetric block-diagonal matrix $P=\text{diag}\{P_h,P_v\}\in R^{n\times n}$ such that the following LMIs hold:

$$EPE^T\geq 0, \tag{15}$$

$$\begin{bmatrix} APA^T-EPE^T & L-APH^T \\ L^T-HPA^T & -(G+G^T-HPH^T) \end{bmatrix}<0, \tag{16}$$

where $P_h\in R^{n_1\times n_1}$ and $P_v\in R^{n_2\times n_2}$.

Proof: From (16), it is easy to see that

$$APA^T-EPE^T<0.$$

By Lemma 2, it follows that 2-D SRM (6)-(7) is acceptable, internally stable and jump modes free. This implies that $\Im(z,w)$ is analytic in $|z|\geq 1$, $|w|\geq 1$. Next, we will show that

$$\Im(e^{j\omega_1},e^{j\omega_2})+\Im^*(e^{j\omega_1},e^{j\omega_2})>0$$

for $\omega_1,\omega_2\in[0,2\pi)$. By the Schur complements, it follows from (16) that

$$G+G^T-HPH^T>0 \tag{17}$$

and

$$APA^T-EPE^T+(L-APH^T)(G+G^T-HPH^T)^{-1}(L^T-HPA^T)<0. \tag{18}$$

Denote

$$z=e^{j\omega_1},\ w=e^{j\omega_2},\ \omega_1,\omega_2\in[0,\ 2\pi),$$
$$\Pi(z,w)=EI(z,w)-A.$$

Now, partition A as

$$A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

with compatible dimensions to $I(z,w)$. Then some elementary algebraic manipulations show that for all $\omega_1,\omega_2\in[0,\ 2\pi)$

$$\Pi\ (z,w)P\Pi(z^{-1},w^{-1})^T+AP\Pi(z^{-1},w^{-1})^T$$
$$+\Pi(z,w)PA^T=-APA^T-EPE^T.$$

Note that the matrix $\Pi(z,w)$ is invertible for all $\omega_1,\omega_2 \in [0,\ 2\pi)$. Hence, it follows from the above equality that

$$HPH^T + H\Pi(z,w)^{-1}APH^T + HPA^T\Pi(z^{-1},w^{-1})^{-T}H^T = H\Pi(z,w)^{-1}(APA^T - EPE^T)\times\Pi(z^{-1},w^{-1})^{-T}H^T. \quad (19)$$

On the other hand, inequality (18) implies that there exists a matrix R>0 such that

$$R + (APA^T - EPE^T) + (L - APH^T) \times(G + G^T - HPH^T)^{-1}(L^T - HPA^T) < 0. \quad (20)$$

Pre- and post-multiplying (20) by $\Pi(z^{-1},\ w^{-1})^{-T}$ H^T and $H\Pi(z,w)^{-1}$ respectively, then we have that for all $\omega_1,\omega_2 \in [0,\ 2\pi)$,

$$H\Pi(z,w)^{-1}\ (A^TPA - E^TPE)\ \Pi(z^{-1},w^{-1})^{-T}H^T + H\Pi(z,w)^{-1}\ \Omega\ \Pi(z^{-1},w^{-1})^{-T}H^T \le 0, \quad (21)$$

where

$$\Omega = R + (L - APH^T)(G+G^T - HPH^T)^{-1}\times(L^T - HPA^T).$$

Now, substituting (19) into (21) gives that for all $\omega_1,\omega_2 \in [0,\ 2\pi)$,

$$-HPH^T - H\Pi(z,w)^{-1}APH^T - HPA^T\Pi(z^{-1},w^{-1})^{-T}H^T + H\Pi(z,w)^{-1}\ \Omega\ \Pi(z^{-1},w^{-1})^{-T}H^T \le 0.$$

Hence by the last inequality, we have that for all $\omega_1,\omega_2 \in [0,\ 2\pi)$,

$$\begin{aligned}
&\Im(z,w) + \Im^*(z,w) = G + G^T + H\Pi(z,w)^{-1}L + L^T\Pi(z^{-1},w^{-1})^{-T}H^T\\
&= (G + G^T - HPH^T) + H\Pi(z,w)^{-1}L + L^T\Pi(z^{-1},w^{-1})^{-T}H^T + HPH^T\\
&\ge (G + G^T - HPH^T) + H\Pi(z,w)^{-1}(L - APH^T) + (L^T - HPA^T)\ \Pi(z^{-1},w^{-1})^{-T}H^T\\
&\quad + H\Pi(z,w)^{-1}\ \Omega\ \Pi(z^{-1},w^{-1})^{-T}H^T\\
&= (G + G^T - HPH^T) - (L^T - HPA^T)\ \Omega^{-1}(L - APH^T)\\
&\quad + [\Pi(z^{-1},w^{-1})^{-T}H^T + \Omega^{-1}(L - APH^T)] \times\Omega\ [H\Pi(z,w)^{-1} + (L^T - HPA^T)\ \Omega^{-1}]\\
&\ge (G + G^T - HPH^T) - (L^T - HPA^T)\ \Omega^{-1}\times(L - APH^T).
\end{aligned} \quad (22)$$

Note that

$$\begin{bmatrix} G + G^T - HPH^T & L^T - HPA^T \\ L - APH^T & \Omega \end{bmatrix} > 0.$$

Then from the Schur complements, it follows that

$$(G + G^T - HPH^T) - (L^T - HPA^T)\ \Omega^{-1} \times(L - APH^T) > 0.$$

This together with (15) shows that for all $\omega_1,\omega_2 \in [0,\ 2\pi)$, $\Im(e^{j\omega_1},e^{j\omega_2}) + \Im^*(e^{j\omega_1},e^{j\omega_2}) > 0$. Hence 2-D SRM (6)-(7) is ESPR. This completes the proof.

Remark 2: Theorem 1 provides an LMI condition for the 2-D SRM (6)-(7) to be acceptable, internally stable, jump modes free, and ESPR. When the 2-D SRM (6)-(7) reduces to a 1-D singular system, it is easy to show that Theorem 1 coincides with Theorem 3 in [22]. Therefore, Theorem 1 in this paper can be regarded as an extension of existing results on positive realness for 1-D singular systems to 2-D singular systems described by the Roesser model.

4. POSITIVE REAL CONTROL OF UNCERTAIN 2-D SRM VIA STATE FEEDBACK

In this section, a sufficient condition for the solvability of the positive real control problem is proposed, and a BMI approach is developed to design state feedback controllers.

Theorem 2: Consider the uncertain 2-D SRM (1)-(2). If there exist scalars $\varepsilon > 0$, matrices $S \in R^{n\times n}$, $R_1 \in R^{n\times n}$, $R_2 \in R^{l\times n}$, $R_3 \in R^{k\times n}$ and symmetric block-diagonal matrix $P = diag\{P_h,\ P_v\} \in R^{n\times n}$ with $P_h \in R^{n_1\times n_1}$ and $P_v \in R^{n_2\times n_2}$ such that

$$EPE^T \ge 0, \quad (23)$$

$$\varepsilon I - NPN^T > 0, \quad (24)$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ \Psi_{12}^T & \Psi_{22} & \Psi_{23} & \Psi_{24} \\ \Psi_{13}^T & \Psi_{23}^T & \Psi_{33} & \Psi_{34} \\ \Psi_{14}^T & \Psi_{24}^T & \Psi_{34}^T & \Psi_{44} \end{bmatrix} < 0, \quad (25)$$

where

$$\Psi_{11} = -EPE^T + \varepsilon MM^T + R_1A_k^T + A_kR_1^T,$$

$$\Psi_{12} = L - R_1H^T + A_kR_2^T,\ \Psi_{13} = R_1N^T + A_kR_3^T,$$

$$\Psi_{14} = A_kS - R_1,\ \Psi_{22} = -HR_2^T - R_2H^T - (G + G^T),$$

$$\Psi_{23} = R_2N^T - HR_3^T,\ \Psi_{24} = -HS - R_2,$$

$$\Psi_{33} = R_3N^T + NR_3^T - \varepsilon I,\ \Psi_{34} = NS - R_3,$$

$\Psi_{44} = P - S - S^T$, $A_k = A + BK$,

then there exists a state feedback such that, for all admissible uncertainties ΔA defined as in (4)-(5), the closed loop system (10)-(11) is acceptable, internally stable, jump modes free, and ESPR.

Proof: Denote

$$W = \begin{bmatrix} -EPE^T + \varepsilon MM^T & L & 0 \\ L^T & -(G+G^T) & 0 \\ 0 & 0 & -\varepsilon I \end{bmatrix},$$

$$\Psi = \begin{bmatrix} A_k^T & -H^T & N^T \end{bmatrix}, \ R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}.$$

Then (25) can be written as

$$\begin{bmatrix} W + \Psi^T R^T + R\Psi & -R + \Psi^T S \\ -R^T + S^T \Psi & P - S - S^T \end{bmatrix} < 0.$$

Note that this is in turn equivalent to [26]

$$W + \Psi^T P \Psi < 0.$$

Let $\Theta = \begin{bmatrix} A_k \\ -H \end{bmatrix}$, $\Gamma = \begin{bmatrix} M \\ 0 \end{bmatrix}$,

$$Z = \begin{bmatrix} -EPE^T & L \\ L^T & -(G+G^T) \end{bmatrix}.$$

Then

$$\begin{aligned} & W + \Psi^T P \Psi \\ &= \begin{bmatrix} -EPE^T + \varepsilon MM^T & L & 0 \\ L^T & -(G+G^T) & 0 \\ 0 & 0 & -\varepsilon I \end{bmatrix} \\ &+ \begin{bmatrix} A_k \\ -H \\ N \end{bmatrix} P \begin{bmatrix} A_k^T & -H^T & N^T \end{bmatrix} \qquad (26) \\ &= \begin{bmatrix} Z + \Theta P \Theta^T + \varepsilon \Gamma \Gamma^T & \Theta P N^T \\ N P \Theta^T & -\varepsilon I + NPN^T \end{bmatrix} < 0. \end{aligned}$$

Using Schur complements, (26) is equivalent to

$$Z + \Theta P \Theta^T + \varepsilon \Gamma^T \Gamma + \Theta P N^T (\varepsilon I - NPN^T)^{-1} N P \Theta^T < 0. \qquad (27)$$

By Lemma 3, we have

$$\begin{aligned} & Z + \Theta P \Theta^T + \varepsilon \Gamma^T \Gamma + \Theta P N^T (\varepsilon I - NPN^T)^{-1} N P \Theta^T \\ & \geq Z + [\Theta + \Gamma F N] P [\Theta + \Gamma F N]^T \\ &= \begin{bmatrix} A_c P A_c^T - EPE^T & L - A_c P H^T \\ L^T - HPA_c^T & -(G + G^T - HPH^T) \end{bmatrix}. \end{aligned}$$

Therefore, the desired result follows immediately from (27) and Theorem 1. This completes the proof.

From Theorem 2, the following iterative algorithm can be implemented to solve the uncertain 2-D SRM positive real control problem.

Algorithm.

Step1: Choose an initial K, solve the following convex optimization problem:

$$\min_{(\varepsilon, P, S, R)} \{\mu\}.$$

Subject to:

$$\begin{bmatrix} W + \Psi^T R^T + R\Psi & -R + \Psi^T S \\ -R^T + S^T \Psi & P - S - S^T \end{bmatrix} < \mu I$$

and (23)-(24) are hold. If $\mu \leq 0$, then problem is solved; Otherwise, go to Step 2.

Step 2: With the obtained scalars ε and matrices P, S, R, solve the above optimization with respect to K. Again, if $\mu \leq 0$, the problem is solved; otherwise, go to Step 1.

Now, we present an illustration example to the proposed design approach in Theorem 2. Consider a 2-D SRM (1)-(2) with parameters:

$$E = \left[\begin{array}{c|c} 1 & 1 \\ \hline 0 & 0 \end{array}\right], \ A = \left[\begin{array}{c|c} 2 & 1 \\ \hline 0 & 1 \end{array}\right], \ B = \left[\begin{array}{c} 3 \\ \hline 2 \end{array}\right], \ L = \left[\begin{array}{cc} 1 & 0 \\ \hline 1 & 1 \end{array}\right],$$

$$G = \begin{bmatrix} 3 & 1 \\ 3 & 5 \end{bmatrix}, \ H = \left[\begin{array}{c|c} 1 & 0 \\ 0 & 1 \end{array}\right], \ M = [0.1 \ 0.1]^T, \ N = [0.1 \ 0.1].$$

It is easy to see by Remark 1 and Lemma 1 that this system is an acceptable, unstable system with jump-mode and not ESPR. By the above algorithm, the solution to BMIs (23)-(25) is as follows:

$$P = \begin{bmatrix} -1.2534 & 0 \\ 0 & 3.2377 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} -0.0055 & 0.0806 \\ 0.0806 & 0.1053 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.0449 & 0.2034 \\ 0.2034 & -2.0038 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 0.0001 & 0.0035 \end{bmatrix},$$

$$\varepsilon = 2.8157.$$

The corresponding state feedback controller can be obtained as

$$u(i,j) = \begin{bmatrix} -4.4 & -0.8 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}.$$

By direct computations, it can be verified that the above controller makes the closed loop system satisfies all the required performances.

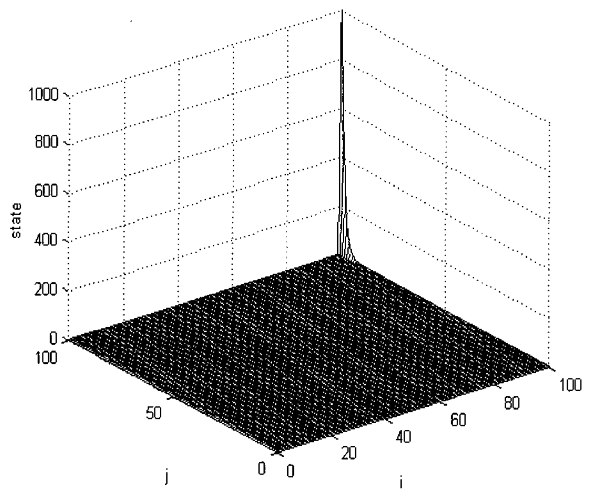

Fig. 1. The open-loop state response of x^h.

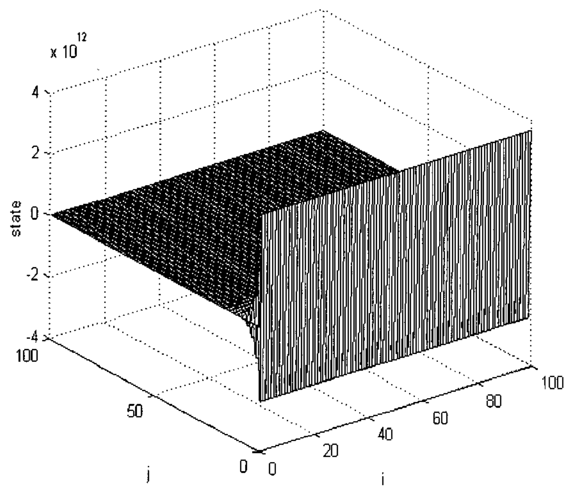

Fig. 2. The closed-loop state response of x^h.

A comparison of state responses of x^h between the open loop system and the closed-loop system are shown in Figs. 1 and 2, respectively. It can be observed that the closed-loop state response is stabilized by the controller designed via the BMI approach to the positive real control of uncertain 2-D singular systems in Section 3 while the open loop system is unstable. The state responses of x^v is similar and thus omitted.

5. CONCLUSIONS

This paper has studied the positive real control problem for uncertain 2-D singular systems described by the Roesser model with time-invariant norm-bounded parameter uncertainties. The proposed feedback law not only makes the corresponding closed-loop acceptable, jump modes free, stable, but also guarantees that the closed-loop transfer function from the disturbance to the controlled output is *extended strictly positive real.*

REFERENCES

[1] T. Kaczorek, "The singular general model of 2-D systems and its solutions," *IEEE Trans. on Automatic Control*, vol. 33, no. 11, pp. 1060-1061, 1988.

[2] T. Kaczorek, "General response formula and minimums energy control for the general singular model for 2-D systems," *IEEE Trans. on Automatic Control*, vol. 35, no. 4, pp. 433-436, 1990.

[3] A. V. Karamanciogle and F. R. Lewis, "Geometric theory for singular Roesser model," *IEEE Trans. on Automatic Control*, vol. 37, no. 6 pp. 801-806, 1992.

[4] T. Kaczorek, "Acceptable input sequences for singular 2-D linear systems," *IEEE Trans. on Automatic Control*, vol. 38, no. 9, pp. 1391-1394 1993.

[5] Y. Zou and S. L. Campbell, "The jump behavior and stability analysis for 2-D singular systems," *Multidimensional Systems & Signal Processing*, vol. 11, pp. 321-338, 2000.

[6] C. Cai, W. Wang, and Y. Zou. "A note on the internal stability for 2-D acceptable linear singular discrete systems," Accepted by *Multidimensional Systems & Signal Processing.*

[7] Y. Zou, W. Wang, and S. Xu, "Structural stability of 2-D singular systems-Part I: basic properties," *Proc. of the Fourth International Conference on Control and Automation*, Montreal, Canada, June, 2003.

[8] Y. Zou, W. Wang, and S. Xu, "Structural stability of 2-D singular systems Part II: a lyapunov approach," *Proc. of the Fourth International Conference on Control and Automation*, Montreal, Canada, June, 2003.

[9] W. Wang and Y. Zou, "The detectability and observer design of 2-D singular systems," *IEEE Trans. on Circuits & Systems*, vol. 49, no. 5, pp. 698-703, 2002.

[10] Y. Zou, W. Wang, and S. Xu, "Regular state observers design for 2-D singular Roesser models," *Proc. of the Fourth International Conference on Control and Automation,* Montreal, Canada, June, 2003.

[11] T. Kaczorek, "The linear-quadratic optimal regulator for singular 2-D systems with variable coefficients," *IEEE Trans. on Automatic Control*, vol. 34, no. 5, pp. 565-566, 1989.

[12] H. Xu, Y. Zou, S. Xu, and J. Lam, "Bounded real lemma and robust H_∞ control of 2-D singular Roesser models," Submitted to *Systems & Control Letters.*

[13] S. Xu, J. Lam, and C. Yang, "Robust H_∞ control of uncertain discrete singular systems with pole placement in a disk," *Systems & Control Letters*, vol. 43, pp. 85-93, 2001.

[14] B. D. O. Anderson and S. Vongpanitlerd, *Network Analysis and Synthesis: A Modern System Theory Approach,* Prentice-Hall, Upper Saddle River, NJ, 1973.
[15] W. M. Haddad and D. S. Bernstein, "Robust stabilization with positive real uncertainty: beyond the small gain theorem," *Systems & Control Letters,* vol. 17, pp. 191-208, 1991.
[16] M. Vidyasagar, *Nonlinear Systems Analysis.* Prentice-Hall, Englewood Cliffs, NJ, 1993.
[17] P. Molander and J. C. Willems, "Synthesis of state feedback control laws with a specified gain and phase margin," *IEEE Trans. on Automatic Control*, vol. 25, no. 5, pp. 928-931, 1980.
[18] W. Sun, P. P. Khargonekar, and D. Shim, "Solution to the positive real control problem for linear time-invariant systems," *IEEE Trans. on Automatic Control*, vol. 39, no. 10, pp. 2034-2046, 1994.
[19] M. S. Mahmoud, Y. C. Soh, and L. Xie, "Observer-based positive real control of uncertain linear systems," *Automatica,* vol. 35, pp. 749-754, 1999.
[20] W. M. Haddad and D. S. Bernstein, "Explicit construction of quadratic Lyapunov functions for the small gain, positivity, circle, and Popov theorems and their application to robust stability. part II: discrete-time theory," *International Journal of Robust Nonlinear Control*, vol. 4, pp. 249-265, 1994.
[21] M. S. Mahmoud and L. Xie, "Positive real analysis and synthesis of uncertain discrete time systems," *IEEE Trans. on Circuits & Systems*, vol. 47, no. 3, pp. 403-406, 2000.
[22] S. Xu and J. Lam, "Positive real control of uncertain singular systems with state delay," *Dynamics of Continuous, Discrete and Impulsive Systems, Series B: Applications and Algorithms,* vol. 9, pp. 387-402, 2002.
[23] L. Zhang, J. Lam, and S. Xu, "On positive realness of descriptor systems," *IEEE Trans. on Circuits & Systems-I: Fundamental Theory and Applications*, vol. 49, no. 3, pp. 401-407, 2002.
[24] C. Du and L. Xie, *H_∞ Control and Filtering of Two-dimensional Systems,* Springer-Verlag, Berlin, 2002.
[25] T. Kaczorek, *Two-Dimension Linear Systems*, Spring-Verlag, New York ,1985.
[26] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou, "A new robust *D*-stability condition for real convex polytopic uncertainty," *Systems & Control Letters,* vol. 40, pp. 21-30, 2000.

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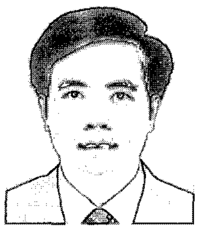

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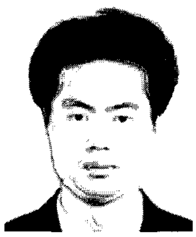

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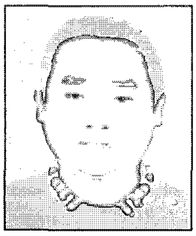

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