Observer Design for A Class of UncertainState-Delayed Nonlinear Systems

Junwei Lu, Chunmei Feng, Shengyuan Xu*, and Yuming Chu

Abstract: This paper deals with the observer design problem for a class of state-delayed nonlinear systems with or without time-varying norm-bounded parameter uncertainty. The nonlinearities under consideration are assumed to satisfy the global Lipschitz conditions and appear in both the state and measured output equations. The problem we address is the design of a nonlinear observer such that the resulting error system is globally asymptotically stable. For the case when there is no parameter uncertainty, a sufficient condition for the solvability of this problem is derived in terms of linear matrix inequalities and the explicit formula of a desired observer is given. Based on this, the robust observer design problem for the case when parameter uncertainties appear is considered and the solvability condition is also given. Both of the solvability conditions obtained in this paper are delay-dependent. A numerical example is provided to demonstrate the applicability of the proposed approach.

Keywords: Linear matrix inequality, nonlinear systems, robust observer, time-delay systems, uncertain systems.

1. INTRODUCTION

In dealing with the problem of observer design, various approaches, such as transfer-function, geometric, algebraic, singular value decomposition and so on, have been successfully proposed and many results on the observer design for linear systems have been reported in the literature [1-3]. When parameter uncertainty appears in a system model, the stability of the resulting error system cannot be guaranteed by the classical observer theory. This has motivated the study of robust observer design. In [4], the problem of robust observer design for uncertain systems with norm-bounded parameter uncertainties was considered and some robust results for both control and estimation were presented. By using the factorization approach, a similar problem was addressed in [5], while [6] studied the problem of robust Kalman filtering design for uncertain linear systems by using a Riccati equation approach.

It is known that time delay arises quite naturally in propagation phenomena, population dynamics or engineering systems such as chemical processes, long transmission lines in pneumatic systems, and so on. Many results related to delay systems have been reported in the literature [7-11]. For observer design for time-delay systems, the work in [12] proposed a general form of linear observers by using the factorization approach, in which a necessary and sufficient condition for the existence of state functional observers for such systems was obtained. For discrete delay systems, a memoryless state observer was designed by the state augmentation approach in [13]. It is worth noting that in both [13] and [12], parameter uncertainties in the system matrices have not been considered.

On the other hand, it is known that one of the most popular ways to deal with the observer design problem for nonlinear systems is the one based on differential geometric approach [14]. It turns out that the observer problem for nonlinear systems is much more difficult than the controller problem [14,15]. Recently, the observer design problem for a class of nonlinear systems was addressed in [16-18], and an algebraic Riccati equation approach was developed.

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Junwei Lu and Chunmei Feng are with the School of Electrical and Automation Engineering, Nanjing Normal University, 78 Bancang Street, Nanjing 210042, P. R. China (e-mails: we889@yahoo.com.cn, fengchunmei@njnu.edu.cn).

Shengyuan Xu is with the Department of Automation, Nanjing University of Science and Technology, Nanjing 210094, P. R. China (e-mail: syxu02@yahoo.com.cn).

Yuming Chu is with the Department of Mathematics, Huzhou Teacher's College, Huzhou, Zhejiang 313000, P. R. China (e-mail: chuyuming@hutc.zj.cn).

* Corresponding author.

Very recently, the observer problem for a class of uncertain nonlinear systems with state delay was considered in [19], however, the observer structure proposed in [19] involves parameter uncertainties, which makes the design of such observers difficult in practical applications.

In this paper, we consider the problem of observer design for a class of state-delayed nonlinear systems with, or without, parameter uncertainty. The class of systems under consideration is described by a linear delayed state space model with the addition of known nonlinearities which depend on state as well as delayed state and satisfy the global Lipschitz conditions. The nonlinearities appear in both the state and measured output equations. The parameter uncertainties are assumed to be time-varying and unknown, but are norm-bounded. We first consider the nonlinear observer design for the above class of nonlinear systems without parameter uncertainty. A linear matrix inequality (LMI) design approach is developed. Then the problem of robust observer design for the above class of nonlinear systems with parameter uncertainty is investigated. The problem to be addressed is the design of a nonlinear observer such that the error system remains globally asymptotically stable for all admissible uncertainties and addressed nonlinearities. A sufficient condition for the solvability of this problem is proposed in terms of LMIs. Furthermore, an explicit formula of a desired robust observer is given. Both of the solvability conditions proposed in this paper are delay-dependent, which will be less conservative than delay-independent ones.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The notation M^T represents the transpose of the matrix M. $\|x\|$ stands for the Euclidean norm of the vector x. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM FORMULATION

Consider the following class of uncertain nonlineartime-delay systems:

$$(\Sigma):\ \dot{x}(t) = (A + \Delta A(t))\, x(t) + (A_d + \Delta A_d(t))\, x(t-\tau) + Gg(x(t),\ x(t-\tau)), \quad (1)$$

$$y(t) = (C + \Delta C(t))\, x(t) + (C_d + \Delta C_d(t))\, x(t-\tau) + Hh(x(t),\ x(t-\tau)), \quad (2)$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \quad (3)$$

where $x(t) \in \mathrm{R}^n$ is the system state, $y(t) \in \mathrm{R}^m$ is the measurement, $g(\cdot,\cdot)$: $\mathrm{R}^n \times \mathrm{R}^n \to R^{n_g}$ and $h(\cdot,\cdot)$: $\mathrm{R}^n \times \mathrm{R}^n \to R^{n_h}$ are known nonlinear functions, A, A_d, C, C_d, G and H are known real constant matrices, $\varphi(t)$ is a real-valued continuous initial function on $[-\tau, 0]$, $\tau > 0$ is a known time delay of the system, $\Delta A(t)$, $\Delta A_d(t)$, $\Delta C(t)$ and $\Delta C_d(t)$ are unknown matrices representing time-varying parameter uncertainties, which are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta A_d(t) \\ \Delta C(t) & \Delta C_d(t) \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(t)[N_1\ N_2], \quad (4)$$

where M_1, M_2, N_1, and N_2 are known real constant matrices and $F(\cdot)$: $\mathrm{R} \to \mathrm{R}^{k\times l}$ is unknown real-valued time-varying matrix satisfying

$$F(t)^T F(t) \leq I, \quad \forall\, t. \quad (5)$$

The parameter uncertainties $\Delta A(t)$, $\Delta A_d(t)$, $\Delta C(t)$ and $\Delta C_d(t)$ are said to be admissible if both (4) and (5) hold.

Throughout the paper, we make the following assumption on the nonlinear functions in system (Σ).

Assumption 1:

i) $g(0, 0) = 0$;

ii) $\|g(x_1, x_2) - g(y_1, y_2)\| \leq \|S_{1g}(x_1 - y_1)\| + \|S_{2g}(x_2 - y_2)\|$, $\|h(x_1, x_2) - h(y_1, y_2)\| \leq \|S_{1h}(x_1 - y_1)\| + \|S_{2h}(x_2 - y_2)\|$,

for all $x_1, x_2, y_1, y_2 \in \mathrm{R}^n$, where S_{1g}, S_{2g}, S_{1h} and S_{2h} are known real constant matrices.

The problem we address in this paper is the design of a state observer such that the error system is globally asymptotically stable. More specifically, we address the following observer design problems:

(a) Nominal observer design problem: For the nominal system of (Σ), i.e., setting $\Delta A(t) \equiv 0$, $\Delta A_d(t) \equiv 0$, $\Delta C(t) \equiv 0$, and $\Delta C_d(t) \equiv 0$, we are concerned with obtaining an estimate $\hat{x}(t)$ of the state $x(t)$ by using the measurement such that the error system is globally asymptotically stable for all the nonlinearities satisfying Assumption 1.

(b) Robust observer design problem: For the uncertain nonlinear system (Σ), we are concerned with obtaining an estimate $\hat{x}(t)$ of the state $x(t)$ by using the measurement such that the error system remains globally asymptotically stable for all admissible uncertainties satisfying (4) and (5) and the nonlinearities satisfying Assumption 1.

3. MAIN RESULTS

In this section, an LMI approach is proposed to solve both the nominal and robust observe design problems formulated in the previous section and delay-dependent solvability conditions will be developed. Before presenting the main results, we give the following lemma which will be used in the proof of our main results.

Lemma 1 [20]: Let A, D, E, F and P be real matrices of appropriate dimensions with $P > 0$ and F satisfying $F^TF \leq I$. Then we have:

1) For any scalar $\varepsilon > 0$,
$DFE + (DFE)^T \leq \varepsilon^{-1}DD^T + \varepsilon E^TE$

2) $AD + (AD)^T \leq APA^T + D^TP^{-1}D.$

The following theorem provides a solution to the nominal observer design problem for system (Σ).

Theorem 1: Consider the system (Σ) with $\Delta A(t) \equiv 0$, $\Delta A_d(t) \equiv 0$, $\Delta C(t) \equiv 0$, and $\Delta C_d(t) \equiv 0$ under Assumption 1. If there exist a scalar $\varepsilon > 0$, matrices $P > 0$, $Q > 0$, $W > 0$, $X > 0$, Y and Z such that the following LMIs hold:

$$\begin{bmatrix} \Psi & PA_d - ZC_d - Y & PG & -ZH & \tau A^TP - \tau C^TZ^T \\ A_d^TP - C_d^TZ^T - Y^T & 2\epsilon S_2^TS_2 - Q & 0 & 0 & \tau A_d^TP - \tau A_d^TZ^T \\ G^TP & 0 & -\epsilon I & 0 & \tau G^TP \\ -H^TZ^T & 0 & 0 & -\epsilon I & -\tau H^TZ^T \\ \tau PA - \tau ZC & \tau PA_d - \tau ZC_d & \tau PG & \tau ZH & \tau W - 2\tau P \end{bmatrix} < 0, \quad (6)$$

$$\begin{bmatrix} X & Y \\ Y^T & W \end{bmatrix} \geq 0, \quad (7)$$

where

$$\Psi = PA + A^TP + Q - ZC - C^TZ^T + \tau X + Y + Y^T + 2\varepsilon S_1^T S_1, \quad (8)$$

$$S_1 = \begin{bmatrix} S_{1g} \\ S_{1h} \end{bmatrix}, S_2 = \begin{bmatrix} S_{2g} \\ S_{2h} \end{bmatrix}, \quad (9)$$

then the nominal observer design problem is solvable. Furthermore, when (6) and (7) are feasible, a suitable nonlinear observer is given as follows:

$$\begin{aligned}\dot{\hat{x}} = A\hat{x}(t) + A_d\hat{x}(t-\tau) + Gg(\hat{x}(t), \hat{x}(t-\tau)) \\ + L_o[y(t) - C\hat{x}(t) - C_d\hat{x}(t-\tau) \\ - Hh(\hat{x}(t), \hat{x}(t-\tau))],\end{aligned} \quad (10)$$

where

$$L_o = P^{-1}Z. \quad (11)$$

Proof: Let

$$e(t) = x(t) - \hat{x}(t). \quad (12)$$

Then from the system (Σ) and the observer (10), it is easy to show that

$$\begin{aligned}\dot{e}(t) = (A - L_oC)e(t) + (A_d - L_oC_d)e(t-\tau) \\ + \bar{G}\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)),\end{aligned} \quad (13)$$

where

$$\bar{G} = [G - L_oH], \quad (14)$$

and

$$\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)) = \begin{bmatrix} g(x(t), x(t-\tau)) - g(\hat{x}(t), \hat{x}(t-\tau)) \\ h(x(t), x(t-\tau)) - h(\hat{x}(t), \hat{x}(t-\tau)) \end{bmatrix}, \quad (15)$$

By Assumption 1, we have

$$\|\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))\|^2 \leq 2\|S_1e(t)\|^2 + 2\|S_2e(t-\tau)\|^2. \quad (16)$$

To show the asymptotic stability of (13), we define the following Lyapunov functional candidate:

$$V(e_t, t) = e(t)^TPe(t) + \int_{t-\tau}^{t} e(s)^T Qe(s)ds + \int_{-\tau}^{0}\int_{t+\alpha}^{t} \dot{e}(s)^T W\dot{e}(s)\, ds d\alpha,$$

where

$$e_t = e(t+\beta), \beta \in [-2\tau, 0].$$

Then, the time-derivative of $V(e_t, t)$ along the solution of (13) is given by

$$\begin{aligned}\dot{V}(e_t, t) = 2e(t)^TP\,[(A - L_oC)\,e(t) \\ + (A_d - L_oC_d)\,e(t-\tau) \\ + \bar{G}\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))] \\ + e(t)^TQe(t) - e(t-\tau)^TQe(t-\tau) \\ + \tau\dot{e}(t)^T W\dot{e}(t) \\ - \int_{t-\tau}^{t} \dot{e}(s)^T W\dot{e}(s)\,ds.\end{aligned} \quad (17)$$

Let

$$a(\alpha) = e(t), b(\alpha) = \dot{e}(\alpha), N = 0.$$

Then, by (7) and Lemma 1 in [21], we have

$$\begin{aligned}-2\int_{t-\tau}^{t} a(\alpha)^T N b(\alpha)d\alpha \\ \leq \tau e(t)^TXe(t) + e(t)^T(Y + Y^T)e(t) \\ - 2e(t)^TYe(t-\tau) + \int_{t-\tau}^{t} \dot{e}(s)^T W\dot{e}(s)\,ds.\end{aligned}$$

This together with (17) gives

$$\begin{aligned}\dot{V}(e_t, t) \leq 2e(t)^TP[(A - L_oC)\,e(t) \\ + (A_d - L_oC_d)\,e(t-\tau) \\ + \bar{G}\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))\end{aligned}$$

$$
\begin{aligned}
&+ e(t)^T (Q + \tau X + Y + Y^T)e(t) \\
&- 2e(t)^T Ye(t-\tau) - e(t-\tau)^T Qe(t-\tau) \\
&+ \tau \dot{e}(t)^T W \dot{e}(t).
\end{aligned} \tag{18}
$$

Then, it follows from (16) and (18) that

$$
\begin{aligned}
\dot{V}(e_t,t) \le\ & 2e(t)^T P[(A - L_o C)\, e(t) \\
&+ (A_d - L_o C_d)\, e(t-\tau) \\
&+ \bar{G}\, \xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)) \\
&+ e(t)^T (Q + \tau X + Y + Y^T)e(t) \\
&- 2e(t)^T Ye(t-\tau) - e(t-\tau)^T Qe(t-\tau) \\
&+ \tau \dot{e}(t)^T W \dot{e}(t) \\
&- \varepsilon \xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))^T \\
&\quad \xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)) \\
&+ 2\varepsilon[e(t)^T S_1^T S_1 e(t) \\
&+ e(t-\tau)^T S_2^T S_2 e(t-\tau)].
\end{aligned}
$$

That is

$$
\dot{V}(e_t,t) \le \varsigma(t)^T \Phi \varsigma(t), \tag{19}
$$

where

$$
\Phi = \begin{bmatrix}
P(A-L_oC)+(A-L_oC)^T P+Q+\tau X+Y+Y^T+2\epsilon S_1^T S_1 & P(A_d - L_oC_d) - Y & P\bar{G} \\
(A_d - L_oC_d)^T P - Y^T & 2\epsilon S_2^T S_2 - Q & 0 \\
\bar{G}^T P & 0 & -\epsilon I
\end{bmatrix}
$$

$$
+\tau \begin{bmatrix} (A - L_oC)^T \\ (A_d - L_oC_d)^T \\ \bar{G}^T \end{bmatrix} W \begin{bmatrix} (A - L_oC)^T \\ (A_d - L_oC_d)^T \\ \bar{G}^T \end{bmatrix}^T, \tag{20}
$$

$$
\varsigma(t) = [e(t)^T\ e(t-\tau)^T\ \xi(x(t),x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))^T]^T. \tag{21}
$$

Now, by (6) it is easy to see that there exists a scalar $a > 0$ such that

$$
\begin{bmatrix}
\Psi + aI & PA_d - ZC_d - Y & PG & -ZH & \tau A^T P - \tau C^T Z^T \\
A_d^T P - C_d^T Z^T - Y^T & 2\epsilon S_2^T S_2 - Q & 0 & 0 & \tau A_d^T P - \tau A_d^T Z^T \\
G^T P & 0 & -\epsilon I & 0 & \tau G^T P \\
-H^T Z^T & 0 & 0 & -\epsilon I & -\tau H^T Z^T \\
\tau PA - \tau ZC & \tau PA_d - \tau ZC_d & \tau PG & \tau ZH & \tau W - 2\tau P
\end{bmatrix} < 0. \tag{22}
$$

Noting $W > 0$ and using Lemma 1, we have

$$
W - 2P \ge -PW^{-1}P. \tag{23}
$$

This together with (22) gives

$$
\begin{bmatrix}
\Psi + aI & PA_d - ZC_d - Y & PG & -ZH & \tau A^T P - \tau C^T Z^T \\
A_d^T P - C_d^T Z^T - Y^T & 2\epsilon S_2^T S_2 - Q & 0 & 0 & \tau A_d^T P - \tau A_d^T Z^T \\
G^T P & 0 & -\epsilon I & 0 & \tau G^T P \\
-H^T Z^T & 0 & 0 & -\epsilon I & -\tau H^T Z^T \\
\tau PA - \tau ZC & \tau PA_d - \tau ZC_d & \tau PG & \tau ZH & -\tau PW^{-1}P
\end{bmatrix} < 0. \tag{24}
$$

Considering (11) and Schur complement, it follows from (24) that

$$
\Phi + \begin{bmatrix} aI & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0, \tag{25}
$$

where Φ is given in (20). From (19) and (25), it is easy to see that

$$
\dot{V}(e_t,t) \le -a\|e(t)\|^2. \tag{26}
$$

By the stability theory in [10], it follows from (26) that (13) is asymptotically stable. This completes the proof.

The following theorem presents a result on the robust observer design for uncertain nonlinear system (Σ).

Theorem 2: Consider the uncertain nonlinear system (Σ) satisfying the Assumption 1. If there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, matrices $P_1 > 0$, $P_2 > 0$, $Q > 0$, $W > 0$, $X > 0$, Y and Z such that the following LMIs hold:

$$
\begin{bmatrix}
\Xi & U_1 - Y + \epsilon_1 \tilde{N}_1^T \tilde{N}_2 & U_2 & \tau \Lambda^T & U_3 \\
U_1^T - Y^T + \epsilon_1 \tilde{N}_2^T \tilde{N}_1 & 2\epsilon_2 \tilde{S}_2^T \tilde{S}_2 + \epsilon_1 \tilde{N}_2^T \tilde{N}_2 - Q & 0 & \tau U_1^T & 0 \\
U_2^T & 0 & -\epsilon_2 I & \tau U_2^T & 0 \\
\tau \Lambda & \tau U_1 & \tau U_2 & \tau W - 2\tau P & 0 \\
\tau U_3^T & 0 & 0 & 0 & -\epsilon_1 I
\end{bmatrix} < 0, \tag{27}
$$

$$\begin{bmatrix} X & Y \\ Y^T & W \end{bmatrix} \geq 0, \tag{28}$$

where $P = \text{diag}(P_1, P_2)$, and

$$\Xi = \Lambda + \Lambda^T + Q + \tau X + Y + Y^T + \varepsilon_1 \tilde{N}_1^T \tilde{N}_1 + 2\varepsilon_2 \tilde{S}_1^T \tilde{S}_1, \tag{29}$$

$$\Lambda = \begin{bmatrix} P_1 A & 0 \\ 0 & P_2 A - ZC \end{bmatrix}, \tag{30}$$

$$U_1 = \begin{bmatrix} P_1 A_d & 0 \\ 0 & P_2 A_d - ZC_d \end{bmatrix}, \tag{31}$$

$$U_2 = \begin{bmatrix} P_1 G & 0 & 0 \\ 0 & P_2 G & -ZH \end{bmatrix}, \tag{32}$$

$$U_3 = \begin{bmatrix} P_1 M_1 \\ P_2 M_1 - ZM_2 \end{bmatrix}, \tag{33}$$

$$\tilde{N}_1 = [\, N_1 \; 0 \,], \quad \tilde{N}_2 = [\, N_2 \; 0 \,], \tag{34}$$

$$\tilde{S}_1 = \begin{bmatrix} S_{1g} & 0 \\ 0 & S_1 \end{bmatrix}, \quad \tilde{S}_2 = \begin{bmatrix} S_{2g} & 0 \\ 0 & S_2 \end{bmatrix}, \tag{35}$$

and S_1 and S_2 are given in (9), then the robust observer design problem is solvable. Furthermore, when (27) and (28) are feasible, a suitable nonlinear observer is given as follows:

$$\begin{aligned} \dot{\hat{x}} = {} & A\hat{x}(t) + A_d \hat{x}(t-\tau) + Gg(\hat{x}(t), \hat{x}(t-\tau)) \\ & + L_r[y(t) - C\hat{x}(t) - C_d \hat{x}(t-\tau) \\ & - Hh(\hat{x}(t), \hat{x}(t-\tau))], \end{aligned} \tag{36}$$

where

$$L_r = P_2^{-1} Z. \tag{37}$$

Proof: Define

$$\tilde{x}(t) = x(t) - \hat{x}(t).$$

Then from (1)-(3) and (36), we obtain

$$\begin{aligned} \dot{\tilde{x}} = {} & (A - L_r C)\tilde{x}(t) + (A_d - L_r C_d)\tilde{x}(t-\tau) \\ & + [(\Delta A(t) - L_r \Delta C(t)]x(t) \\ & + [(\Delta A_d(t) - L_r \Delta C_d(t)]x(t-\tau) \\ & + \bar{G}\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)), \end{aligned} \tag{38}$$

where $\bar{G}$ and $\xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))$ are defined in (14) and (15), respectively.

Considering (1)-(3) and (38), we have

$$\begin{aligned} \dot{\eta}(t) = {} & (A_c + \Delta A_c(t))\eta(t) + (A_{cd} + \Delta A_{cd}(t))\eta(t-\tau) \\ & + G_c \xi_c(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)), \end{aligned} \tag{39}$$

where

$$\eta(t)^T = [\, x(t)^T \tilde{x}(t)^T \,]^T,$$

$$A_c = \begin{bmatrix} A & 0 \\ 0 & A - L_r C \end{bmatrix},$$

$$\Delta A_c(t) = \begin{bmatrix} \Delta A(t) & 0 \\ \Delta A(t) - L_r \Delta C(t) & 0 \end{bmatrix},$$

$$A_{cd} = \begin{bmatrix} A_d & 0 \\ 0 & A_d - L_r C_d \end{bmatrix},$$

$$\Delta A_{cd} = \begin{bmatrix} \Delta A_d(t) & 0 \\ \Delta A_d(t) - L_r \Delta C_d(t) & 0 \end{bmatrix},$$

$$G_c = \begin{bmatrix} G & 0 \\ 0 & \bar{G} \end{bmatrix},$$

and

$$\xi_c(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)) = [\, g(x(t), x(t-\tau))^T \xi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))^T \,]^T$$

By Assumption 1, it is easy to show that

$$\|\xi_c(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))\|^2 \leq 2\|\tilde{S}_1 \eta(t)\|^2 + 2\|\tilde{S}_2 \eta(t-\tau)\|^2, \tag{40}$$

Now we define the following Lyapunov functional candidate for (39):

$$V(\eta_t, t) = \eta(t)^T P\eta(t) + \int_{t-\tau}^{t} \eta(s)^T Q\eta(s)ds + \int_{-\tau}^{0}\int_{t+\alpha}^{t} \eta(s)^T W\eta(s)dsd\alpha.$$

Then, the time-derivative of $V(\eta_t, t)$ along the solution of (39) is given by

$$\begin{aligned} \dot{V}(\eta_t, t) = {} & 2\eta(t)^T P[(A_c + \Delta A_c(t))\eta(t) \\ & + (A_{cd} + \Delta A_{cd}(t))\eta(t-\tau) \\ & + G_c \xi_c(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))] \\ & + \eta(t)^T Q\eta(t) - \eta(t-\tau)^T Q\eta(t-\tau) \\ & + \tau\dot{\eta}(t)^T W\dot{\eta}(t) - \int_{t-\tau}^{t} \dot{\eta}(s)^T W\dot{\eta}(s)\, ds. \end{aligned}$$

By (7) and Lemma 1 in [21], we have

$$\begin{aligned} \dot{V}(\eta_t, t) \leq {} & 2\eta(t)^T P[(A_c + \Delta A_c(t))\eta(t) \\ & + (A_{cd} + \Delta A_{cd}(t))\eta(t-\tau) \\ & + G_c \xi_c(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)) \\ & + \eta(t)^T(Q + \tau X + Y + Y^T)\eta(t) \\ & - 2\eta(t)^T Y\eta(t-\tau) - \eta(t-\tau)^T Q\eta(t-\tau) \\ & + \tau\dot{\eta}(t)^T W\dot{\eta}(t). \end{aligned} \tag{41}$$

Note that

$$[\Delta A_c(t)\; \Delta A_{cd}(t)] = \tilde{M}_1 F(t)[\, \tilde{N}_1 \; \tilde{N}_2 \,],$$

where $\tilde{N}_1$ and $\tilde{N}_2$ are given in (34), and

$$\tilde{M}_1 = \begin{bmatrix} M_1 \\ M_1 - L_r M_2 \end{bmatrix}.$$

Then, by Lemma 1, it can be seen that

$$\begin{aligned}&2\eta(t)^T P[\Delta A_c(t)\eta(t) + \Delta A_{cd}(t)\eta(t-\tau)]\\&= 2\eta(t)^T P\tilde{M}_1 F(t)[\tilde{N}_1\eta(t)+\tilde{N}_2\eta(t-\tau)]\\&\leq \epsilon_1^{-1}\eta(t)^T P\tilde{M}_1\tilde{M}_1^T P\eta(t)\\&\quad+\varepsilon_1[\tilde{N}_1\eta(t)+\tilde{N}_2\eta(t-\tau)]^T\\&\quad[\tilde{N}_1\eta(t)+\tilde{N}_2\eta(t-\tau)].\end{aligned}\tag{42}$$

It follows from (40), (41), and (42) that

$$\begin{aligned}\dot{V}(\eta_t,t)\leq{}& 2\eta(t)^T P[(A_c\eta(t)+A_{cd}\eta(t-\tau)\\&+G_c\xi_c(x(t),x(t-\tau),\hat{x}(t),\hat{x}(t-\tau))]\\&+\eta(t)^T(Q+\tau X+Y+Y^T)\eta(t)\\&-2\eta(t)^T Y\eta(t-\tau)-\eta(t-\tau)^T Q\eta(t-\tau)\\&+\tau\dot{\eta}(t)^T W\dot{\eta}(t)\\&+\epsilon_1^{-1}\eta(t)^T P\tilde{M}_1\tilde{M}_1^T P\eta(t)\\&+\varepsilon_1[\tilde{N}_1\eta(t)+\tilde{N}_2\eta(t-\tau)]^T\\&\quad[\tilde{N}_1\eta(t)+\tilde{N}_2\eta(t-\tau)]\\&-\varepsilon_2\xi_c(x(t),x(t-\tau),\hat{x}(t),\hat{x}(t-\tau))^T\\&\quad\xi_c(x(t),x(t-\tau),\hat{x}(t),\hat{x}(t-\tau))\\&+2\varepsilon_2[\eta(t)^T\tilde{S}_1^T\tilde{S}_1\eta(t)\\&+\eta(t-\tau)^T\tilde{S}_2^T\tilde{S}_2\eta(t-\tau)\\={}&\theta(t)^T\Gamma\theta(t),\end{aligned}$$

where

$$\theta(t)=[\eta(t)^T\ \eta(t-\tau)^T\ \xi_c(x(t),x(t-\tau),\ \hat{x}(t),\ \hat{x}(t-\tau))^T]^T,$$

and

$$\Gamma=\begin{bmatrix}\Theta & PA_{cd}-Y & PG_c\\ A_{cd}^T P-Y^T & 2\epsilon_2\tilde{S}_2^T\tilde{S}_2-Q & 0\\ G_c^T P & 0 & -\epsilon_2 I\end{bmatrix}+\tau\begin{bmatrix}A_c^T\\A_{cd}^T\\G_c^T\end{bmatrix}W\begin{bmatrix}A_c^T\\A_{cd}^T\\G_c^T\end{bmatrix}^T+\varepsilon_1\begin{bmatrix}\tilde{N}_1^T\\\tilde{N}_2^T\\0\end{bmatrix}\begin{bmatrix}\tilde{N}_1^T\\\tilde{N}_2^T\\0\end{bmatrix}^T,$$

$$\begin{aligned}\Theta={}&PA_c+A_c^T P+Q+\tau X+Y+Y^T\\&+2\varepsilon_2\tilde{S}_1^T\tilde{S}_1+\epsilon_1^{-1}P\tilde{M}_1\tilde{M}_1^T P.\end{aligned}$$

Then, by using (27), (28) and following a similar line as in the proof of Theorem 1, we can deduce that there exists a scalar $b > 0$ such that

$$\dot{V}(\eta_t,t)\leq -b\|\eta(t)\|^2.$$

Therefore, we have that (39) is asymptotically stable for all admissible uncertainties satisfying (4) and (5) and nonlinearities satisfying the Assumption 1. This completes the proof.

Remark 1: Theorems 1 and 2 provide a method for designing nominal and robust observers for system (Σ), respectively, and the desired observer can be constructed by solving certain LMIs. The LMI conditions in Theorems 1 and 2 are delay-dependent, which will be less conservative than delay-independent ones. It is also worth pointing out that the proposed LMIs can be solved efficiently, and no tuning of parameters is required [22] although there are several parameters to be determined.

In the case when there are not nonlinearities in system (Σ), that is, the system (Σ) reduces to the following uncertain time-delay system:

$$(\Sigma_1):\ \begin{aligned}\dot{x}(t)&=(A+\Delta A(t))x(t)+(A_d+\Delta A_d(t))x(t-\tau) &(43)\\ y(t)&=(C+\Delta C(t))x(t)+(C_d+\Delta C_d(t))x(t-\tau) &(44)\\ x(t)&=\varphi(t),\quad\forall t\in[-\tau,0], &(45)\end{aligned}$$

where $\Delta A(t)$, $\Delta A_d(t)$ and $\Delta C(t)$ are unknown matrices satisfying (4) and (5). Then, from Theorem 2, we have the following robust observer design result for the above system.

Corollary 1: Consider the uncertain linear system (Σ_1) satisfying the Assumption 1. If there exist a scalar $\varepsilon_1 > 0$, matrices $P_1 > 0$, $P_2 > 0$, $Q > 0$, $W > 0$, $X > 0$, Y and Z such that the following LMIs hold:

$$\begin{bmatrix}\hat{\Xi} & U_1-Y+\epsilon_1\tilde{N}_1^T\tilde{N}_2 & \tau\Lambda^T & U_3\\ U_1^T-Y^T+\epsilon_1\tilde{N}_2^T\tilde{N}_1 & \epsilon_1\tilde{N}_2^T\tilde{N}_2-Q & \tau U_1^T & 0\\ \tau\Lambda & \tau U_1 & \tau W-2\tau P & 0\\ U_3^T & 0 & 0 & -\epsilon_1 I\end{bmatrix}<0,\tag{46}$$

$$\begin{bmatrix}X & Y\\ Y^T & W\end{bmatrix}\geq 0,\tag{47}$$

where $P=\text{diag}(P_1,P_2)$, *and*

$$\hat{\Xi}=\Lambda+\Lambda^T+Q+\tau X+Y+Y^T+\varepsilon_1\tilde{N}_1^T\tilde{N}_1,$$

and Λ, U_1 and U_3 are given in (30), (31), and (33), respectively, then the robust observer design problem is solvable. In this case, a desired robust observer for system (Σ_1) is given as follows:

$$\dot{\hat{x}}=A\hat{x}(t)+A_d\hat{x}(t-\tau)+L_c[y(t)-C\hat{x}(t)-C_d\hat{x}(t-\tau)],\tag{48}$$

where

$$L_c=P_2^{-1}Z.$$

In the case when there are not nonlinearities, neither delays nor parameter uncertainties in system (Σ), it can be found that the designed observer reduces to the usual observer of Luenberger type.

4. NUMERICAL EXAMPLE

In this section, we shall give a numerical example to demonstrate the effectiveness of the proposed method.

Consider a simplified mathematical model of the Mach number dynamic response to guide vane angle changes with $u(t) = 0$, which can be described by [23]:

$$\dot{x}_1(t) = -ax_1 + kax_2(t-\tau), \quad (49)$$

$$\dot{x}_2(t) = x_3(t), \quad (50)$$

$$\dot{x}_3(t) = -\omega^2 x_2 - 2\zeta\omega x_3. \quad (51)$$

As in [23], we choose

$$a = \frac{1}{1.964}, \quad k = -0.0117\text{deg}^{-1},$$

$$\tau = 0.33s, \quad \omega = 6.0\text{rad/s}, \quad \zeta = 0.8.$$

In this example, we suppose the measurement $y(t)$ is

$$\begin{aligned} y(t) = {} & [1 + 0.1f(t)]x_1(t) - 0.5x_2(t) + 0.8x_3(t) \\ & + [0.1f(t) - 0.2]x_2(t-\tau) + 0.3x_3(t-\tau) \\ & + 0.2\sin[0.2x_2(t) + 0.3x_1(t-\tau)], \end{aligned} \quad (52)$$

where $f(t)$ is unknown but satisfies $|f(t)| \le 1$. Then it is easy to see that the system in (49)-(52) satisfies Assumption 1 and has the form in (1) and (2).

To design a robust observer for system (49)-(52), we use Matlab LMI Control Toolbox to solve the LMIs in (27) and (28), and obtain the solution as follows:

$$P_1 = \begin{bmatrix} 10.8219 & 0.1017 & 0.0139 \\ 0.1017 & 6.6886 & 0.7544 \\ 0.0139 & 0.7544 & 0.1945 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 10.3391 & 0.0762 & -0.0524 \\ 0.0762 & 8.7695 & 0.7945 \\ -0.0524 & 0.7945 & 0.1879 \end{bmatrix},$$

$$Q = \text{diag}\left(\begin{bmatrix} 4.8566 & 0.0818 & 0.0005 \\ 0.0818 & 5.1571 & 0.2987 \\ 0.0005 & 0.2987 & 0.2566 \end{bmatrix}, \begin{bmatrix} 5.5738 & 0.0536 & 0.0024 \\ 0.0536 & 4.7562 & 0.2057 \\ 0.0024 & 0.2057 & 0.3562 \end{bmatrix}\right),$$

$$W = \text{diag}\left(\begin{bmatrix} 5.5088 & -0.0362 & -0.0049 \\ -0.0362 & 1.6347 & 0.1376 \\ -0.0049 & 0.1376 & 0.0307 \end{bmatrix}, \begin{bmatrix} 5.3098 & 0.0236 & -0.0109 \\ 0.0236 & 3.3328 & 0.1879 \\ -0.0109 & 0.1879 & 0.0294 \end{bmatrix}\right),$$

$$X = \text{diag}\left(\begin{bmatrix} 5.3139 & -0.1463 & 0.0055 \\ -0.1463 & 4.9421 & 0.3263 \\ 0.0055 & 0.3263 & 0.4507 \end{bmatrix}, \begin{bmatrix} 5.2965 & 0.0389 & -0.0075 \\ 0.0389 & 5.1752 & 0.4145 \\ -0.0075 & 0.4145 & 0.5727 \end{bmatrix}\right),$$

$$Y = \text{diag}\left(\begin{bmatrix} -0.8929 & 0.3876 & 0.0246 \\ -0.0091 & -0.5879 & -0.0439 \\ 0.0057 & -0.0694 & -0.0459 \end{bmatrix}, \begin{bmatrix} -0.7055 & 0.0088 & 0.0030 \\ 0.0536 & -0.5977 & -0.0741 \\ 0.0073 & 0.0048 & -0.0484 \end{bmatrix}\right),$$

$$Z = \begin{bmatrix} -0.4419 \\ -4.1967 \\ -0.3408 \end{bmatrix}, \quad \varepsilon_1 = 4.8440, \quad \varepsilon_2 = 5.7527.$$

Therefore, by Theorem 2 we have that a desired nonlinear observer can be chosen as in (36) with

$$L_r = \begin{bmatrix} -0.0374 \\ -0.5073 \\ 0.3207 \end{bmatrix}.$$

5. CONCLUSIONS

In this paper, we have studied the problem of observer design for a class of state-delayed nonlinear systems with time-varying norm-bounded parameter uncertainties. For both cases with and without parameter uncertainties, the solvability conditions of the problem have been presented and an LMI design approach has been developed. Both of the solvability conditions are delay-dependent. A numerical example has shown the effectiveness of the proposed approach.

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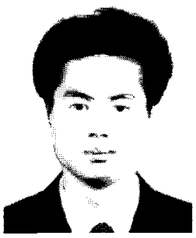
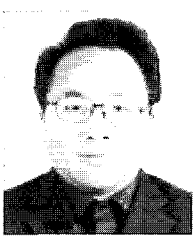
Junwei Lu received the B.Eng. degree from the Nanjing University of Aeronautics and Astronautics, Nanjing, China in 2001. She is working toward a M.Eng. degree in the Nanjing University of Science and Technology, Nanjing, China. Her current research interests include robust filtering and control, time-delay systems and nonlinear systems.

Chunmei Feng received the B.Eng. degree from Shandong University of Science and Technology, Jinan, China, in 1993. She is working toward a M.Eng. degree in the Nanjing University of Science and Technology, Nanjing, China. Her current research interests include robust filtering and control, time-delay systems and nonlinear systems.



Shengyuan Xu received the Ph.D. degree from Nanjing University of Science and Technology in 1999. His research interests include robust filtering and control, singular systems, time-delay systems, multidimensional systems and nonlinear systems.



Yuming Chu received the B.Sc. degree from the Hangzhou Normal University, Hangzhou, China, in 1988, the M.Sc. and Ph.D. degrees from the Hunan University, Changsha, China in 1991 and 1994, respectively. His current research interests include robust filtering and control, quasi-conformal mapping and complex dynamic systems.