Stability Condition of Robust and Non-fragile H^∞ Hovering Control with Real-time Tuning Available Fuzzy Compensator

Joon Ki Kim, Do Hyung Lim, Won Ki Kim, Soon Ju Kang, and Hong Bae Park

Abstract: In this paper, we describe the synthesis of robust and non-fragile H^∞ state feedback controllers for linear systems with affine parameter uncertainties, as well as a static state feedback controller with polytopic uncertainty. The sufficient condition of controller existence, the design method of robust and non-fragile H^∞ static state feedback controller with fuzzy compensator, and the region of controllers that satisfies non-fragility are presented. We show that the resulting controller guarantees the asymptotic stability and disturbance attenuation of the closed loop system in spite of controller gain variations within a resulted polytopic region.

Keywords: Parameterized LMI, relaxation technique, robust and non-fragile H^∞ control.

1. INTRODUCTION

Most plants in the industry have severe nonlinearity and uncertainties. Thus, they post additional difficulties to the control theory of general nonlinear systems and the design of their controllers. It is generally known that feedback systems designed for robustness with respect to plant parameters, or for optimization of a single performance measure, may require very accurate controllers [1]. However, in practice, controllers do have a certain degree of variation due to finite word length and round-off errors in digital systems, as well as the imprecision inherent in analog systems and the need for additional tuning of parameters in the final controller. Therefore, it is necessary that any controller should be able to tolerate some uncertainty in the controller as well as in the plant [1-9].

The control of a helicopter is more difficult than the control of a fixed airfoil aircraft. The difficulties that arise in the control of a helicopter can be broadly classified under three categories: nonlinearity, uncertainty, and instability. The control of a helicopter, therefore, represents a challenge to any method of control system design. Helicopters are nonlinear, multi-variable, and have coupled systems. The question of how to control different nonlinear variables to achieve desirable performance cannot be answered by using a conventional approach. To preserve stability and performance in the presence of helicopter model uncertainties and exogenous disturbances, robust control techniques such as H^2, H^∞, and μ-analysis are useful tools to meet the control requirements. Cho *et al.* [10] proposed a robust and non-fragile H^∞ controller design method for uncertain systems. However, helicopters have severe nonlinearity and uncertainties. Hence, it has need of a compensator for nonlinearity.

In this paper, we propose a robust and non-fragile H^∞ controller design method with real-time tuning available fuzzy compensator. Also the sufficient condition of controller existence, the design method of robust and non-fragile H^∞ static state feedback controller, and the region of controllers that satisfies non-fragility are presented. The sufficient condition is presented using PLMIs, that is, LMIs whose coefficients are functions of a parameter confined to a compact set. However, in contrast to LMIs, PLMI feasibility problems involve infinitely many more LMIs, hence are inherently difficult to solve numerically. Therefore PLMIs are transformed into finitely many LMI problems using relaxation techniques [11,12].

For practical control design, a simple fuzzy control design with guaranteed control performance is more appealing for uncertain nonlinear systems. In this work, we use fuzzy logic controllers of Mamdani type to compensate the nonlinearity. This type of controller has a heuristic nature, which reflects the experience of a human pilot.

The paper is organized as follows. The definition of

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PLMI and basic lemma are described in Section 2 while Section 3 presents the control structure. Finally, Section 5 discusses simulation results for the helicopter hovering problem.

2. PRELIMINARIES

We consider parameterized LMIs (PLMIs), that is, LMIs depending on a parameter θ evoloving in a compact set. The parameter θ can designate parameter uncertainties or system operations but virtually appears. In this case, a particular emphasis is placed on PLMIs of the form

$$M_0(z)+\sum_{i=1}^{L}\theta_i M_i(z)+\sum_{1\le i\le j\le L}\theta_i\theta_j M_{ij}(z)<0, \quad (1)$$

where z is the decision variable, $M_i(z)$, $M_{ij}(z)$ are affine symmetric matrix-valued functions of z, and θ is a parameter confined to either the polytope

$$\theta\in\Gamma:=\{\theta=(\theta_1,\theta_2,\cdots,\theta_L): \sum_{i=1}^{L}\theta_i=1, \theta_i\ge 0, i=1,2,\cdots,L\} \quad (2)$$

or the parameter hyper-rectangle

$$\theta\in\Gamma:=[\alpha\ \beta];\ \alpha,\beta\in\Re^L, \quad \alpha\ge 0, \beta>0, \alpha_i\ge 0, \beta_i>0, i=1,2,\cdots,L, \quad (3)$$

where α_i and β_i are elements of vector α,β each other.

However, PLMI feasibility problems involve an infinite amount of LMIs according to the variations of parameters, hence are very difficult to solve numerically. Computational efforts for locating feasible points are expected to be much greater than those of LMIs. In this paper, we use relaxation techniques where PLMIs are replaced by a finite number of LMIs. Such approaches are potentially conservative but often provide practically exploitable solutions of difficult problems with a reasonable computational effort.

Lemma 1 [11]: The PLMI problem (1) and (2) has a solution z whenever the following quadratic conditions hold,

$$x^T M_0(z)x+\sum_{i=1}^{L}\theta_i x^T M_i(z)x + \sum_{1\le i\le j\le L}\max\left\{-x^T M_{ij}(z)x\cdot\left(\frac{\theta_i^2+\theta_j^2}{2}-\frac{\theta_i+\theta_j}{2}+0.125\right),\ x^T M_{ij}(z)x\cdot\frac{\theta_i^2+\theta_j^2}{2}\right\}<0,$$

$$\theta\in vert\,\Gamma. \quad (4)$$

The latter conditions are readily rewritten as LMIs and can be easily expressed as an LMI feasibility problem. The third term is a tight upper bound of $\theta_i\theta_j x^T M_{ij}(z)x$ with $\theta_i+\theta_j\le 1$. Therefore, if the set Γ is alternatively defined as

$$\theta\in\Gamma:=\{\theta=(\theta_1,\theta_2,\cdots,\theta_L): \sum_{i=1}^{L}\theta_i=\upsilon, \theta_i\ge 0, i=1,2,\cdots,L\}, \quad (5)$$

with $\upsilon>1$, one can use the change of variable $\bar\theta_i=\theta_i/\upsilon$ to recover the case $\bar\theta_i+\bar\theta_j\le 1$. Analogously, applying the change of variable $\theta_i+\theta_j\le 1$ to the constraint (3) yields the relation $\bar\theta\in[0\ 1]^L$.

3. CONTROLLER STRUCTURE

Consider a linear time-varying delayed system with affine parameter uncertainties

$$\begin{aligned}\dot x(t)&=A(t,\xi)x(t)+A_d(t,\xi)x(t-\tau(t))\\&\quad+B_1(t,\xi)w(t)+B_2(t,\xi)u(t),\\ z(t)&=C(t,\xi)x(t),\end{aligned} \quad (6)$$

where $x(t)\in\Re^n$ is the state, $u(t)\in\Re^m$ is the control input, $w(t)\in\Re^l$ is the disturbance input, and $z(t)\in\Re^p$ is the controlled output. The system matrices $A(t,\xi)$, $A_d(t,\xi)$, $B_1(t,\xi)$, $B_2(t,\xi)$, and $C(t,\xi)$ are supposed to have appropriate dimension and the following time-varying structured uncertainties:

$$\begin{aligned}A(t,\xi)&=A_0+\sum_{i=1}^{L}\xi_i(t)A_i,\\ A_d(t,\xi)&=A_{d0}+\sum_{i=1}^{L}\xi_i(t)A_{di},\\ B_1(t,\xi)&=B_{10}+\sum_{i=1}^{L}\xi_i(t)B_{1i},\\ B_2(t,\xi)&=B_{20}+\sum_{i=1}^{L}\xi_i(t)B_{2i},\\ C(t,\xi)&=C_0+\sum_{i=1}^{L}\xi_i(t)C_i.\end{aligned} \quad (7)$$

Also, the time-delay is time-varying and satisfies

$$0\le\tau(t)\le h \quad \dot\tau(t)\le d<1. \quad (8)$$

Although one finds the robust H^∞ state feedback controller $u(t)=Kx(t)$, the actual controller with additive perturbations implemented is assumed as

$$u(t)=[K_0+\Delta K(t)]x(t)=K(t,\xi)x(t), \quad (9)$$

where $K(t,\xi)$ is the region of controller variations, and K_i is the vertices of polytope. And the region of controller variations is rewritten as

$$K(t,\xi) = K_0 + \sum_{i=1}^{L} \xi_i(t)\tilde{K}_i(t), \ \tilde{K}_i = K_i - K_0, \\ \xi_i(t) \geq 0, \ \sum_{i=1}^{L} \xi_i(t) = 1, \ i = 1, 2, \cdots, L. \quad (10)$$

Here, the value of $\tilde{K}_i$ indicates the measure of non-fragility against controller gain variations.

System (6) without time-delay is transformed to the closed loop system with fuzzy compensator of affine form as

$$\dot{x}(t) = \Big[A(t,\xi) + diag\big\{ \Delta u_{f_1}(t), \Delta u_{f_2}(t), \cdots, \Delta u_{f_n}(t) \big\} \\ + B_2(t,\xi)\Big\{ K_0 + \sum_{i=1}^{L} \xi_i(t)\tilde{K}_i \Big\} \Big] x(t). \quad (11)$$

Here,

$$A(t,\xi) = A_0 + \sum_{i=1}^{L} \xi_i(t) A_i \\ + diag\big\{ \Delta u_{f_1}(t), \Delta u_{f_2}(t), \cdots, \Delta u_{f_n}(t) \big\} \quad (12) \\ = \bar{A}(t,\xi).$$

Theorem 1: Consider the linear parameter uncertain system (6) without time-delay. If there exists the positive definite matrix Q, matrices Y_0, and Y_i ($i = 1, 2, \cdots, L$) such that

$$\begin{bmatrix} \Psi & B_1^T(t,\xi) & QC^T(t,\xi) \\ B_1(t,\xi) & -\rho I & 0 \\ C(t,\xi)Q & 0 & -I \end{bmatrix} < 0,$$

$$\Psi = Q\bar{A}^T(t,\xi) + \bar{A}(t,\xi)Q + B_2(t,\xi)Y_0 + Y_0^T B_2^T(t,\xi) \\ + \sum_{i=1}^{L} \xi_i(t)\Big[B_2(t,\xi)Y_i + Y_i^T B_2^T(t,\xi) \Big], \quad (13)$$

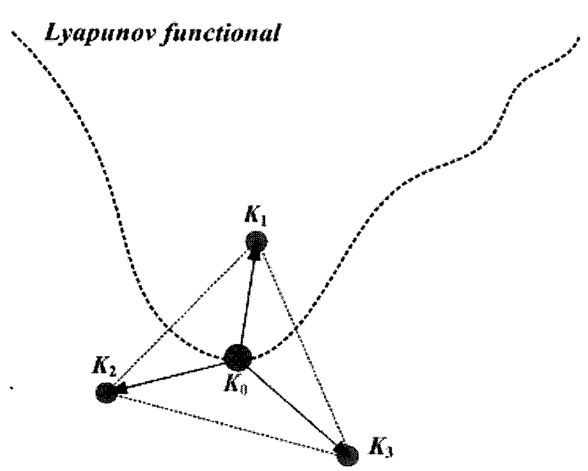

Fig. 1. The basic concept of non-fragile control.

then the closed loop system (11) is asymptotically stable with disturbance attenuation γ and non-fragility. Here, some variables are defined as follows:

$$Q = P^{-1}, \ \rho = \gamma^2, \ Y_0 = K_0 Q, \ Y_i = \tilde{K}_i Q. \quad (14)$$

Proof: When Lyapunov derivative corresponding to the closed loop system with Lyapunov functional $V(x(t),t) = x^T(t)Px(t)$ is negative, suppose that the disturbance input is zero for all time. The closed loop system is asymptotically stable.

Under zero initial condition, let us introduce

$$J = \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]\, dt. \quad (15)$$

Then performance measure (15) for any nonzero $w(t) \in L_2[0 \ \infty)$,

$$J = \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t) \\ + \frac{d}{dt}\{x^T(t)Px(t)\}]dt - x^T(\infty)Px(\infty), \quad (16)$$

then robust H^∞ condition

$$\begin{bmatrix} x^T(t) & w^T(t) \end{bmatrix} \begin{bmatrix} \Xi & PB_1(t,\xi) \\ B_1^T(t,\xi)P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} < 0,$$

$$\Xi = \bar{A}^T(t,\xi)P + P\bar{A}(t,\xi) + C^T(t,\xi)C(t,\xi) \\ + PB_2(t,\xi)K(t,\xi) + K(t,\xi)^T B_2^T(t,\xi)P \quad (17)$$

implies $\| z(t) \|_2 \leq \gamma \| w(t) \|_2$ for all nonzero disturbances. Also, the inequality (17) can be transformed to (13) using Schur complements and change variables in (14). □

The proposed sufficient condition of existence for robust and non-fragile H^∞ static feedback controller (11) is presented using PLMIs, that is LMIs whose coefficients are functions of a parameter confined to a compact set. However, in contrast to LMIs, PLMI feasibility problems involve an infinite number of LMIs, hence are transformed into finitely many LMI problems using relaxation techniques.

Theorem 2: The linear parameter uncertain system (6) is asymptotically stable with disturbance attenuation γ and non-fragility whenever there exist matrices Y_0, Y_i ($i = 1, 2, \cdots, L$), positive definite matrix Q, and positive constant ρ such that

$$x^T M_0(z)x + \sum_{i=1}^{L} \xi_i x^T M_i(z)x + \sum_{j=1}^{L} \xi_j x^T N_j(z)x \\ + \sum_{1 \leq i \leq j \leq L} \max\left\{ -x^T M_{ij}(z)x \cdot \left(\frac{\xi_i^2 + \xi_j^2}{2} \right. \right.$$

$$-\frac{\xi_i+\xi_j}{2}+0.125\Bigg),\ x^T M_{ij}(z)x\cdot\frac{\xi_i^2+\xi_j^2}{2}\Bigg\}<0$$

$$\forall\|x\|=1,\ \left(\xi_i,\ \xi_j\right)\in vert\ \Gamma \tag{18}$$

holds for z, $M_i(z)$, $N_j(z)$, and $M_{ij}(z)$ defined below:

$$M_0(z)=\begin{bmatrix} Q\bar{A}_0^{\,T}+\bar{A}_0Q+B_{20}Y_0+Y_0^TB_{20}^{\,T} & B_{10}^{\,T} & QC_0^T \\ B_{10} & -\rho I & 0 \\ C_0Q & 0 & -I \end{bmatrix},$$

$$M_i(z)=\begin{bmatrix} Q\bar{A}_i^{\,T}+\bar{A}_iQ+B_{2i}Y_i+Y_i^TB_{2i}^{\,T} & B_{1i}^{\,T} & QC_i^T \\ B_{1i} & 0 & 0 \\ C_iQ & 0 & 0 \end{bmatrix},$$

$$N_j(z)=\begin{bmatrix} B_{20}Y_j+Y_j^TB_{20}^{\,T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{19}$$

$$M_{ij}(z)=\begin{bmatrix} B_{2i}Y_j+Y_j^TB_{2i}^{\,T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Proof: Using the modified PLMI form and applying lemma 1, the proof is easily obtained. □

Remark 1: The inequality (13) is converted to *a finite number of LMI problems* in terms of Q, ρ, Y_0, and $Y_i\ (i=1,2,\cdots,L)$ using the relaxation technique of lemma 1. Therefore, the proposed robust and non-fragile H^∞ state feedback controller K_0 and the region of controllers that satisfy non-fragility can be calculated from $\tilde{K}_i=Y_iQ^{-1}\ (i=1,2,\cdots,L)$ after determining the LMI solutions from (18). In addition, the value of disturbance attenuation γ can be obtained by $\gamma=\sqrt{\rho}$ in (12).

Because the controller implementation is subject to imprecision inherent in analog-digital and digital-analog conversion, finite word length, finite resolution measuring instruments and round-off errors in numerical computations, as well as a useful design procedure should generate a controller which also has sufficient space for readjustment of its coefficients. The inequality (18) provides a sufficient condition for the existence of the robust controller under additive control gain perturbations of the form (10).

Remark 2: The proposed robust H^∞ controller is not fragile under additive control gain perturbations and less conservative than controller design algorithms regarding control gain perturbations as system uncertainties. Because control gain perturbations should be independent of system uncertainties, the proposed sufficient condition is *less conservative* than a conventional robust H^∞ controller design algorithm for a linear uncertain system.

Corollary 1: Consider the linear system with affine parameter uncertainties in (6) and the time-varying delay (8). If there exist three positive-definite matrices P, X_1, and X_2 such that

$$\begin{bmatrix} \Pi & \tilde{h}A_d(t,\xi)P & \tilde{d}_hA_d(t,\xi)P & B_1(t,\xi)P & C^T(t,\xi) \\ \tilde{h}PA_d^T(t,\xi) & -X_1 & 0 & 0 & 0 \\ \tilde{d}_hPA_d^T(t,\xi) & 0 & -X_2 & 0 & 0 \\ PB_1^T(t,\xi) & 0 & 0 & -\rho I & 0 \\ C(t,\xi) & 0 & 0 & 0 & -I \end{bmatrix}<0,$$

$$\begin{aligned}\Pi=\bar{A}^T(t,\xi)P+P\bar{A}(t,\xi)+A_d^T(t,\xi)P+A_d^T(t,\xi)P \\ +K^T(t,\xi)B_2^T(t,\xi)P+PB_2(t,\xi)K(t,\xi),\end{aligned} \tag{20}$$

then the closed-loop system is asymptotically delay-dependent stable with disturbance attenuation γ and non-fragility, where $\tilde{h}=\sqrt{h^2+1}$ and $\tilde{d}_h=h/(1-d)$.

Proof: Using the modified PLMI form and applying Lemma 1, the PLMI (20) are transformed into the LMI problems. □

4. HELICOPTER DYNAMICS

Flying a helicopter is a complex control problem [13], due to the multiple inputs and outputs which in addition are partially coupled. A helicopter has a nonlinear system, which is inherently unstable and very sensitive to external disturbances such as wind and ground effects.

Linearization is essential to derive simplified working models, considering the inherent instability under hover condition. Small scale helicopters have very similar characteristics; therefore, it is beneficial to derive a generalized linear model for such types of helicopters, common to all controller designs.

The helicopter system can be considered as a lumped model consisting of a main rotor, a tail rotor, a horizontal stabilizer, a vertical stabilizer, and a fuselage, which are denoted by the subscripts M, T, H, V, and F, respectively.

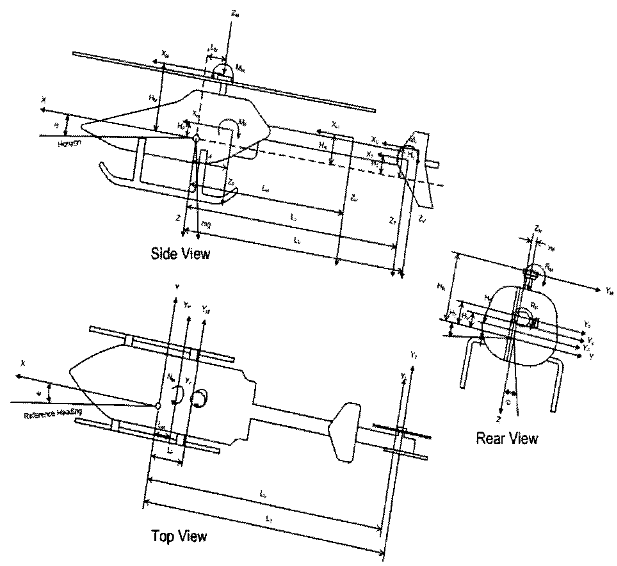

Fig. 2. Free body diagram of helicopter.

Then, we define that $\dot{x}, \dot{y}, \dot{z}, q, p, r, B_1$, and A_1 are longitudinal, lateral, vertical, pitching, rolling, yawing velocity, longitudinal cyclic pitch, and lateral cyclic pitch, respectively. The force experienced by the helicopter is the resultant force of the thrust generated by the main and tail rotors, damping forces from the horizontal and vertical stabilizer, aerodynamic force due to fuselage, and gravitational force. In hover or forward flight with slow velocity, the velocity is so slow that we can ignore the drag contributed from the horizontal, vertical stabilizers, and fuselage. We derive the dynamic equation of the helicopter.

In this work, the helicopter model used to simulate the flight in hover position is an ERGO50 helicopter [14]. The general form used for the state matrix model is shown by (21).

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\\ \dot{\phi}\\ \dot{p}\\ \dot{\theta}\\ \dot{q}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}
\frac{1}{m}\frac{\partial X}{\partial \dot{x}} & \frac{1}{m}\frac{\partial X}{\partial \dot{y}} & 0 & 0 & \frac{1}{m}\frac{\partial X}{\partial p} & -g & \frac{1}{m}\frac{\partial X}{\partial q} & 0\\
\frac{1}{m}\frac{\partial Y}{\partial \dot{x}} & \frac{1}{m}\frac{\partial Y}{\partial \dot{y}} & 0 & g & \frac{1}{m}\frac{\partial Y}{\partial p} & 0 & \frac{1}{m}\frac{\partial Y}{\partial q} & \frac{1}{m}\frac{\partial Y}{\partial r}\\
0 & 0 & \frac{1}{m}\frac{\partial Z}{\partial \dot{z}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
\frac{1}{I_{xx}}\frac{\partial R}{\partial \dot{x}} & \frac{1}{I_{xx}}\frac{\partial R}{\partial \dot{y}} & \frac{1}{I_{xx}}\frac{\partial R}{\partial \dot{z}} & 0 & \frac{1}{I_{xx}}\frac{\partial R}{\partial p} & 0 & \frac{1}{I_{xx}}\frac{\partial R}{\partial q} & \frac{1}{I_{xx}}\frac{\partial R}{\partial r}\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
\frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{x}} & \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{y}} & \frac{1}{I_{yy}}\frac{\partial M}{\partial \dot{z}} & 0 & \frac{1}{I_{yy}}\frac{\partial M}{\partial p} & 0 & \frac{1}{I_{yy}}\frac{\partial M}{\partial q} & \frac{1}{I_{yy}}\frac{\partial M}{\partial r}\\
0 & \frac{1}{I_{zz}}\frac{\partial N}{\partial \dot{y}} & \frac{1}{I_{zz}}\frac{\partial N}{\partial \dot{z}} & 0 & \frac{1}{I_{zz}}\frac{\partial N}{\partial p} & 0 & 0 & \frac{1}{I_{zz}}\frac{\partial N}{\partial r}
\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \phi\\ p\\ \theta\\ q\\ r\end{bmatrix}$$

$$+\begin{bmatrix}
\frac{1}{m}\frac{\partial X}{\partial \theta_M} & 0 & \frac{1}{m}\frac{\partial X}{\partial A_1} & \frac{1}{m}\frac{\partial X}{\partial B_1}\\
\frac{1}{m}\frac{\partial Y}{\partial \theta_M} & \frac{1}{m}\frac{\partial Y}{\partial \theta_T} & \frac{1}{m}\frac{\partial Y}{\partial A_1} & \frac{1}{m}\frac{\partial Y}{\partial B_1}\\
\frac{1}{m}\frac{\partial z}{\partial \theta_M} & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
\frac{1}{I_{xx}}\frac{\partial R}{\partial \theta_M} & \frac{1}{I_{xx}}\frac{\partial R}{\partial \theta_T} & \frac{1}{I_{xx}}\frac{\partial R}{\partial A_1} & \frac{1}{I_{xx}}\frac{\partial R}{\partial B_1}\\
0 & 0 & 0 & 0\\
\frac{1}{I_{yy}}\frac{\partial M}{\partial \theta_M} & \frac{1}{I_{yy}}\frac{\partial M}{\partial \theta_T} & \frac{1}{I_{yy}}\frac{\partial M}{\partial A_1} & \frac{1}{I_{yy}}\frac{\partial M}{\partial B_1}\\
\frac{1}{I_{zz}}\frac{\partial N}{\partial \theta_M} & \frac{1}{I_{zz}}\frac{\partial N}{\partial \theta_T} & 0 & 0
\end{bmatrix}\begin{bmatrix}\theta_M\\ \theta_T\\ A_1\\ B_1\end{bmatrix} \quad (21)$$

There are four control inputs ($\theta_M, \theta_T, A_1, B_1$), corresponding to the main rotor pitch angle, tail rotor pitch angle, lateral cyclic pitch, and longitudinal cyclic pitch.

5. SIMULATION RESULTS

Consider a linear system (6) with affine parameter uncertainties satisfying

$$\begin{aligned}A(t,\xi) &= A_0 + \xi_1(t)\cdot A_1 + \xi_2(t)\cdot A_2,\\ B_2(t,\xi) &= B_{20} + \xi_1(t)\cdot B_{21} + \xi_2(t)\cdot B_{22},\end{aligned} \quad (22)$$

and parameters $\xi_1(t)$ and $\xi_2(t)$ satisfying

$$\xi \in \Gamma := \left\{ \xi = (\xi_1,\ \xi_2) : \sum_{i=1}^{2} \xi_i(t) = 1,\ \xi_i(t) \ge 0 \right\}. \quad (23)$$

Since the moment of inertia (I_{xx}, I_{yy}, I_{zz}) was acquired by measurement, it may have the uncertainty.

Hence, we assume that the moment of inertia and the total mass has the error of 10 percents and 5 percents, respectively. Also, the mathematical modeling of a helicopter has some error in linearization such as $\sin\theta \cong \theta$ and $\cos\theta \cong 1$. We assume that the uncertainties are

$$\begin{aligned} &\sin\overline{\theta} - \overline{\theta} \le \Delta_\theta \le \underline{\theta} - \sin\underline{\theta}, \quad -\underline{\theta} \le \theta \le \overline{\theta}, \\ &-\max\left\{\left|\cos\underline{\phi} - 1\right|, \ \left|\cos\overline{\phi} - 1\right|\right\} \le \Delta_\phi \le 0, \\ &-\underline{\phi} \le \phi \le \overline{\phi}, \end{aligned} \tag{24}$$

where θ and ϕ are the pitch and roll angle, respectively.

The robust and non-fragile H^∞ state feedback gain and vertex of perturbation satisfying non-fragility are represented in (25) using Theorem 2.

$$K_0 = \begin{bmatrix} 0.6306 & 0.1103 & 8.0094 & 10.3277 & 0.8955 & -4.6973 & 0.5309 & 2.1660 \\ 10.1357 & 10.1645 & 20.0893 & 55.6685 & -2.9021 & -59.0670 & 3.8548 & 177.0386 \\ 14.9696 & -33.9123 & 1.5648 & -299.8408 & -17.6717 & -135.1994 & -9.6092 & -27.2139 \\ -30.7624 & -1.4399 & 4.4261 & -18.6963 & -4.7545 & 290.7262 & 20.6915 & 34.1174 \end{bmatrix},$$

$$\tilde{K}_1 = \begin{bmatrix} -0.6754 & -2.4136 & -0.6764 & -20.8111 & -1.0757 & 4.1905 & -1.2236 & -2.6015 \\ -22.7772 & -7.1312 & 5.8185 & -26.8321 & -4.1029 & 166.4240 & 12.8395 & 96.3336 \\ -5.1597 & 10.8536 & -2.0966 & 96.4131 & 3.5835 & 54.2444 & 2.5501 & -2.4807 \\ 2.7301 & -5.0618 & 3.0832 & -38.1736 & -1.8894 & -38.9025 & -0.2748 & 31.0842 \end{bmatrix},$$

$$\tilde{K}_2 = \begin{bmatrix} -0.9919 & -2.6082 & 0.0707 & -22.2489 & -1.0102 & 6.8802 & -1.1269 & -2.8333 \\ 10.0757 & 16.7581 & 14.2640 & 153.9468 & 3.2078 & -102.8227 & -1.0740 & 89.2251 \\ -8.3054 & 5.6192 & -2.4833 & 48.4277 & -0.7497 & 77.1509 & 2.5460 & -4.2375 \\ 9.5374 & 0.5089 & 5.1360 & 4.6866 & 0.1908 & -87.4895 & -0.3977 & 26.9150 \end{bmatrix},$$

$$\gamma = 0.2646. \tag{25}$$

This simulation shows that the vertices of hovering controller polytope guarantee the asymptotic stability. And the disturbance $w(k)$ is defined by (26).

$$w(k) = \begin{cases} 1, & \text{if } 2 \le t \le 5 \text{ sec}, \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

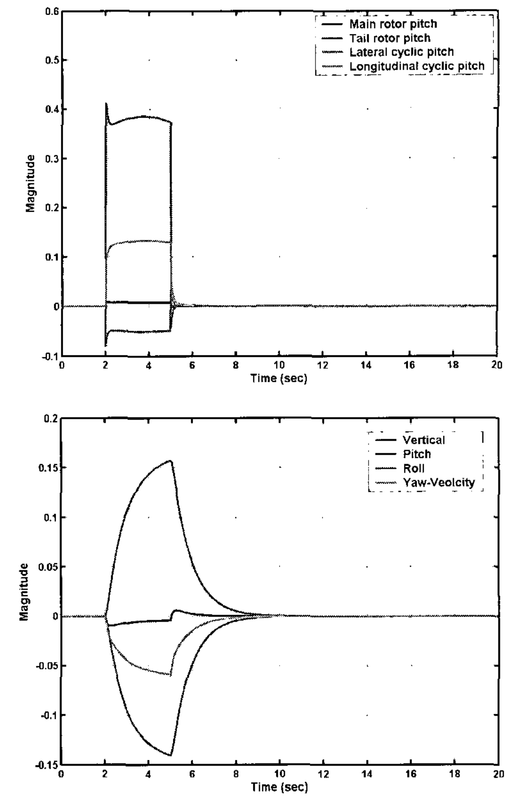

Fig. 3. The hovering response of a helicopter for the nominal controller K_0 with fuzzy compensator.

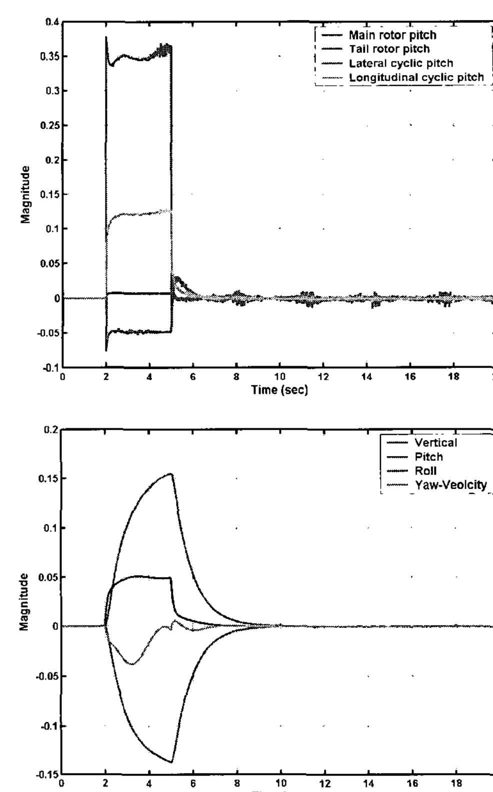

Fig. 4. The hovering response of a helicopter for the actual controller with time-varying additive perturbation $K = K_0 + \sin^2 t \cdot \tilde{K}_1 + \cos^2 t \cdot \tilde{K}_2$.

6. CONCLUSIONS

In this paper, we presented the robust and non-fragile H^∞ hovering controller design method for a helicopter system with affine parameter uncertainties and state feedback controller with polytopic uncertainties. Also, the robust and non-fragile hovering controller and the region of controllers which satisfies non-fragility were calculated at the same time using the PLMI approach.

Because the effects of the input membership functions are not taken into account in the stability conditions, the proposed controller design algorithm allows the input of membership functions in real-time tuning. Although the designed fuzzy rule is erroneous, the proposed controller guarantees the stability of the helicopter in flight.

In spite of the controller gain variations within the resulted polytopic region and incorrect rules, the obtained robust and non-fragile H^∞ hovering controller guaranteed the asymptotic stability and disturbance attenuation from the closed loop system.

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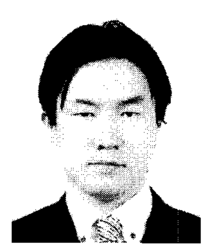

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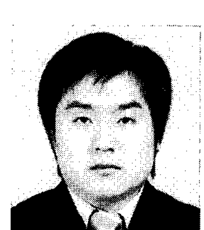

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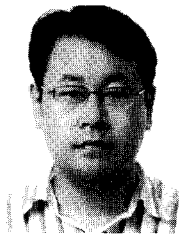

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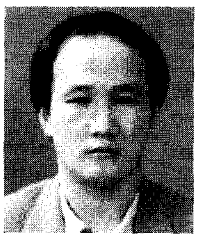

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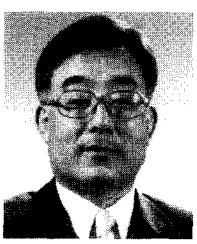

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