

Comments on "Time-Delayed State Estimator for Linear Systems with Unknown Inputs"

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Abstract: A recently published paper demonstrates the utility of allowing delays in state estimators for linear systems with unknown inputs. In this note, we point out an error in the derivation of the estimator.

Keywords: Colored noise, Kalman filters, time-delayed state estimation, unknown input observers.

1. INTRODUCTION

In a recently published paper [1], Jin and Tahk consider the problem of estimating the state of the system

$$\begin{aligned}x_{k+1} &= Ax_k + Bu_k + Mf_k + v_k, \\y_k &= Cx_k + w_k,\end{aligned}$$

where x_k is the state vector, u_k is a known input, f_k is an unknown input, and v_k and w_k are independent zero mean white noise processes with covariance matrices Q_k and R_k respectively. Jin and Tahk propose an estimator of the form

$$\begin{aligned}\hat{x}_{k+1} &= Ax_k + Bu_k \\ &+ [K_0 \quad K_1 \quad \dots \quad K_d] \begin{bmatrix} y_k - \hat{y}_k \\ y_{k+1} - \hat{y}_{k+1} \\ \vdots \\ y_{k+d} - \hat{y}_{k+d} \end{bmatrix},\end{aligned}$$

where d is the estimator delay, and $\hat{y}_k = C\hat{x}_k$,

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$\hat{y}_{k+1} = CA\hat{x}_{k+1} + CBu_k$, and so on. It is shown in [1] that under certain conditions, the gain matrices K_1, K_2, \dots, K_d can be chosen to decouple the estimation error from the unknown input f_k . The estimation error (defined as $e_{k+1} \equiv x_{k+1} - \hat{x}_{k+1}$) is then given by

$$e_{k+1} = (\bar{A} - K_0 C) e_k - K_0 w_k - \bar{K} \bar{w}_{k/1} + \bar{S} \bar{v}_k,$$

where

$$\bar{A} = (I - K_1 C - \dots - K_d CA^{d-1}) A,$$

$$\bar{K} = [K_1 \quad \dots \quad K_d],$$

$$\bar{S} = [I \quad 0 \quad \dots] - \bar{K} \begin{bmatrix} C & 0 & \dots & 0 \\ CA & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{d-1} & CA^{d-2} & \dots & C \end{bmatrix},$$

$$\bar{w}_k = [w_k^T \quad \dots \quad w_{k+d}^T]^T,$$

$$\bar{w}_{k/1} = [w_{k+1}^T \quad \dots \quad w_{k+d}^T]^T,$$

$$\bar{v}_k = [v_k^T \quad \dots \quad v_{k+d-1}^T]^T.$$

Based on this expression, the estimation error covariance matrix (defined as $P_{k+1} \equiv E\{e_{k+1}e_{k+1}^T\}$) is specified in equation (15) of [1] to be

$$\begin{aligned}P_{k+1} &= [\bar{A} - K_0 C] P_k [\bar{A} - K_0 C]^T + K_0 R_k K_0^T \\ &\quad + \bar{K} \bar{R}_k \bar{K}^T + \bar{S} \bar{Q}_k \bar{S}^T.\end{aligned}$$

The above expression for the estimation error covariance matrix is incorrect since it neglects the correlation between e_k and the noise vectors \bar{w}_k

and \bar{v}_k . This implies that the subsequent equations in [1] for the optimal filter gains are also erroneous. The correlation exists because the delays in the estimator force e_k to become a function of multiple time-samples of the noise processes w_k and v_k . In other words, the noise becomes colored from the perspective of the error. This situation also arises when constructing zero-delay estimators for systems where the unknown inputs affect the output equation, and was studied by Darouach et al. in [2]. The proper expression for the error covariance matrix can be obtained by using Kalman filtering equations for colored noise as described in [2] and [3].

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