Differential Space Time Coding based on Different Unitary Matrices Sets

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Abstract

This paper investigates a distinct set of complex unitary matrices for QPSK differential space time coding. After properly selecting the initial transmission matrix and unitary matrices we find that the different combinations of them could lead different BER performance over slow/fast Rayleigh fading channels and antennas correlated channels. The numerical results show that the proper selection of the initial transmission matrix and the set of unitary matrices can efficiently improve the bit error rate performance, especially for the antennas correlated fading channel. The computer simulations are evaluated over slow and fast Rayleigh fading channels.

Key words : Space Time Coding, Unitary Matrices, QPSK, Rayleigh Fading.

Ⅰ. Introduction

Space time coding with multiple transmit and receive antennas minimizes the effect of multipath fading and improves the performance and capacity of digital transmission over wireless radio channels. Thus far, it has been assumed that perfect channel estimates, e.g. perfect channel state information, are available at the receiver and coherent detection is employed. However, in some situations, such as high mobility environment or channel fading conditions changing rapidly, it may be difficult or costly to estimate the channel accurately. For such situations, it is useful to develop space time coding techniques that do not require channel estimates either at the receiver or at the transmitter. Recently, various schemes have been proposed, such that they use the differential coding and noncoherent decoding algorithm without channel estimates[1]~[4].

Especially, the differential space time coding based on group codes and unitary matrices has attracted much attention because of its simple construction[3]~[6]. In this work, we investigate a distinct set of complex unitary matrices for QPSK differential space time coding. After properly selecting the initial transmission matrix and unitary matrices we find that the different combinations of them could lead different BER performance over slow/fast Rayleigh fading channels and antennas correlated channels.

Ⅱ. Unitary Matrices for Differential Space Time Coding

In Reference [3], the authors proposed the space time coding with unitary matrices modulation. Let G be a set of $L\times L$ unitary matrices, where $L\geq n_T$[5].

$$g^H g = g g^H = I_L \quad \text{for} \quad g \in G \tag{1}$$

where the entries in g belong to the modulation constellation set A, $(\cdot)^H$ denotes the Hermitian of a matrix, I_L is an $L\times L$ identity matrix and the system is with n_T transmit antennas. We assume that there is one $n_T\times L$ matrix D, such that for any unitary matrix g in the set, Dg generates one $n_T\times L$ matrix, whose entries are the elements of the signal constellation set A. That is $Dg\in A^{n_T\times L}$ for all $g\in G$. Assuming that the number of the codeword is denoted by $|G|$, the spectral efficiency of the code is given by

$$\eta = \log_2 |G|/L \tag{2}$$

If the matrix D satisfies

$$DD^H = LI_{n_T} \tag{3}$$

then we have

$$(Dg)(Dg)^H = LI_{n_T}. \tag{4}$$

In this case, D is called the initial transmission matrix, and g is the unitary matrix for differential space time coding. In References [3], [5], the authors introduced a differential space time code for $n_T=L=2$ and $D=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, with a set of unitary matrices,

$$G=\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix}, \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix}\right\} \tag{5}$$

which is used for QPSK modulation constellation $A=\{\pm1, \pm j\}$, $j=\sqrt{-1}$. The differential encoding/decoding

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principles for unitary space time modulation schemes discussed in References [3]~[6] can be applied to the differential space time codes. At the tth encoding block, $\log_2|G|$ bits are mapped into the set G and a unitary matrix g_{z_t}, where $z_t=\{0,1,\ldots,|G|-1\}$.

To initialize the differential transmission, $X_0=D$ is sent from n_T transmit antennas over L symbol periods. The encoding rule is given by [5]

$$X_t = X_{t-1}\, g_{z_t} \tag{6}$$

where we assume that the fading coefficients are constant across every two transmission blocks. The received signals for the $t-th$ transmission block are represented by an $n_R \times L$ matrix R_t.

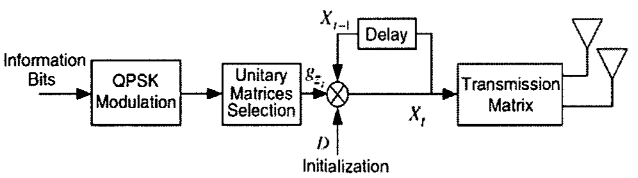
The differential space time decoding based on the current and previous received signal matrices is given by [5]

$$\begin{aligned}\hat{z}_t &= \arg\max_{l\in Q} \operatorname{Re} Tr\{R_{t-1}\, g_l\, R_t^H\} \\ &= \arg\max_{l\in Q} \operatorname{Re} Tr\{g_l R_t^H R_{t-1}\}\end{aligned} \tag{7}$$

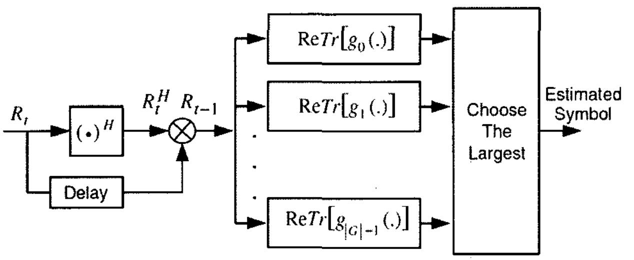
where ReTr denotes the real part of the trace. The simple transmission model of the differential space time encoder and decoder is illustrated in Fig. 1. In this work, we use only two transmit antennas and one receiver antenna; the spectral efficiency $\eta=1$.

III. Distinct Complex Unitary Matrices for Space Time Coding

In this section, we introduce some new complex unitary matrices, which are distinct from the previous ones in (5). Different from the conventional designs, the



(a) Encoder



(b) Decoder

Fig. 1. The system model of differential space time.

elements in the new set include both the $\{\pm 1\}$ and $\{\pm j\}$ with QPSK modulation constellation set A. It forms the j-rotation space time block code construction and Jacket matrix[7],[8].

$$G_D=\left\{\begin{bmatrix}1 & 0\\ 0 & -j\end{bmatrix},\begin{bmatrix}0 & 1\\ j & 0\end{bmatrix},\begin{bmatrix}0 & j\\ 1 & 0\end{bmatrix},\begin{bmatrix}-j & 0\\ 0 & 0\end{bmatrix}\right\} \tag{8}$$

where we have

$$g_D^H \cdot g_D = g_D \cdot g_D^H = I_L \quad \text{for } g_D \in G_D \tag{9}$$

and the determinant of the different code matrix in G_D is the same as that of the set G,

$$|det\,(g^i - g^j)| = 2 \ \text{ for } i \neq j,\ \ g^i, g^j \in G \tag{10}$$

$$|det(g^i_D - g^j_D)| = 2 \ \text{ for } i \neq j,\ \ g^i_D, g^j_D \in G \tag{11}$$

In References [3], [4], (10) and (11) imply that these two unitary sets (US) have the same upper bound. However, in our simulations, we find that proper selection of the initial transmission matrix for the different US will affect the performance.

3-1 Unitary Matrices Multiplications

By combining with the differential encoding function, we need to calculate the unitary matrices multiplications.

Property 1: The multiplications of unitary matrix g in the set G are also a unitary matrix g' in $\{\pm G^*\}$

$$g' = g^i \times g^j \times \cdots \times g^t,\ \ g' \in \{\pm G^*\}\ \ (g^i, g^j, \cdots, g^t) \in G \tag{12}$$

where $g^i, g^j, \ldots g^t$ are the unitary matrices in G, and $\{\pm G^*\}$ denotes the set including $\{\pm G\}$ and their permuted cases.

Property 2: The multiplications of unitary matrices g_D in the set G_D are also a unitary matrix g'_D.

(i) If the number of g_D in multiplications is even, g'_D is a matrix in set $\{\pm G^*\}$.

(ii) If the number of g_D in multiplications is odd, g'_D is a matrix in set $\{\pm G^*_D\}$.

$$g'_D = \underbrace{g^i_D \times g^j_D \times \ldots \times g^t_D}_{t\ times}$$

$$\begin{cases} g'_D \in \{\pm G^*_D\}, & n = odd \\ g'_D \in \{\pm G^*\}, & n = even \end{cases} \quad (g^i_D, g^j_D, \ldots, g^t_D) \in G_D \tag{13}$$

where $g^i_D, g^j_D, \ldots, g^t_D$ are unitary matrices in G_D. Equations (12) and (13) imply that the multiplications of the unitary matrices will lead to different properties of differential space time coding. The details of these results will be presented in the rest of this work.

3-2 Transmission Model 1

Let us define G as US 1, and G_D as US 2, respectively. When the initial transmission matrix $D=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, it is defined as transmission model 1. These two sets used transmission model 1 have different performances with the same noncoherent decoding algorithm over slow Rayleigh fading channel, as shown in Fig. 2. As illustrated in Fig. 2, these two sets have a similar performance over lower signal to noise ratio(SNR) by using E_b/N_o, where E_b is the spectral energy density of the bit and N_o is the power spectral density of the noise, respectively. However, US 1 (G) leads to a sharp degradation compared with US 2 (G_D) over higher SNR. The main reason of this phenomenon is that the transmitted symbols are different.

Case A: By considering US 1 (G), each unitary matrix $g \in G$ has only two symbols, $\{\pm 1\}$ or $\{\pm j\}$. If the initial transmission matrix is set as $D=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, the combined group Dg has

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} = \begin{bmatrix} j & j \\ j & -j \end{bmatrix} = j\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} = \begin{bmatrix} -j & j \\ j & j \end{bmatrix} = j\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}.$$

The results show that the coded matrices only consist of the Hadamard matrices and their rotated cases, and every matrix uses the anti-polar symbols $\{\pm 1\}$ or $\{\pm j\}$.

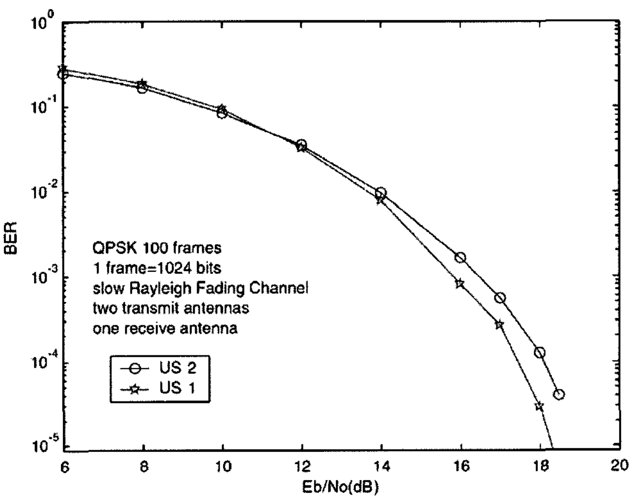


Fig. 2. Performance of transmission model 1 by using different US over slow Rayleigh fading channel.

Obviously, those symbols have largest Euclidean distance in the QSPK constellation. Further multiplications by using $X_t = X_{t-1} g_{z_t}$, $g_{z_t} \in G$; will have the same result as the above function, since the multiplication of the unitary matrix g is also a unitary matrix in unitary group 1 $\{\pm G\}$ and its permuted cases, as shown in Property 1.

Case B: By considering US 2 (G_D), and the initial transmission matrix $D=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, we have

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix} = \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ j & 0 \end{bmatrix} = \begin{bmatrix} -j & 1 \\ j & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & j \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & j \\ 1 & j \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} -j & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -j & -1 \\ -j & 1 \end{bmatrix}$$

The results denote that the coded matrices are complex unitary matrices, and every matrix has three different symbols at least. However, further multiplications by using $X_t = X_{t-1} g_{z_t}$, $g_{z_t} \in G_D$, have different results, such as

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ j & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 0 & j \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} j & j \\ -j & j \end{bmatrix} = j\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} -j & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -j & j \\ -j & -j \end{bmatrix} = j\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$$

These coded matrices are the same as the transmitted symbols in Case A, which have the largest Euclidean distance used the anti-polar symbols $\{\pm 1\}$ or $\{\pm j\}$. Generally, the further differential space time coding has

$$X_t = X_{t-1} g_{z_t} = D \times \underbrace{g_1 \times g_2 \times \ldots \times g_t}_{t \ times} \tag{14}$$

where $t=\{1,2,3,..\}$ and . $(g_1, g_2, \ldots, g_t) \in G_D$ According to Property 2, we obtain that

$$g_{multi} = \underbrace{g_1 \times g_2 \times \ldots \times g_t}_{t \ times}, \begin{cases} g_{multi} \in \{\pm G_D^*\}, & n = odd \\ g_{multi} \in \{\pm G^*\}, & n = even \end{cases} \tag{15}$$

Therefore, the transmitted coded matrices in Case B have two different patterns. One is the complex unitary matrix, which has three different symbols at least; the other is the Hadamard matrices or their rotated cases as shown in Case A That is, the transmitted coded matrices.

$X_t = Dg_{multi}$ have only anti-polar symbols $\{\pm 1\}$ or $\{\pm j\}$ when t=even; $X_t = Dg_{multi}$ has three different transmitted symbols at least, when t=odd: Normally, for a random message sequence, half of them can generate the coded matrices as Hadamard matrices or their rotated cases, if Case B is used.

In transmission model 1, the transmitted symbols generated from G and G_D are different. And it is clear that the symbols from US 1 (G) have the largest Euclidean distances for QPSK transmission, since each time only the anti-polar symbols are transmitted. The detection error can be protected.

3-3 Transmission Model 2

The initial transmission matrix is now set as $D' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, a complex unitary matrix. The simulation results generated from G and G_D will be contrary to the previous one (transmission model 1) over slow Rayleigh fading channel, as shown in Fig. 3. The main reason will be described in Cases C and D.

Case C: As shown in Fig. 3, the set G case has slow BER degradation, since the anti-polar symbols do not exist. The coded matrices $D'g$ are now changed as

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -j \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -j & 1 \\ j & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix} = \begin{bmatrix} j & 1 \\ j & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} = \begin{bmatrix} -1 & j \\ 1 & j \end{bmatrix}$$

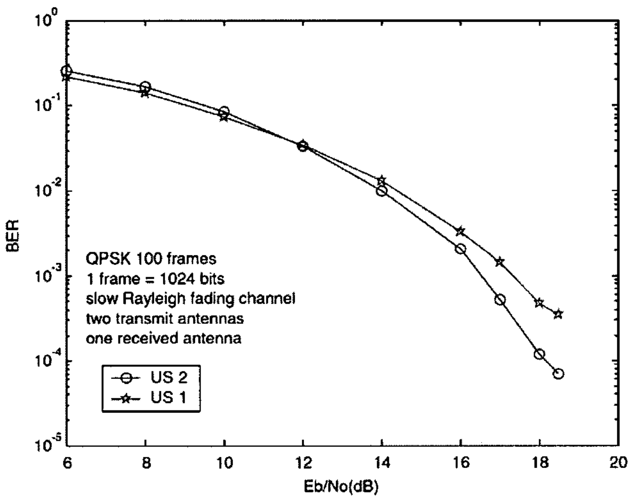


Fig. 3. Performance of transmission model 2 by using different US over slow Rayleigh fading channel.

The results show that the coded matrices are now complex unitary matrices, which are similar to Case B when n=odd: The further multiplications by using $X_t = X_{t-1} g_{z_t}$, $g_{z_t} \in G$, will have the same result, since the multiplication of the unitary matrix g is also a unitary matrix in US 1 $\{\pm G^*\}$, as shown in Property 1.

Case D: Now, by considering the US G_D in transmission model 2, we first have

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ j & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} 0 & j \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} j & j \\ -j & j \end{bmatrix} = j\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \times \begin{bmatrix} -j & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -j & j \\ -j & -j \end{bmatrix} = j\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$$

The results show that the coded matrices are Hadamard matrices and their rotated cases, where only anti-polar symbols $\{\pm 1\}$ or $\{\pm j\}$ exist for every matrix. Secondly, similar to Case B, the further multiplications of the unitary matrices have another output, such as

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix} = \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ j & 0 \end{bmatrix} = \begin{bmatrix} j & 1 \\ -j & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 0 & j \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & j \\ -1 & j \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} -j & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -j & 1 \\ -j & -1 \end{bmatrix}.$$

The results denote that the coded matrices are complex unitary matrices, and every matrix has three different symbols at least. Similarly, the further differential space time coding has

$$X_t = X_{t-1} g_{z_t} = D' \times \underbrace{g_1 \times g_2 \times \ldots \times g_t}_{t\ times},\ t \in \{1,2,3,\ldots\} \tag{16}$$

where $(g_1, g_2, \ldots, g_t) \in G_D$.. According to (15), $X_t = D' g_{multi}$ has only anti-polar symbols $\{\pm 1\}$ or $\{\pm j\}$, when t=odd; $X_t = D' g_{multi}$ has three different symbols at least, when t=even. Normally, for a random message sequence, half of them can generate the coded matrices as Hadamard matrices or their rotated cases, which is the same as Case B.

3-4 Numerical Results

The multiplications of the unitary matrices from G and G_D, are listed in Table 1. It demonstrates that the

Table 1. The multiplications of different unitary matrices.

	$G=\left\{\begin{bmatrix}1&0\\0&1\end{bmatrix},\begin{bmatrix}1&0\\-1&0\end{bmatrix},\begin{bmatrix}j&0\\0&-j\end{bmatrix},\begin{bmatrix}0&j\\j&0\end{bmatrix}\right\}$		$G_D=\left\{\begin{bmatrix}1&0\\0&-j\end{bmatrix},\begin{bmatrix}0&1\\j&0\end{bmatrix},\begin{bmatrix}0&j\\1&0\end{bmatrix},\begin{bmatrix}-j&0\\0&1\end{bmatrix}\right\}$
		n=even	n=odd
$g^i \times g^i$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}\times\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}1&0\\-1&0\end{bmatrix}\times\begin{bmatrix}0&1\\-1&0\end{bmatrix}=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ $\begin{bmatrix}j&0\\0&-j\end{bmatrix}\times\begin{bmatrix}j&0\\0&-j\end{bmatrix}=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ $\begin{bmatrix}0&j\\j&0\end{bmatrix}\times\begin{bmatrix}0&j\\j&0\end{bmatrix}=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&-j\end{bmatrix}\times\begin{bmatrix}1&0\\0&-j\end{bmatrix}=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$ $\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}=\begin{bmatrix}j&0\\0&j\end{bmatrix}$ $\begin{bmatrix}0&j\\1&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}=\begin{bmatrix}j&0\\0&j\end{bmatrix}$ $\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}-j&0\\0&1\end{bmatrix}=\begin{bmatrix}-1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&-j\end{bmatrix}\times\begin{bmatrix}1&0\\0&-j\end{bmatrix}\times\begin{bmatrix}1&0\\0&-j\end{bmatrix}=\begin{bmatrix}1&0\\0&j\end{bmatrix}$ $\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}=\begin{bmatrix}0&j\\-1&0\end{bmatrix}$ $\begin{bmatrix}0&j\\1&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}=\begin{bmatrix}0&-1\\j&0\end{bmatrix}$ $\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}-j&0\\0&1\end{bmatrix}=\begin{bmatrix}j&0\\0&1\end{bmatrix}$
$g^j \times g^j,\ i \neq j$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}\times\begin{bmatrix}0&1\\-1&0\end{bmatrix}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$ $\begin{bmatrix}0&1\\-1&0\end{bmatrix}\times\begin{bmatrix}0&j\\j&0\end{bmatrix}=\begin{bmatrix}j&0\\0&-j\end{bmatrix}$ $\begin{bmatrix}j&0\\0&-j\end{bmatrix}\times\begin{bmatrix}0&1\\-1&0\end{bmatrix}=\begin{bmatrix}0&j\\j&0\end{bmatrix}$ $\begin{bmatrix}0&j\\j&0\end{bmatrix}\times\begin{bmatrix}j&0\\0&-j\end{bmatrix}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$	$\begin{bmatrix}1&0\\0&-j\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ $\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$ $\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}=\begin{bmatrix}0&-1\\j&0\end{bmatrix}$ $\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}=\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}1&0\\0&-j\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}=\begin{bmatrix}j&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}=\begin{bmatrix}0&j\\-1&0\end{bmatrix}$ $\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}\times\begin{bmatrix}0&1\\j&0\end{bmatrix}=\begin{bmatrix}1&0\\0&j\end{bmatrix}$ $\begin{bmatrix}-j&0\\0&1\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}\times\begin{bmatrix}0&j\\1&0\end{bmatrix}=\begin{bmatrix}1&0\\0&j\end{bmatrix}$

functions are obtained from Property 1 and 2.

Some other numerical results from the simulations of transmission models 1 and 2 over slow Rayleigh fading channel are shown in Table 2. As illustrated in Table 2, the BER performance of Case A has 0.5 dB improvement from that of Cases B and D, but it gained about 1.1 dB from that of Case C. Same as the analysis of the multiplications of unitary matrices, the BER performance of Case B is the same as that of Case D. The simulations demonstrate that the multiplications of the unitary matrices and the anti-polar transmitted symbols with largest Euclidean distances are the main reason for the differences of the different cases.

The similar results can also be achieved over fast Rayleigh fading channel, as shown in Figs. 4 and 5. In this situation, f_d is Doppler frequency and T is the duration of the coded symbols.

When transmission model 1 over fast Rayleigh fading channel is used, US 1 leads to a sharp degradation compared with US 2 over higher SNR, but US 2 has a slightly better performance over lower SNR case, as illustrated in Fig. 4.

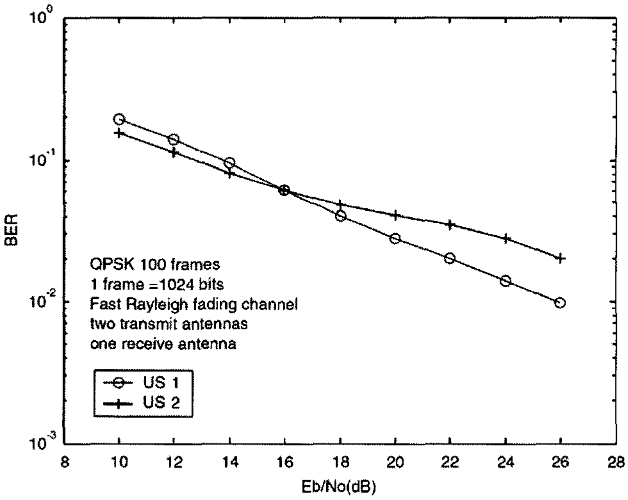


Fig. 4. The performance of transmission model 1 over fast Rayleigh fading channel with f_dT=0.05.

If transmission model 2 over fast Rayleigh fading channel is used, we find that the performance of US 1 and US 2 will be better than that of transmission model 1, under lower SNR cases, as shown in Fig. 5. For

Table 2. Performances of the different cases over slow Rayleigh fading channel(two transmit antennas and one receiver antenna).

E_b/N_0 (dB)	Case A	Case B	Case C	Case D
18	BER=0.0000292	BER=0.0001269	BER=0.0004785	BER=0.0001719
16	BER=0.0008203	BER=0.0016	BER=0.0033	BER=0.0021
14	BER=0.0079	BER=0.0096	BER=0.0132	BER=0.0102

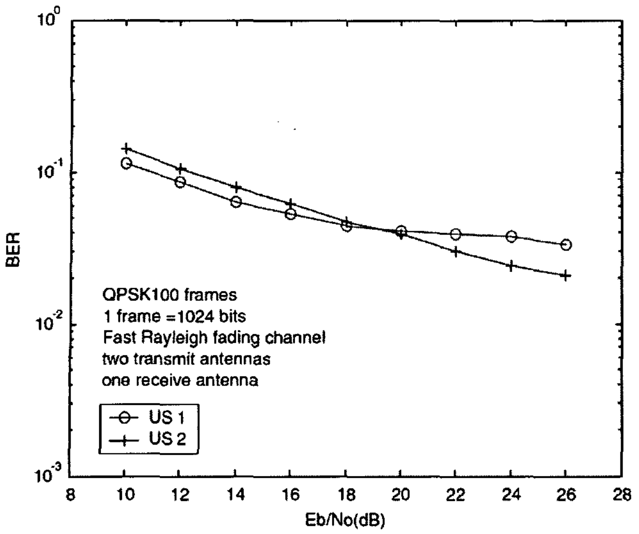


Fig. 5. The performance of transmission model 2 over fast Rayleigh fading channel with f_dT=0.05.

example, if E_b/N_0=10 dB; the BER of US 1 and the transmission model 2 is about 0.085, but the same case of transmission model 1 is about 0.112. Moreover, the cases from set G have slow BER degradation in the simulations, according to the multiplications of unitary matrices in Cases C and D.

Similar results are also obtained in the antennas correlated channels. Assuming that the antenna correlation matrix is given by

$$\Theta_R=\begin{bmatrix} 1 & \theta \\ \theta & 1 \end{bmatrix} \tag{17}$$

where θ is the correlation factor(CF) between the transmit antennas. In the simulation, the correlation factor is chosen to be 0.25, 0.5, and 0.75. It can be observed

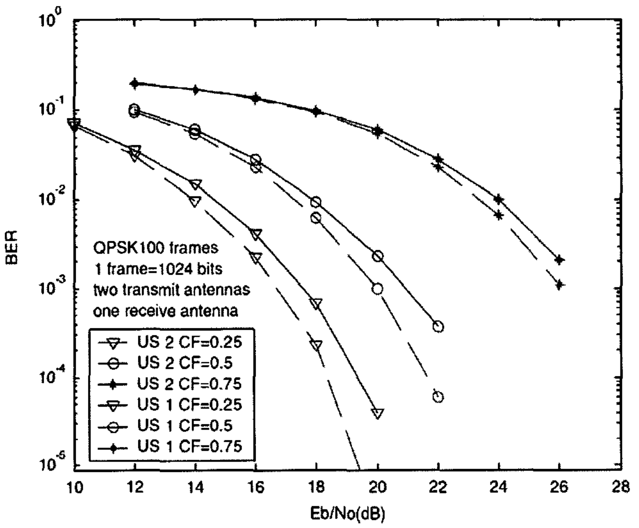


Fig. 6. Correlated antennas for different US used transmission model 1.

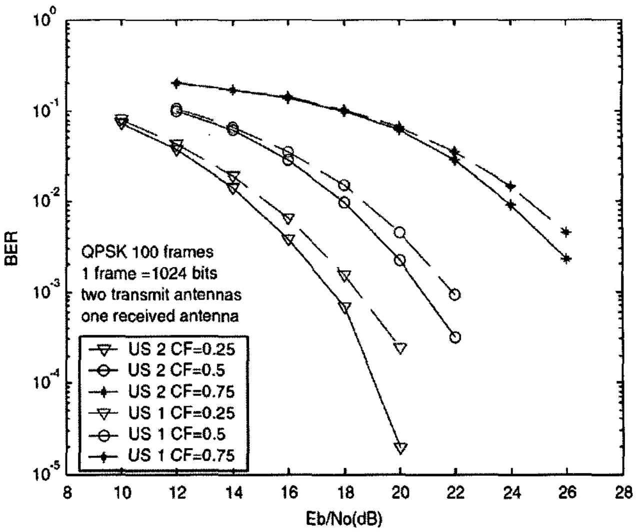


Fig. 7. Correlated antennas for different US used transmission model 2.

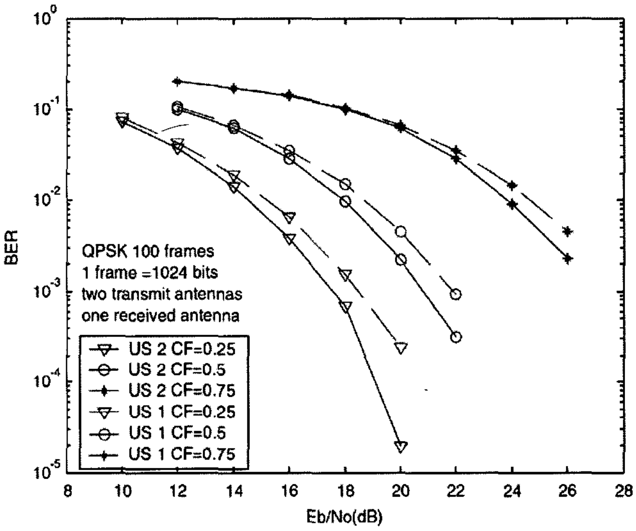


Fig. 8. CF v.s.. E_b/N_0 (BER target=10^{-3}).

that the code performance is sharply degraded when the correlation factor decreases, as shown in Figs. 6 and 7. And as illustrated in Fig. 8, we find that US 2 will lead to the same performance on different transmission models.

Ⅳ. Conclusion

This paper investigates a distinct set of complex unitary matrices for QPSK differential space time coding. The numerical results show that the proper selection of the initial transmission matrix and the US can efficiently improve the BER performance. And transmission model 2 could lead to better performance over lower SNR if the system is operated in the fast Rayleigh fading channel. As a result, we could generalize several criteria for differential space time coding as follows:

(1) If D is the real unitary matrix, the US 1 (G) is the best choice for the differential encoding.

(2) If D is not the real unitary matrix or unknown, the US 2 (G_D) is the best choice for the differential encoding.

(3) If the system is in lower SNR, transmission model 2 is a good choice.

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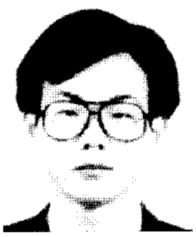

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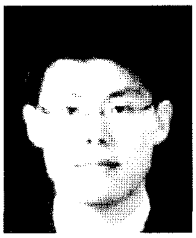

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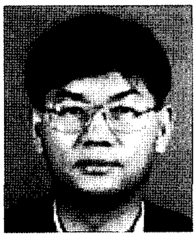

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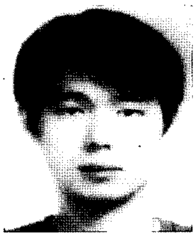

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