A New Excitation Control for Multimachine Power Systems I: Decentralized Nonlinear Adaptive Control Design and Stability Analysis

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Abstract: In this paper a new excitation control scheme that improves the transient stability of multimachine power systems is proposed. To this end the backstepping technique is used to transform the system to a suitable partially linear form. On this system, a combination of both feedback linearization and adaptive control techniques are used to confront the nonlinearities. As shown in the paper, the resulting nonlinear control law ensures the uniform boundedness of all the state and estimated variables. Furthermore, it is proven that all the error variables are uniformly ultimately bounded (UUB) i.e. they converge to arbitrarily selected small regions around zero in finite-time. Simulation tests on a two generator infinite bus power system demonstrate the effectiveness of the proposed control.

Keywords: Multimachine power system control, adaptive control, backstepping design, decentralized control.

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1. INTRODUCTION

Power systems are continuously growing in size and complexity with increasing interconnections. They consist of several generating units while the power demands vary incessantly. Additionally, small or large disturbances such as power changes or short-circuits (faults) may transpire. One of the most crucial operation demands is the maintenance of system stability. In particular, when a fault occurs, large currents and torques are produced and control action must be taken promptly if system stability is to be sustained. This is an imperative solution to the power system transient stability problem defined as that of assessing whether or not the system will reach an acceptable steady-state following the fault. However, power systems are large scale highly nonlinear systems that include a number of synchronous machines as producers. One of the main goals of the excitation control of each machine is the enhancement of power system stability.

Conventional excitation controllers are mainly designed by using linear control theory. Especially for the case of a single machine to infinite-bus power system, a method that has been extensively used is one based on the linearization around an operating point and the design of linear excitation controllers [1,2]. The main disadvantages of this design such as lack of reliability and robustness are well-known.

In the last decade, nonlinear control theory has also been widely used to account for the nonlinearities of the controlled power systems [3,4]. The majority of these controllers are based on the feedback linearization technique [5]. Feedback linearization is recently enhanced by using robust control designs such as H_∞ control and L_2 disturbance attenuation [6-12]. In recent years, new approaches have been proposed for power stability designs according to other sophisticated schemes such as fuzzy logic control [13], adaptive control [14,15] and neuro-control [16-19]. Combinations of the above techniques are also proposed [20-22] in order to exploit the advantages of each method under the cost of the increase in complexity.

In this paper, we consider a multimachine power system wherein each machine is represented by its third order nonlinear dynamic model and the transmission net is described by the admittance matrix. On this model the well-known backstepping technique [23,24] is used in order to obtain the most possible partially linear form of the system. On this form we use the most simplified feedback linearization scheme in order to obtain a local feedback control law i.e., a control law that is dependent from the local measurable states of the system and a local measurable variable. Simultaneously, all the other nonlinearities that are dependent from locally immeasurable variables or variables that are not states are left on an unknown nonlinear term. An adaptive

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control mechanism is then used to estimate suitable bounds of this unknown nonlinear term. Finally, we propose a nonlinear feedback controller (Theorem 1) based on both the effective adaptive operation and the suitable selection of some design constants.

By this control design we prove that the third error variable is driven in finite-time in a neighborhood of the origin of arbitrary small dimensions. As soon as this happens, the other two error variables insert in finite-time in a sphere around the origin of arbitrarily small radius (Theorem 2). The boundedness of all signals is proved. The adaptation mechanism used belongs to the class of direct adaptive algorithms in the sense that it guarantees the uniform ultimate boundness (UUB) of the error variables while the estimated parameter errors remain bounded. Furthermore, as ascertained (Theorem 3), the power angle deviations converge to an even smaller region as time passes. This is crucial for the selection of the design constants since it leads to significantly lesser values for the control gains. Simulation results after a symmetrical three-phase short circuit fault on a two machine-infinite bus test system demonstrate the effectiveness of the proposed scheme.

2. DYNAMIC MODEL

After reducing the multimachine power system into a network with generator nodes only, the classical third-order single-axis dynamic generator model is used for the design of the excitation controller, whereas differential equations that represent dynamics with very short time constants have been neglected. In general, for a n-generator power system, the dynamic model of the i-th generator is

$$\dot{\delta}_i(t) = \omega_i(t) - \omega_0, \tag{1}$$

$$\dot{\omega}_i(t) = -\frac{D_i}{M_i}(\omega_i(t) - \omega_0) + \frac{\omega_0}{M_i}(P_{mi} - P_{ei}(t)), \tag{2}$$

$$\dot{E}'_{qi}(t) = \frac{1}{T'_{d0i}}(E_{fi}(t) - E_{qi}(t)), \tag{3}$$

where

$$E_{qi}(t) = E'_{qi}(t) + (x_{di} - x'_{di})I_{di}(t), \tag{4}$$

$$E_{fi}(t) = k_{ci}u_{fi}(t), \tag{5}$$

$$I_{qi}(t) = \sum_{j=1}^{n} E'_{qj}(B_{ij}\sin\delta_{ij}(t) + G_{ij}\cos\delta_{ij}(t)), \tag{6}$$

$$I_{di}(t) = \sum_{j=1}^{n} E'_{qj}(G_{ij}\sin\delta_{ij}(t) - B_{ij}\cos\delta_{ij}(t)), \tag{7}$$

$$P_{ei}(t) = E'_{qi}(t)I_{qi}(t), \tag{8}$$

$$Q_{ei} = E'_{qi}I_{di}(t), \tag{9}$$

$$E_{qi}(t) = x_{adi}I_{fi}(t), \tag{10}$$

$$V_{tqi}(t) = E'_{qi}(t) - x'_{di}I_{di}(t), \tag{11}$$

$$V_{tdi}(t) = x'_{di}I_{qi}(t), \tag{12}$$

$$V_{ti}(t) = \sqrt{V_{tqi}^2(t) + V_{tdi}^2(t)}. \tag{13}$$

The symbols used in the above equations are explained in the Appendix.

3. BACKSTEPPING DESIGN

Introducing the first error variable for the i-th machine

$$z_{i1} = \Delta\delta_i \tag{14}$$

and viewing $\Delta\omega_i$ as a virtual control, we define the second error variable

$$z_{i2} = \Delta\omega_i - \alpha_{i1}(\Delta\delta_i), \tag{15}$$

where α_{i1} is a function to be designed.

Consider the first candidate Lyapunov function

$$V_1 = \frac{1}{2}\sum_{i=1}^{n} z_{i1}^2. \tag{16}$$

Then, selecting

$$\alpha_{i1}(\Delta\delta_i) = -c_{i1}z_{i1} = -c_{i1}\Delta\delta_i, \tag{17}$$

where $c_{i1} > 0$ is a constant that can be suitably selected and taking into account that

$$\dot{z}_{i1} = \Delta\omega_i, \tag{18}$$

we have

$$\dot{V}_1 = -\sum_{i=1}^{n} c_{i1}z_{i1}^2 + \sum_{i=1}^{n} z_{i1}z_{i2}. \tag{19}$$

For the second error variable the dynamics are

$$\dot{z}_{i2} = -\frac{D_i}{M_i}\Delta\omega_i - \frac{\omega_0}{M_i}\Delta P_{ei} - \frac{\partial\alpha_{i1}}{\partial\Delta\delta_i}\Delta\omega_i. \tag{20}$$

Viewing ΔP_{ei} as a virtual control we introduce the third error variable

$$z_{i3} = \Delta P_{ei} - \alpha_{i2}(\Delta\delta_i, \Delta\omega_i), \tag{21}$$

where α_{i2} is a function to be designed.

For the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{n} z_{i2}^2, \tag{22}$$

we have

$$\dot{V}_2 = -\sum_{i=1}^{n} c_{i1}z_{i1}^2 - \sum_{i=1}^{n}\frac{\omega_0}{M_i}z_{i2}z_{i3} + \sum_{i=1}^{n} z_{i2}\left[z_{i1} - \left(\frac{D_i}{M_i} + \frac{\partial\alpha_{i1}}{\partial\Delta\delta_i}\right)\Delta\omega_i - \frac{\omega_0}{M_i}\alpha_{i2}\right]. \tag{23}$$

If one defines

$$\alpha_{i2}\left(\Delta\delta_i,\Delta\omega_i\right)=\frac{M_i}{\omega_0}\left[z_{i1}+c_{i2}z_{i2}-\left(\frac{D_i}{M_i}+\frac{\partial\alpha_{i1}}{\partial\Delta\delta_i}\right)\Delta\omega_i\right].$$

or equivalently in terms of $\Delta\delta_i$ and $\Delta\omega_i$ as

$$\alpha_{i2}\left(\Delta\delta_i,\Delta\omega_i\right)=\frac{M_i}{\omega_0}\left(1+c_{i1}c_{i2}\right)\Delta\delta_i+\frac{M_i}{\omega_0}\left(c_{i1}+c_{i2}-\frac{D_i}{M_i}\right)\Delta\omega_i, \tag{24}$$

where $c_{i2}>0$ is a constant that can be arbitrarily selected, then

$$\dot{V}_2=-\sum_{i=1}^{n}c_{i1}z_{i1}^2-\sum_{i=1}^{n}c_{i2}z_{i2}^2-\sum_{i=1}^{n}\frac{\omega_0}{M_i}z_{i2}z_{i3} \tag{25}$$

and therefore (24) leads to

$$\frac{\partial\alpha_{i2}}{\partial\Delta\delta_i}=\frac{M_i}{\omega_0}\left(1+c_{i1}c_{i2}\right),\quad \frac{\partial\alpha_{i2}}{\partial\Delta\omega_i}=\frac{M_i}{\omega_0}\left(c_{i1}+c_{i2}-\frac{D_i}{M_i}\right). \tag{26}$$

For the third error variable it holds true that

$$\dot{z}_{i3}=\tilde{f}_i(t)+\frac{1}{T'_{d0i}}I_{qi}u_{fi}(t)-\frac{\partial\alpha_{i2}}{\partial\Delta\delta_i}\Delta\omega_i-\frac{\partial\alpha_{i2}}{\partial\Delta\omega_i}\left[-\frac{D_i}{M_i}\Delta\omega_i-\frac{\omega_0}{M_i}\Delta P_{ei}\right], \tag{27}$$

where

$$\tilde{f}_i(t)=E'_{qi}(t)\dot{I}_{qi}(t)-\frac{1}{T'_{d0i}}\left[E'_{qi}(t)+\left(x_{di}-x'_{di}\right)I_{di}(t)\right]I_{qi}(t)$$

is obviously a complex nonlinear function. However, for decentralized control purposes this is an unknown function since it cannot be reconstructed from the local i-th machine's variables. Considering now, the Lyapunov function

$$V_3=V_2+\frac{1}{2}\sum_{i=1}^{n}z_{i3}^2, \tag{28}$$

its time derivative results in

$$\dot{V}_3=-\sum_{i=1}^{n}c_{i1}z_{i1}^2-\sum_{i=1}^{n}c_{i2}z_{i2}^2+\sum_{i=1}^{n}z_{i3}\Big[\tilde{f}_i(t)-\frac{\partial\alpha_{i2}}{\partial\Delta\delta_i}\Delta\omega_i+\frac{1}{T'_{d0i}}I_{qi}u_{fi}(t)-\frac{\omega_0}{M_i}z_{i2}+\frac{\partial\alpha_{i2}}{\partial\Delta\omega_i}\left(\frac{D_i}{M_i}\Delta\omega_i+\frac{\omega_0}{M_i}\Delta P_{ei}\right)\Big]. \tag{29}$$

Using on (27), the feedback linearization technique, we select the input

$$u_{fi}(t)=\frac{T'_{d0i}}{I_{qi}}\left[-c_{i3}z_{i3}+\frac{\partial\alpha_{i2}}{\partial\Delta\delta_i}\Delta\omega_i+v_i-\frac{\partial\alpha_{i2}}{\partial\Delta\omega_i}\left(\frac{D_i}{M_i}\Delta\omega_i+\frac{\omega_0}{M_i}\Delta P_{ei}\right)\right]. \tag{30}$$

where $c_{i3}>0$ is a constant that can be suitably chosen.

4. CONTROLLER DESIGN AND STABILITY ANALYSIS

The proposed excitation control law of each machine, for $k_{ci}=1$ as it is resulted from (30) is

$$E_{fi}(t)=\frac{T'_{d0i}}{I_{qi}}\left(k_{i1}\Delta\delta_i+k_{i2}\Delta\omega_i-k_{i3}\Delta P_{ei}+v_i\right), \tag{31}$$

where $v_i=v_i(t)$ is an arbitrary external input and the constant gains are given by

$$k_{i1}=\frac{M_i}{\omega_0}c_{i3}\left(1+c_{i1}c_{i2}\right), \tag{32}$$

$$k_{i2}=\frac{M_i}{\omega_0}\left[\left(c_{i3}-\frac{D_i}{M_i}\right)\left(c_{i1}+c_{i2}-\frac{D_i}{M_i}\right)+c_{i1}c_{i2}+1\right], \tag{33}$$

$$k_{i3}=c_{i1}+c_{i2}+c_{i3}-\frac{D_i}{M_i}. \tag{34}$$

For this control law, (29) becomes

$$\dot{V}_3=-\sum_{i=1}^{n}c_{i1}z_{i1}^2-\sum_{i=1}^{n}c_{i2}z_{i2}^2-\sum_{i=1}^{n}c_{i3}z_{i3}^2+\sum_{i=1}^{n}z_{i3}\left[\tilde{f}_i(t)-\frac{\omega_0}{M_i}z_{i2}+v_i(t)\right]. \tag{35}$$

and the z_{i3}-dynamics are as follows

$$\dot{z}_{i3}=-c_{i3}z_{i3}+\tilde{f}_i(t)+v_i(t). \tag{36}$$

Let the function

$$f_{i1}(t)=\tilde{f}_i(t)-\frac{\omega_0}{M_i}z_{i2}=E'_{qi}(t)\dot{I}_{qi}(t)-\frac{1}{T'_{d0i}}P_{ei}-\frac{x_{di}-x'_{di}}{T'_{d0i}}I_{di}(t)I_{qi}(t)-\frac{\omega_0}{M_i}z_{i2}. \tag{37}$$

Since the electrical power P_{ei} and the reactive power Q_{ei} of each generator as well as the electrical power flow through its transmission line are all bounded and given that the excitation voltage E_{fi} may raise up to 5 times of the E_{qi} when there is no load in the system, we can easily establish that there are large enough unknown positive constants $\sigma_i, \xi_{i1}, \xi_{i2}, \xi_{i3}$ such that

$$\left|f_{i1}(t)\right| \le \sigma_i + \sum_{j=1}^{3} \xi_{ij}\left|z_{ij}\right|, \quad i = 1,2,\ldots,n. \tag{38}$$

Now using adaptive control techniques we derive estimates for these constants that can be effectively used in the design of the control law $v_i(t)$ in such a way that the third error variable z_{i3} converges in finite-time in an arbitrary small neighborhood of the origin while all signals remain bounded. Specifically we will prove the following theorem.

Theorem 1: For the n-machine system described by equations (1)-(13) under the assumption of (38), let the excitation input be chosen as in (31) with gains given by (32)-(34) and v_i given by the nonlinear expression

$$v_i = -\frac{\rho_i^2 z_{i3}}{\rho_i\left|z_{i3}\right| + l_i}, \tag{39}$$

where l_i is a small positive scalar and

$$\rho_i = \left|c_{i3} - \lambda_i\right|\left|z_{i3}\right| + \hat{\sigma}_i(t) + \sum_{j=1}^{3} \hat{\xi}_{ij}(t)\left|z_{ij}\right| \tag{40}$$

with λ_i a constant related to the adaptive control design through the following estimates' update laws

$$\dot{\hat{\sigma}}_i(t) = \begin{cases} \alpha_i\left|z_{i3}\right|, & \text{if} \quad \left|z_{i3}\right| > \sqrt{\dfrac{2l_i}{\lambda_i}}, \\ 0, & \text{otherwise} \end{cases} \tag{41}$$

$$\alpha_i > 0 \quad i = 1,2,\ldots,n \quad \text{with} \quad \hat{\sigma}_i(t_0) \ge 0$$

$$\dot{\hat{\xi}}_{ij}(t) = \begin{cases} \gamma_{ij}\left|z_{i3}z_{ij}\right|, & \text{if} \quad \left|z_{i3}\right| > \sqrt{\dfrac{2l_i}{\lambda_i}}, \\ 0, & \text{otherwise} \end{cases} \tag{42}$$

$$\gamma_{ij} > 0 \quad j = 1,2,3 \quad i = 1,2,\ldots,n, \hat{\xi}_{ij}(t_0) \ge 0.$$

Then, there exists $T_i > t_0$ such that

$$\left|z_{i3}(t)\right| \le \sqrt{\frac{2l_i}{\lambda_i}} \quad \forall t \ge T_i, \quad i = 1,2,\ldots,n \tag{43}$$

and the other two error variables $z_{i1}(t), z_{i2}(t)$ are uniformly bounded for $t \in [t_0, T_i]$ and the signals $\hat{\sigma}_i(t)$, $\hat{\xi}_{i1}(t)$, $\hat{\xi}_{i2}(t)$, $\hat{\xi}_{i3}(t)$ are all uniformly bounded for $t \ge t_0$.

Proof: For the i-th subsystem consider the nonnegative function

$$V_i = \frac{z_{i1}^2}{2} + \frac{z_{i2}^2}{2} + \frac{z_{i3}^2}{2} + \frac{\left[\hat{\sigma}_i(t) - \sigma_i\right]^2}{2\alpha_i} + \sum_{j=1}^{3} \frac{\left[\hat{\xi}_{ij}(t) - \xi_{ij}\right]^2}{2\gamma_{ij}}. \tag{44}$$

For an excitation input given by eqs. (31)-(34) and update laws given by (41)-(42), the time derivative of V_i for the case wherein

$$\left|z_{i3}\right| > \sqrt{\frac{2l_i}{\lambda_i}} \tag{45}$$

is as follows

$$\begin{aligned} \dot{V}_i &= -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 - c_{i3}z_{i3}^2 + z_{i3}\left[f_{i1}(t) + v_i(t)\right] \\ &\quad + \left[\hat{\sigma}_i(t) - \sigma_i\right]\left|z_{i3}\right| + \sum_{j=1}^{3}\left[\hat{\xi}_{ij}(t) - \xi_{ij}\right]\left|z_{i3}z_{ij}\right|. \end{aligned} \tag{46}$$

Therefore, the following inequality results

$$\begin{aligned} \dot{V}_i &\le -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 - \lambda_i z_{i3}^2 \\ &\quad + \left|z_{i3}\right|\left[\left|f_{i1}(t)\right| - \sigma_i - \sum_{j=1}^{3}\xi_{ij}\left|z_{ij}\right|\right] + z_{i3}v_i(t) \\ &\quad + \left|z_{i3}\right|\underbrace{\left[\left|\lambda_i - c_{i3}\right|\left|z_{i3}\right| + \hat{\sigma}_i(t) + \sum_{j=1}^{3}\hat{\xi}_{ij}(t)\left|z_{ij}\right|\right]}_{\rho_i}. \end{aligned} \tag{47}$$

From (38) one can easily see that the fourth term on the right-hand side of (47) is always nonpositive and therefore (47) provides the following simplified inequality

$$\dot{V}_i \le -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 - \lambda_i z_{i3}^2 + z_{i3}v_i(t) + \left|z_{i3}\right|\rho_i. \tag{48}$$

Selecting $v_i(t)$ as in (39) we arrive at

$$\begin{aligned} \dot{V}_i &\le -\lambda_i z_{i3}^2 - \frac{\rho_i^2 z_{i3}^2}{\rho_i\left|z_{i3}\right| + l_i} + \left|z_{i3}\right|\rho_i \\ &= -\lambda_i z_{i3}^2 + \frac{l_i\rho_i\left|z_{i3}\right|}{\rho_i\left|z_{i3}\right| + l_i}. \end{aligned} \tag{49}$$

However, since from (45) we have $-\lambda_i z_{i3}^2 < -2l_i$ we result in

$$\dot{V}_i \le -2l_i + \frac{l_i\rho_i\left|z_{i3}\right| + l_i^2}{\rho_i\left|z_{i3}\right| + l_i} = -2l_i + l_i = -l_i. \tag{50}$$

From (50) it is clear that there exists $T_i \le t_0 + V_i(t_0)/l_i$ such that for every $t \ge T_i$ it holds true that $\left|z_{i3}(t)\right| \le \sqrt{2l_i/\lambda_i}$. The uniform boundedness of $z_{i1}(t), z_{i2}(t), z_{i3}(t), \hat{\sigma}_i(t), \hat{\xi}_{ij}(t)$, $j = 1,2,3$ in the time interval $[t_0, T_i]$ is obvious from the definition of

V_i in (44). From (41)-(42), it can be seen that for $t > T_i$ we have that $\hat{\sigma}_i(t) = \hat{\sigma}_i(T_i)(i = 1,2,\ldots,n)$ and $\hat{\xi}_{ij}(t) = \hat{\xi}_{ij}(T_i)$ $(j = 1,2,3,\ i = 1,2,\ldots,n)$. Hence, the estimates $\hat{\sigma}_i(t), \hat{\xi}_{ij}(t)$ are uniformly bounded for all $t \geq t_0$. □

Next, we demonstrate that after the third error variable enters into the chosen neighborhood, the other two error variables will also enter in finite-time in a sphere of arbitrary small radius. Actually, the radius of the sphere is directly related to the length of the interval, which ultimately bounds the third error variable. Thus, we prove the following Theorem.

Theorem 2: For the n-machine system defined by (1)-(13) and the excitation input given by (31)-(34) and (39)-(42) there exists $T_{i1} \geq T_i, (i = 1,2,\ldots,n)$ such that for every $t \geq T_{i1}$ the z_{i1}, z_{i2} error variables lie inside the sphere

$$S = \left\{ (z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \leq \left(\frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i \varepsilon_i c_{i2} \lambda_i}} \right)^2 \right\} \tag{51}$$

with center the origin and radius

$$R_{i1} := \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{\min\{c_{i1}, (1-\varepsilon_i)c_{i2}\} \varepsilon_i c_{i2} \lambda_i}}, \tag{52}$$

where ε_i is a design constant with $0 < \varepsilon_i < 1$.

Proof: Let the nonnegative function

$$V_{i1} = \frac{z_{i1}^2}{2} + \frac{z_{i2}^2}{2}. \tag{53}$$

The dynamics for the z_{i1}, z_{i2} variables are given by

$$\begin{cases} \dot{z}_{i1} = z_{i2} - c_{i1} z_{i1} \\ \dot{z}_{i2} = -z_{i1} - c_{i2} z_{i2} - \dfrac{\omega_0}{M_i} z_{i3} \end{cases} \tag{54}$$

and the time derivative of V_{i1} for $t \geq T_i$ is

$$\begin{aligned} \dot{V}_{i1} &= -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 - \frac{\omega_0}{M_i} z_{i2} z_{i3} \\ &\leq -c_{i1} z_{i1}^2 - c_{i2} z_{i2}^2 + \frac{\omega_0}{M_i} \sqrt{\frac{2l_i}{\lambda_i}} |z_{i2}| \\ &\leq -c_{i1} z_{i1}^2 - c_{i2}(1-\varepsilon_i) z_{i2}^2 \\ &\quad -\varepsilon_i c_{i2} \left(|z_{i2}| - \frac{\omega_0}{2\varepsilon_i c_{i2} M_i} \sqrt{\frac{2l_i}{\lambda_i}} \right)^2 + \frac{l_i \omega_0^2}{2\varepsilon_i c_{i2} \lambda_i M_i^2}. \end{aligned} \tag{55}$$

Defining $m_i = \min\{c_{i1}, c_{i2}(1-\varepsilon_i)\}$ we have that

$$\begin{aligned} \dot{V}_{i1} &\leq -m_i \left(z_{i1}^2 + z_{i2}^2 \right) + \frac{l_i \omega_0^2}{2\varepsilon_i c_{i2} \lambda_i M_i^2} \\ &= -2m_i V_{i1} + \frac{l_i \omega_0^2}{2\varepsilon_i c_{i2} \lambda_i M_i^2}, \end{aligned} \tag{56}$$

$$\dot{V}_{i1} \leq -2m_i \left(V_{i1} - \frac{l_i \omega_0^2}{4 m_i \varepsilon_i c_{i2} \lambda_i M_i^2} \right). \tag{57}$$

Using the comparison principle [25] we have

$$\begin{aligned} &V_{i1}(t) - \frac{l_i \omega_0^2}{4 m_i \varepsilon_i c_{i2} \lambda_i M_i^2} \\ &\quad \leq \left[V_{i1}(T_i) - \frac{l_i \omega_0^2}{4 m_i \varepsilon_i c_{i2} \lambda_i M_i^2} \right] e^{-2m_i(t-T_i)}, \end{aligned} \tag{58}$$

$$V_{i1}(t) \leq V_{i1}(T_i) e^{-2m_i(t-T_i)} + \frac{l_i \omega_0^2}{4 m_i \varepsilon_i c_{i2} \lambda_i M_i^2}. \tag{59}$$

Therefore, there exists a T_{i1}

$$T_{i1} = \max\left\{ T_i, T_i + \frac{1}{m_i} \ln \left[\frac{V_{i1}^{1/2}(T_i)}{\dfrac{\omega_0}{2M_i} \sqrt{\dfrac{l_i}{m_i \varepsilon_i c_{i2} \lambda_i}}} \right] \right\}, \tag{60}$$

so that for $t \geq T_{i1}$ it holds true that

$$V_{i1}(t) \leq \frac{l_i \omega_0^2}{2 m_i \varepsilon_i c_{i2} \lambda_i M_i^2}, \tag{61}$$

i.e. the error variables z_{i1}, z_{i2} enter in finite-time inside the sphere

$$S = \left\{ (z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \leq \left(\frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i \varepsilon_i c_{i2} \lambda_i}} \right)^2 \right\}$$

with the center as the origin and radius

$$R_{i1} = \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{\min\{c_{i1}, (1-\varepsilon_i)c_{i2}\} \varepsilon_i c_{i2} \lambda_i}}.$$ □

Apparently, Theorem 2 directly gives a bound for $\Delta\delta_i = z_{i1}$

$$|\Delta\delta_i(t)| \leq \frac{\omega_0}{M_i} \sqrt{\frac{l_i}{m_i \varepsilon_i c_{i2} \lambda_i}} \quad \forall t \geq T_i. \tag{62}$$

However, as $t \to \infty$, Theorem 2 results in a tighter bound for $\lim_{t\to\infty} |\Delta\delta_i(t)|$ as shown by the following Corollary.

Corollary 1: For the n-machine system defined by (1)-(13) and the excitation input given by (31)-(34) and (39)-(42) the following bounds on the angle deviations hold true

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le R_{i2} := \frac{\omega_0}{M_i c_{i1}}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}}. \quad (63)$$

Proof: Since $z_{i2} = \Delta\dot{\delta}_i + c_{i1}\Delta\delta_i$ Theorem 2 gives

$$|z_{i2}(t)| \le \frac{\omega_0}{M_i}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}} \quad \forall t \ge T_i. \quad (64)$$

Therefore

$$\Delta\dot{\delta}_i(t) + c_{i1}\Delta\delta_i(t) \le \frac{\omega_0}{M_i}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}} \quad \forall t \ge T_i$$

and (65)

$$\Delta\dot{\delta}_i(t) + c_{i1}\Delta\delta_i(t) \ge -\frac{\omega_0}{M_i}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}} \quad \forall t \ge T_i.$$

Now using Lemma Growan-Bellman [26] we arrive at

$$\Delta\delta_i(T_i)e^{-c_{i1}(t-T_i)} - \frac{\omega_0}{M_i c_{i1}}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}}\left(1-e^{-c_{i1}(t-T_i)}\right) \le \Delta\delta_i(t)$$
$$\le \Delta\delta_i(T_i)e^{-c_{i1}(t-T_i)} + \frac{\omega_0}{M_i c_{i1}}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}}\left(1-e^{-c_{i1}(t-T_i)}\right) \quad \forall t \ge T_i$$

and taking the limit as $t\to\infty$ the following bound occurs

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le \frac{\omega_0}{M_i c_{i1}}\sqrt{\frac{l_i}{m_i\varepsilon_i c_{i2}\lambda_i}}. \qquad \square$$

Remark 1: Note that for $\varepsilon_i = 1/2$ and $c_{i1} > c_{i2}/2$ we have the simple bound form

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le \frac{2\omega_0}{M_i c_{i1} c_{i2}}\sqrt{\frac{l_i}{\lambda_i}}. \quad (66)$$

However, an even superior bound for $\lim_{t\to\infty}|\Delta\delta_i(t)|$ can be obtained. To this end, the following theorem is proved.

Theorem 3: For the n-machine system defined by (1)-(13) and the excitation input given by (31)-(34) and (39)-(42), the following bounds on the angle deviations hold true

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le R_i := \frac{\omega_0}{M_i(1+c_{i1}c_{i2})}\sqrt{\frac{2l_i}{\lambda_i}}. \quad (67)$$

Proof: As $t\to\infty$ the system tends to its steady state wherein

$$\lim_{t\to\infty}\Delta\dot{\delta}_i = \lim_{t\to\infty}\Delta\dot{\omega}_i = 0.$$

From (1) and (2) one can directly obtain

$$\lim_{t\to\infty}\Delta\omega_i(t) = \lim_{t\to\infty}\Delta P_{ei}(t) = 0$$

and therefore from (21) and (24) we have for z_{i3}

$$\lim_{t\to\infty} z_{i3}(t) = -\frac{M_i}{\omega_0}(1+c_{i1}c_{i2})\lim_{t\to\infty}\Delta\delta_i(t).$$

Finally, using (43) we obtain the bound for $|\Delta\delta_i|$ as $t\to\infty$

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le R_i := \frac{\omega_o}{M_i(1+c_{i1}c_{i2})}\sqrt{\frac{2l_i}{\lambda_i}}. \qquad \square$$

Remark 2: Using the inequality $1/(1+x) < 1/x < \sqrt{2}/x \quad \forall x > 0$, the bounds R_{i1}, R_{i2}, R_i given by (62), (66) and (67) respectively, are proved to be of decreasing order for the common case where $c_{i1} > 1$, i.e. $R_{i1} > R_{i2} > R_i$.

Remark 3: Comparing the bounds given by Theorems 2 and 3 it can be easily seen that if the bound given by Theorem 3 is used, the design results in smaller values for c_{i1}, c_{i2} and consequently in smaller values of the control gains k_{i1}, k_{i2}, k_{i3}.

Remark 4: The positive scalars c_{i1}, c_{i2} and c_{i3} can be arbitrarily selected with reasonable limits coming from inequality (67) in accordance to desirable $\lim_{t\to\infty}|\Delta\delta_i|$, l_i and λ_i.

5. CASE STUDY

The two generator infinite bus power system shown in Fig. 1, is used to demonstrate the efficiency of the proposed controller.

The system parameters are as follows:

$x_{T1} = 0.129$ p.u., $x_{T2} = 0.11$ p.u., $x_{12} = 0.55$ p.u., $x_{13} = 0.53$ p.u., $x_{23} = 0.6$ p.u., $T'_{d01} = 6.9$ sec, $x_{d1} = 1.863$ p.u., $x'_{d1} = 0.257$ p.u., $D_1 = 5.0$ p.u., $M_1 = 8.0$ sec, $M_2 = 10.2$ sec, $D_2 = 3.0$ p.u., $x_{d2} = 2.36$ p.u., $x'_{d2} = 0.319$ p.u., $T'_{d02} = 7.96$ sec, $k_{c1} = 1.0$ p.u., $k_{c2} = 1.0$ p.u..

In the simulations, for a more accurate evaluation of the proposed controller, we take into account the physical limits of the excitation voltage, which are considered to be:

$$|k_{c1}u_{f1}| \le 5.0\,p.u., \qquad |k_{c2}u_{f2}| \le 5.0\,p.u.$$

The following case is simulated.

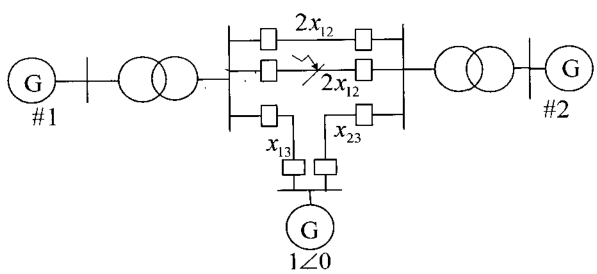

Fig. 1. Two machine infinite bus test system.

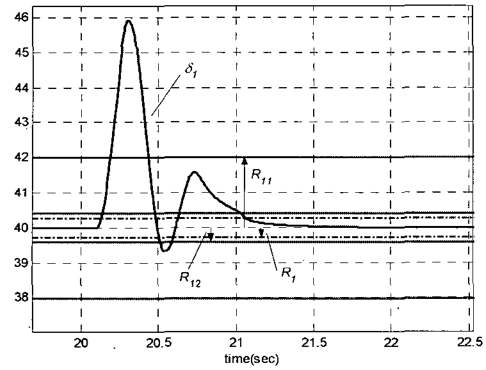

Fig. 2. Power angle deviations (in deg) and its calculated bounds of machine #1.

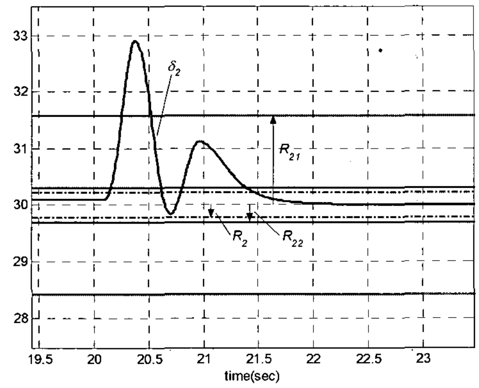

Fig. 3. Power angle deviations (in deg) and its calculated bounds of machine #2.

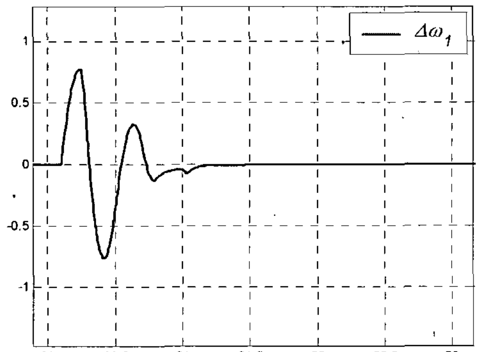

Fig. 4. Nominal speed deviation of machine #1.

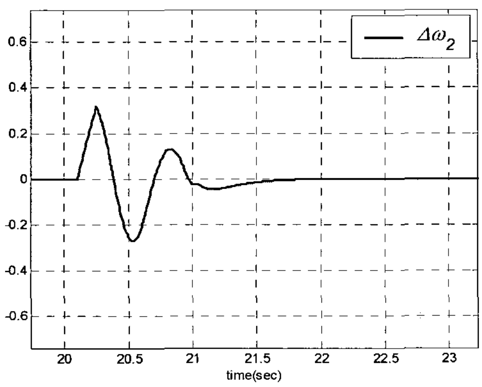

Fig. 5. Nominal speed deviation of machine #2.

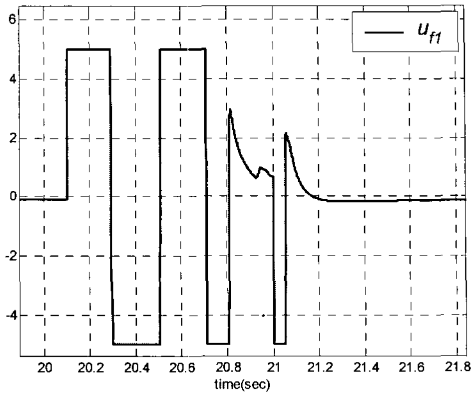

Fig. 6. Excitation voltage of machine #1.

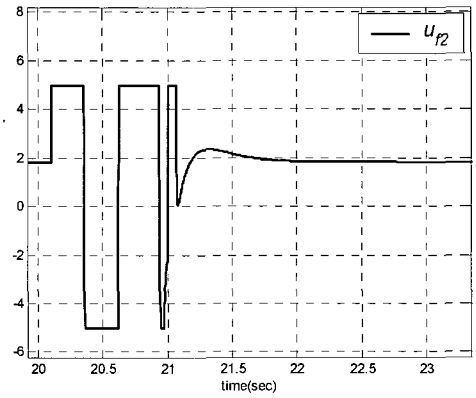

Fig. 7. Excitation voltage of machine #2.

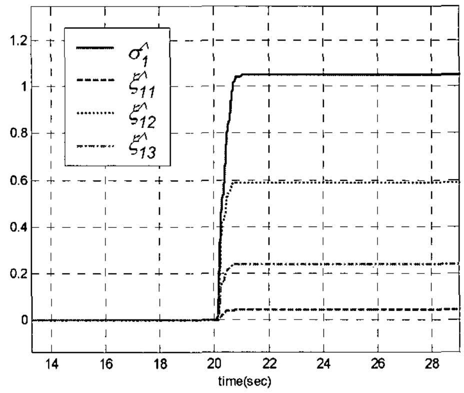

Fig. 8. Estimated parameters of machine #1.

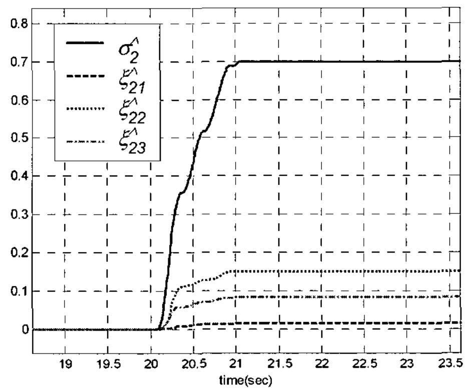

Fig. 9. Estimated parameters of machine #2.

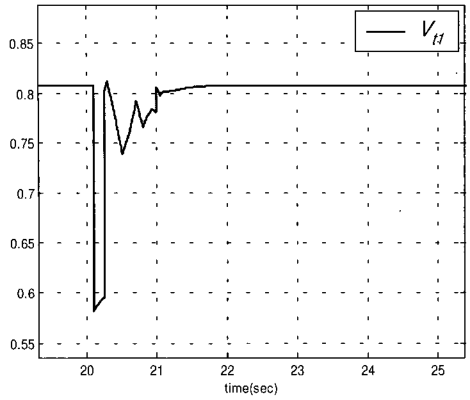

Fig. 10. Terminal voltage of machine #1.

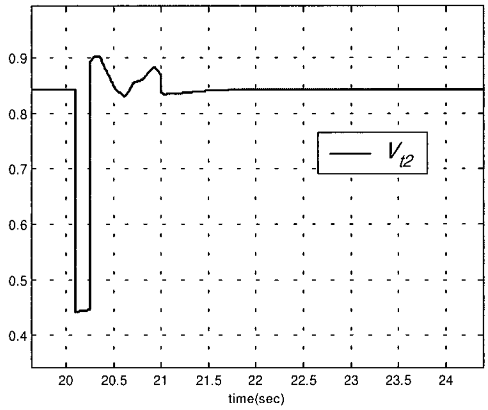

Fig. 11. Terminal voltage of machine #2.

Permanent serious fault: A symmetrical three phase short circuit fault occurs on one of the transmission lines between Generator #1 and Generator #2 at $t = 21.1$ sec. The fault is removed by opening the brakers of the faulted line at $t = 21.25$ sec and the system is restored at $t = 22$ sec. If we use λ to represent the fraction of the fault, simulations are made for $\lambda = 0.6$, i.e. for a fault near the middle of the line and towards Generator #2.

The operating point considered in the simulation is:

$\delta_{10} = 40^0$, $V_{t10} = 0.808$, $P_{m10} = 0.7$,

$\delta_{20} = 30^0$, $V_{t20} = 0.843$, $P_{m20} = 0.6$.

The controllers' parameters are

$c_{11} = 5$, $c_{12} = 10$, $c_{13} = 50$, $\gamma_1 = 10$, $\lambda_1 = 50$, $\delta_1 = 0.001$,

$c_{21} = 5$, $c_{22} = 10$, $c_{23} = 50$, $\gamma_2 = 10$, $\lambda_2 = 50$, $\delta_2 = 0.001$.

The simulation results are given in Figs. 2 to 11. The response of the system appears to be very satisfactory since it does not have undesirable oscillations or overshoots. The system maintains stability after the transient period and returns to the nominal frequency very quickly. This is not an unexpected situation due to the nonlinear action of the proposed excitation control.

6. CONCLUSIONS

The proposed controller is completely decentralized with a rather simple structure

$$E_{fi}(t) = \frac{T'_{d0i}}{I_{qi}}\left(K_{i1}\Delta\delta_i + K_{i2}\Delta\omega_i - K_{i3}\Delta P_{ei} - v_i\right),$$

where the nonlinear input term v_i is given by $v_i = -\frac{\rho_i^2 z_{i3}}{\rho_i |z_{i3}| + l_i}$ (see Theorem 1) with $z_{i3} = \Delta P_{ei} - \frac{M_i}{\omega_0}(1 + c_{i1}c_{i2})\Delta\delta_i - \frac{M_i}{\omega_0}\left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right)\Delta\omega_i$ and constant gains given by (32)-(34).

The proposed control scheme involves only local measurements of P_{ei}, ω_i, δ_i and the current I_{qi} that can be calculated from the measurements.

As it is shown by an extensive analysis this control scheme ensures stability while it permits the selection of the control parameters in such a way that a trade-off between the gain values and the region width R_{i1} is obtained.

APPENDIX

The nomenclature used is as follows.

$\delta_i(t)$: power angle, in radian; $\omega_i(t)$: rotor speed, in rad/sec; ω_0: synchronous machine speed, in rad/sec; P_{mi}: mechanical input power, in p.u; $P_{ei}(t)$: active electrical power, in p.u.; D_i: damping constant, in p.u.; M_i: inertia coefficient, in seconds; $E'_{qi}(t)$: transient EMF in the q-axis in p.u.; $E_{qi}(t)$: EMF in the q-axis, in p.u.; $E_{fi}(t)$: equivalent EMF in excitation coil, in p.u.; T'_{d0i}: d-axis transient short circuit time constant, in seconds; $I_{fi}(t)$: excitation current, in p.u.; $I_{qi}(t)$: q-axis current, in p.u.; $I_{di}(t)$: d-axis current, in p.u.; $Q_{ei}(t)$: reactive electrical power, in p.u.; $V_{ti}(t)$: generator terminal voltage, in p.u.; k_{ci}: gain of generator excitation amplifier, in p.u.; $u_{fi}(t)$: input of the SCR amplifier, in p.u.; x'_{di}: d-axis transient reactance, in p.u.; x_{di}: d-axis reactance, in p.u.; x_{adi}: mutual reactance between the excitation coil and the stator coil, in p.u.; $Y_{ij} = G_{ij} + jB_{ij}$: the ith row and jth column element of nodal admittance matrix, in p.u.; $\Delta\delta_i = \delta_i - \delta_{i0}$; $\Delta\omega_i = \omega_i - \omega_0$; $\Delta P_{ei} = P_{ei} - P_{mi}$.

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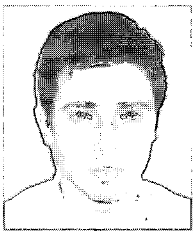

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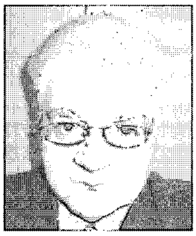

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