Robust H_∞ Control for Uncertain Two-Dimensional Discrete Systems Described by the General Model via Output Feedback Controllers

Huiling Xu, Yun Zou, Shengyuan Xu, and Lei Guo

Abstract: This paper considers the problem of robust H_∞ control for uncertain 2-D discrete systems in the General Model via output feedback controllers. The parameter uncertainty is assumed to be norm-bounded. The purpose is the design of output feedback controllers such that the closed-loop system is stable while satisfying a prescribed H_∞ performance level. In terms of a linear matrix inequality, a sufficient condition for the solvability of the problem is obtained, and an explicit expression of desired output feedback controllers is given. An example is provided to demonstrate the application of the proposed method.

Keywords: Discrete systems, general model, linear matrix inequality, robust H_∞ control, two-dimensional systems, uncertain systems.

1. INTRODUCTION

Two-dimensional (2-D) systems have received much attention during the past decades [10] since 2-D systems have extensive applications in image processing, seismographic data processing, thermal processes, water stream heating, modeling of partial differential equations and other areas [3,8]. Different kinds of 2-D models, such as 2-D Roesser models and 2-D Fornasini-Marchesini models [2] etc., have been proposed. These models have also been extended to multidimensional systems; see, e.g., [11,13]. A great number of fundamental notions and results of one-dimensional (1-D) discrete systems were generalized to 2-D discrete systems [17]. Since the introduction of the general state space model of 2-D systems (2-D GM) in [9], a lot of research topics, such as controllability [5], minimum energy control [4], internal stability [1], computation of 2-D eigenvalues and the transfer function matrix [20] related to 2-D GM have been studied in the literature.

Since the late 1980s, the problem of H_∞ control and filtering for linear systems has drawn considerable attention; many relevant results have been reported in the literature; see, e.g., [14], and the references therein. The robust H_∞ control problem for discrete-time systems was addressed in [19]. Very recently, robust H_∞ control and filtering for 2-D systems described by Roesser models and Fornasini-Marchesini model have been studied; see, e.g., [2,16, 18], and the references therein. However, the problem of robust H_∞ control for 2-D discrete systems described by General models has not been investigated up to date.

In this paper, we are concerned with the problem of robust H_∞ control for 2-D discrete systems in the General model with parameter uncertainties. The parameter uncertainty is assumed to be unknown but norm bounded. The problem to be addressed is to design an output feedback controller such that the resulting closed-loop system is asymptotically stable and satisfies a prescribed H_∞ performance for all admissible uncertainties. A sufficient condition for the solvability of this problem is proposed in terms of a linear matrix inequality (LMI). When this LMI is feasible, an explicit expression of desired output feedback controllers is given. An example is provided to demonstrate the applicability of the proposed methods.

Notation. Throughout this paper, for Hermitian matrices X and Y, the notation $X \geq Y$ (respectively, X

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>Y) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The superscript "T" represents the transpose and the complex conjugate transpose. The notation $\|x\|$ stands for the Euclidean norm of the vector x. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of 2-D discrete-time systems described by the general model [9]:

$$\begin{aligned}(\Sigma): x(i+1,j+1) &= (A_1+\Delta A_1)x(i+1,j) \\ &+(A_2+\Delta A_2)x(i,j+1)+(A_0+\Delta A_0)x(i,j) \\ &+B_1u(i+1,j)+B_2u(i,j+1)+B_0u(i,j) \\ &+L_1\omega(i+1,j)+L_2\omega(i,j+1)+L_0\omega(i,j),\end{aligned} \quad (1)$$

$$z(i,j)=Hx(i,j)+G\omega(i,j)+Bu(i,j), \quad (2)$$

$$y(i,j)=(C+\Delta C)x(i,j)+D\omega(i,j), \quad (3)$$

with the boundary conditions:

$$x(0,j)=x_{0j},\quad x(i,0)=x_{i0},\quad i,j=0,1,2\cdots, \quad (4)$$

where $x(i,j)\in R^n$, $u(i,j)\in R^m$, and $y(i,j)\in R^l$ are respectively, the local state vector, the control input, and measurement output of the plant.

$z(i,j)\in R^q$ is the controlled output, $\omega(i,j)\in R^p$ is the disturbance input which is assumed to belong to $l_2\{[0,\infty),[0,\infty)\}$ A_n, B_n, L_n C, B, D, H and G are known real constant matrices with appropriate dimensions. ΔA_n and ΔC are unknown matrices representing norm-bounded parameter uncertainties, and are assumed to be of the form:

$$\begin{bmatrix}\Delta A_1\\ \Delta A_2\\ \Delta A_0\\ \Delta C\end{bmatrix}=\begin{bmatrix}M_1\\ M_2\\ M_0\\ M_c\end{bmatrix}F(i,j)N, \quad (5)$$

where $F(i,j)\in R^{q\times l}$ is an unknown real matrix satisfying

$$\|F(i,j)\|\le 1, \quad (6)$$

and M_n, $(n=0,1,2)$, M_c and N are known real constant matrices with appropriate dimensions.

An unforced 2-D GM extracted from (1)-(3) is given by

$$\begin{aligned}(\Sigma_1): x(i+1,j+1) &= A_1x(i+1,j)+A_2x(i,j+1) \\ &+A_0x(i,j)+L_1\omega(i+1,j) \quad (7)\\ &+L_2\omega(i,j+1)+L_0\omega(i,j),\end{aligned}$$

$$z(i,j)=Hx(i,j)+G\omega(i,j). \quad (8)$$

Then the transfer function of system is (Σ_1) as follows:

$$T(z_1,z_2)=H(z_1z_2I-z_1A_1-z_2A_2-A_0)^{-1}(z_1L_1+z_2L_2+L_0)+G.$$

Definition 1 [3]: The 2-D GM (Σ_1) is asymptotically stable, if $\sup_{i,j}\|x(i,j)\|<\infty$ and $\lim_{i\to\infty,j\to\infty} x(i,j)=0$ and zero input $\omega(i,j)\equiv 0$ and any boundary condition such that $\sup_i\|x(i,0)\|<\infty$ and $\sup_j\|x(0,j)\|<\infty$.

Definition 2: Consider the 2-D GM (Σ_1) with zero boundary condition. Given a scalar $\gamma>0$, 2-D GM (Σ_1) is said to be asymptotically stable with and H_∞ noise attenuation γ if it is asymptotically stable and satisfies $\|\bar{z}(i,j)\|_2<\gamma\|\bar{\omega}(i,j)\|_2$ for any nonzero $\omega(i,j)\in l_2\{[0,\infty),[0,\infty)\}$, where

$$\bar{z}(i,j)=\left[z(i+1,j)^T,z(i,j+1)^T,z(i,j)^T\right]^T,$$

$$\bar{\omega}(i,j)=\left[\omega(i+1,j)^T,\omega(i,j+1)^T,\omega(i,j)^T\right]^T.$$

Remark 1: Note that the definition is a natural extension of 2-D Fornasini-Marchesini local state-space (FM LSS) model to General Model (GM) case [2].

Remark 2: By the 2-D Parseval's theorem [12], it is easy to see that the condition $\|\bar{z}(i,j)\|_2<\gamma\|\bar{\omega}(i,j)\|_2$ under zero-initial conditions for any nonzero $\omega(i,j)\in l_2\{[0,\infty),[0,\infty)\}$ is equivalent to

$$\|T(e^{j\theta_1},e^{j\theta_2})\|_\infty=\sup_{\theta_1,\theta_2\in[0,2\pi]}\bar{\sigma}[T(e^{j\theta_1},e^{j\theta_2})]<\gamma.$$

where $\bar{\sigma}(\bullet)$ represents the maximum singular value of matrix $(\bullet)$

Now, we consider the following full-order dynamic output feedback controller:

$$(\Sigma_k): \hat{x}(i+1,j+1) = A_{1k}\hat{x}(i+1,j) + A_{2k}\hat{x}(i,j+1) + A_{0k}\hat{x}(i,j) + B_{1k}y(i+1,j) + B_{1k}y(i+1,j) + B_{0k}y(i,j) \quad (9)$$

$$u(i,j) = C_k\hat{x}(i,j) + D_k y(i,j), \quad (10)$$

where $\hat{x}(i,j)$ is the controller state, A_{nk}, B_{nk}, $(n = 0,1,2)$, C_k and D_k are matrices to be determined later. Applying this controller to the uncertain 2D GM $(\sum)$ results in the following closed-loop system:

$$(\bar{\Sigma}): \xi(i+1,j+1) = (\bar{A}_{1c} + \Delta\bar{A}_{1c})\xi(i+1,j) + (\bar{A}_{2c} + \Delta\bar{A}_{2c})\xi(i,j+1) + (\bar{A}_{0c} + \Delta\bar{A}_{0c})\xi(i,j) + \bar{L}_{1c}\omega(i+1,j) + \bar{L}_{2c}\omega(i,j+1) + \bar{L}_{0c}\omega(i,j), \quad (11)$$

$$z(i,j) = (C_c + \Delta C_c)\xi(i,j) + D_c\omega(i,j), \quad (12)$$

where

$$\xi(i,j) = \begin{bmatrix} x(i,j) \\ \hat{x}(i,j) \end{bmatrix}, \quad \bar{M}_n = \begin{bmatrix} M_n + B_n D_k M_c \\ B_{nk} M_c \end{bmatrix},$$

$$\bar{M}_c = BD_k M_c, \quad \bar{N} = \begin{bmatrix} N & 0 \end{bmatrix},$$

$$\Delta\bar{A}_{nc} = \bar{M}_n F\bar{N}, \quad \Delta C_c = \bar{M}_c F\bar{N}.$$

and

$$\bar{A}_{nc} = \begin{bmatrix} A_n + B_n D_k C & B_n C_k \\ B_{nk} C & A_{nk} \end{bmatrix}, \; D_c = G + BD_k D,$$

$$C_c = \begin{bmatrix} H + BD_k C & BC_k \end{bmatrix}, \; \bar{L}_{nc} = \begin{bmatrix} L_n + B_n D_k D \\ B_{nk} D \end{bmatrix},$$

$$n = 0,1,2.$$

The robust H_∞ control problem to be addressed in this paper can be formulated as follows: given a scalar $\gamma > 0$, find an output feedback controller in the form of (Σ_k), such that the resulting closed-loop system $(\bar{\Sigma})$ is asymptotically stable with an H_∞ noise attenuation γ.

We end this section by presenting Lemmas that will be essential in the proof of our main results in the next section.

Lemma 1 [15]**:** Let Υ be a nonsingular symmetric matrix and F satisfy (6). If M and N are constant matrices of appropriate dimensions and there exists a constant $\varepsilon > 0$ such that $\varepsilon I - NTN^T > 0$, then

$$(A + MFN)^T T^{-1}(A + MFN) \le A^T(T - \varepsilon MM^T)^{-1}A + \varepsilon^{-1}N^T N.$$

Lemma 2 [7]**:** The 2-D GM $(\sum_1)$ is asymptotically stable if there exist matrices $P_1 > 0$, $P_2 > 0$, $P_0 > 0$ such that the following LMI holds:

$$A^T PA - \bar{P} < 0,$$

where

$$\bar{P} = diag\{P_1, P_2, P_0\}, \quad A = \begin{bmatrix} A_1 & A_2 & A_0 \end{bmatrix},$$

$$P = P_1 + P_2 + P_0.$$

3. BOUNDED REAL LEMMA

The following bounded real lemma will play an important role in solving the robust H_∞ control problem formulated in next section.

Theorem 1: Given a scalar $\gamma > 0$, the 2-D GM (Σ_1) is asymptotically stable with an H_∞ noise attenuation γ, if there exist matrices $P > 0$, $P_1 > 0$, $P_2 > 0$ such that the following LMI holds:

$$\begin{bmatrix} A^T PA - \bar{P} + \bar{H}^T\bar{H} & A^T PL + \bar{H}^T\bar{G} \\ L^T PA + \bar{G}^T\bar{H} & -\gamma^2 I + L^T PL + \bar{G}^T\bar{G} \end{bmatrix} < 0, \quad (13)$$

where

$$\bar{P} = diag\{P_1 - P_2, P_2, P - P_1\},$$

$$\bar{H} = diag\{H, H, H\}, \bar{G} = diag\{G, G, G\},$$

$$L = \begin{bmatrix} L_1 & L_2 & L_0 \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & A_2 & A_0 \end{bmatrix}.$$

Proof : By (13), it is easy to see that

$$A^T PA - \bar{P} < 0.$$

Noting this and Lemma 2, we have that system (Σ_1) with $\omega(i,j) = 0$ is asymptotically stable. Therefore, $G(z_1, z_2)$ is analytic in $|z_1| \ge 1$, $|z_2| \ge 1$. Next, we shall show

$$U(e^{j\theta_1}, e^{j\theta_2}) = \gamma^2 I - T^*(e^{j\theta_1}, e^{j\theta_2})T(e^{j\theta_1}, e^{j\theta_2}) > 0,$$

for all $\theta_1, \theta_2 \in [0, 2\pi]$.

To this end, we note that (13) implies that there exist a matrix $\Phi > 0$ such that

$$\begin{bmatrix} A^T PA - \bar{P} + \bar{H}^T\bar{H} + \bar{\Phi} & A^T PL + \bar{H}^T\bar{G} \\ L^T PA + \bar{G}^T\bar{H} & -\gamma^2 I + L^T PL + \bar{G}^T\bar{G} \end{bmatrix} < 0, \quad (14)$$

where $\bar{\Phi} = diag\{\Phi, 0, 0\}$.

Pre-multiplying and post-multiplying (14) by

$$\begin{bmatrix} e^{-j\theta_1}I & e^{-j\theta_2}I & I & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-j\theta_1}I & e^{-j\theta_2}I & I \end{bmatrix},$$

and

$$\begin{bmatrix} e^{j\theta_1}I & e^{j\theta_2}I & I & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{j\theta_1}I & e^{j\theta_2}I & I \end{bmatrix}^T,$$

respectively, we obtain that for all $\theta_1,\theta_2 \in [0,2\pi]$

$$\begin{bmatrix} A(-j\theta_1,-j\theta_2)^T \\ L(-j\theta_1,-j\theta_2)^T \end{bmatrix} P\begin{bmatrix} A(j\theta_1,j\theta_2) & L(j\theta_1,j\theta_2) \end{bmatrix} + \begin{bmatrix} -P+\Phi+3H^TH & 3H^TG \\ 3G^TH & -3\gamma^2 I+3G^TG \end{bmatrix} < 0, \tag{15}$$

where

$$A(j\theta_1,j\theta_2) = e^{j\theta_1}A_1 + e^{j\theta_2}A_2 + A_0,$$
$$L(j\theta_1,j\theta_2) = e^{j\theta_1}L_1 + e^{j\theta_2}L_2 + L_0.$$

Therefore, from (15), we have that for all $\theta_1,\theta_2 \in [0,2\pi]$,

$$\Phi - P + 3H^TH + A(-j\theta_1,-j\theta_2)^T \mathrm{P}\, A(j\theta_1,j\theta_2) < 0,$$

and

$$\begin{aligned} &\Phi - P + 3H^TH + A(-j\theta_1,-j\theta_2)^T \mathrm{P}\, A(j\theta_1,j\theta_2) \\ &+[A(-j\theta_1,-j\theta_2)^T PL(j\theta_1,j\theta_2)+3H^TG] \\ &\Lambda^{-1}(j\theta_1,j\theta_2)\times[L(-j\theta_1,-j\theta_2)^T PA(j\theta_1,j\theta_2) \\ &+3G^TH]<0, \end{aligned} \tag{16}$$

where

$$\begin{aligned} &\Lambda(j\theta_1,j\theta_2) \\ &= 3\gamma^2 I - 3G^TG - L(-j\theta_1,-j\theta_2)^T PL(j\theta_1,j\theta_2) > 0. \end{aligned}$$

Let

$$\begin{aligned} &\Omega(j\theta_1,j\theta_2) \\ &= \Phi + [A(-j\theta_1,-j\theta_2)^T PL(j\theta_1,j\theta_2)+3H^TG] \\ &\times \Lambda^{-1}(j\theta_1,j\theta_2)\times[L(-j\theta_1,-j\theta_2)^T \\ &PA(j\theta_1,j\theta_2)+3G^TH]. \end{aligned}$$

It easy to see that $\Omega(j\theta_1,j\theta_2)>0$.

Then, (16) can be rewritten

$$\begin{aligned} &\Omega(j\theta_1,j\theta_2)^T PA(j\theta_1,j\theta_2) - P \\ &+3H^TH + \Omega(j\theta_1,j\theta_2) < 0. \end{aligned} \tag{17}$$

Let

$$R(j\theta_1,j\theta_2) = e^{j\theta_1,j\theta_2} I - A(j\theta_1,j\theta_2).$$

Then, it is east to see that the asymptotic stability of the system (Σ_1) implies that $R(j\theta_1,j\theta_2)$ is invertible for all $\theta_1,\theta_2 \in [0,2\pi]$.

By (16), we have that

$$\begin{aligned} &3U(e^{j\theta_1},e^{j\theta_2}) = 3\gamma^2 I - 3T^*(e^{j\theta_1},e^{j\theta_2})T(e^{j\theta_1},e^{j\theta_2}) \\ &\geq \Lambda(j\theta_1,j\theta_2) - [L(-j\theta_1,-j\theta_2)^T PA(j\theta_1,j\theta_2) \\ &+3G^TH]\Omega(j\theta_1,j\theta_2)^{-1} \\ &\times[A(-j\theta_1,-j\theta_2)^T PL(j\theta_1,j\theta_2)+3H^TG]. \end{aligned} \tag{18}$$

Note that

$$\begin{aligned} &\Omega(j\theta_1,j\theta_2) - [A(-j\theta_1,-j\theta_2)^T PL(j\theta_1,j\theta_2) \\ &+3H^TG]\times\Lambda(j\theta_1,j\theta_2)^{-1}[L(-j\theta_1,-j\theta_2)^T \\ &PA(j\theta_1,j\theta_2)+3G^TH] = \Phi > 0. \end{aligned} \tag{19}$$

Then, by the Schur complement formula, (19) can be written

$$\begin{bmatrix} \Lambda(j\theta_1,j\theta_2) & L(-j\theta_1,-j\theta_2)^T PA(j\theta_1,j\theta_2)+3G^TH \\ A(-j\theta_1,-j\theta_2)^T PL(j\theta_1,j\theta_2)+3H^TG & \Omega(j\theta_1,j\theta_2) \end{bmatrix} > 0. \tag{20}$$

From (18) and (20), we have that $U\left(e^{j\theta_1,j\theta_2}\right)>0$ for all $\theta_1,\theta_2 \in [0,2\pi]$. Thus, the 2-D GM ($\Sigma_1$) has an H_∞ noise attenuation γ. This completes the proof. □

Remark 3: In Theorem 1 degenerate to the existing bounded real lemma for discrete 2-D Fornasini-Marchesini local state-space (FM LSS) model [2]. Thus, Theorem 1 can be regarded as a natural generalization of bounded real lemma of 2-D Fornasini-Marchesini local state-space (LSS) model to General Model (GM) case.

4. ROBUST H_∞ CONTROL

In this section, an LMI approach will be developed to solve the robust H_∞ control problem.

Theorem 2: Consider the uncertain 2-D GM (Σ). Given a scalar $\gamma>0$, then there full-order dynamic output feedback controller in the form of (Σ_k), such that the resulting closed-loop system is asymptotically stable with an H_∞ noise attenuation γ, if there exist matrices $X>0$, $Y>0$, $\Pi_{P_1}>0$, $\Pi_{P_2}>0$, D_k Z_n, Φ_n,Ψ and scalar $\varepsilon_n>0,(n=0,1,2)$ such that

$$\begin{bmatrix} \Theta_1 & 0 & J^T & E^T & 0 & U^T \\ 0 & \Theta_2 & V^T & F^T & 0 & 0 \\ J & V & -\tilde{P} & 0 & Q & 0 \\ E & F & 0 & \Theta_3 & K & 0 \\ 0 & 0 & Q^T & K^T & \Theta_4 & 0 \\ U & 0 & 0 & 0 & 0 & \Theta_5 \end{bmatrix} < 0, \tag{21}$$

where

$$\Theta_1 = diag\left\{-\Pi_{P_1} + \Pi P_2, -\Pi_{P_2}, -\tilde{P} + \Pi_{P_1}\right\},$$

$$\Theta_2 = diag\left\{-\gamma^2 I, -\gamma^2 I, -\gamma^2 I\right\},$$
$$\Theta_3 = diag\{-I, -I, -I\},$$
$$\Theta_4 = diag\{-\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_0 I\},$$
$$\Theta_5 = diag\left\{-\varepsilon_1^{-1} I, -\varepsilon_2^{-1} I, -\varepsilon_0^{-1} I\right\},$$
$$E = diag\{E_1, E_1, E_1\},$$
$$E_1 = \begin{bmatrix} HY + B\Psi & H + BD_K C \end{bmatrix},$$
$$F = diag\{F_1, F_1, F_1\},\ F_1 = G + BD_k D,$$
$$K = diag\{K_1, K_1, K_1\}, K_1 = BD_k M_c,$$
$$U = diag\{U_1, U_1, U_1\}, U_1 = \begin{bmatrix} NY & N \end{bmatrix},$$
$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix},$$
$$J_n = \begin{bmatrix} A_n Y + B_n \Psi & A_n + B_n D_k C \\ Z_n & X^T A_n + \Phi_n C \end{bmatrix},$$
$$V = \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix},\ V_n = \begin{bmatrix} L_n + B_n D_k D \\ X^T L_n + \Phi_n D \end{bmatrix},$$
$$Q = \begin{bmatrix} Q_1 & Q_2 & Q_3 \end{bmatrix},\ \tilde{P} = \begin{bmatrix} Y & I \\ I & X \end{bmatrix},$$
$$Q_n = \begin{bmatrix} M_n + B_n D_k M_c \\ X^T M_n + \Phi_n M_c \end{bmatrix},\quad n = 0,1,2.$$

In this case, a desired dynamic output feedback controller is given in the form of (Σ) with parameters as follows:

$$\begin{aligned} A_{nk} &= S^{-1}(Z_n - X^T A_n Y - SB_{nk} \\ &\quad CY - X^T B_n C_k W^T - X^T B_n D_k CY)W^{-T}, \end{aligned} \tag{22}$$
$$B_{nk} = S^{-1}\left(\Phi_n - X^T B_n D_k\right),$$
$$C_k = \left(\Psi - D_k CY\right)W^{-T}, n = 0,1,2. \tag{23}$$

where S and W are any nonsingular matrices satisfying

$$SW^T = I - XY. \tag{24}$$

Proof : First, from (21) it is easy to see

$$\begin{bmatrix} -Y & -I \\ -I & -X \end{bmatrix} < 0,$$

which $I - XY$ is nonsingular. This ensures that there always exist nonsingular matrices S and W such that (24) is satisfied. Now, we introduce the following nonsingular matrices:

$$\Omega_1 = \begin{bmatrix} Y & I \\ W^T & 0 \end{bmatrix},\quad \Omega_2 = \begin{bmatrix} I & X \\ 0 & S^T \end{bmatrix}. \tag{25}$$

Let

$$P = \Omega_2 \Omega_1^{-1}.$$

Then

$$P = \begin{bmatrix} X & S \\ S^T & \Xi \end{bmatrix}, \tag{26}$$

where

$$\Xi = W^{-1} Y\left(X - Y^{-1}\right) Y W^{-T} > 0.$$

We have $P > 0$. By calculations, it can be verified that the LMI in (21) can be re-written as

$$\begin{bmatrix} \bar{\Theta}_1 & 0 & \bar{J}^T & \bar{E}^T & 0 & \bar{U}^T \\ 0 & \Theta_2 & \bar{V}^T & \bar{F}^T & 0 & 0 \\ \bar{J} & \bar{V} & -\bar{P} & 0 & \bar{Q} & 0 \\ \bar{E} & \bar{F} & 0 & \Theta_3 & \bar{K} & 0 \\ 0 & 0 & \bar{Q}^T & \bar{K}^T & \Theta_4 & 0 \\ \bar{U} & 0 & 0 & 0 & 0 & \Theta_5 \end{bmatrix} < 0, \tag{27}$$

where

$$\bar{\Theta}_1 = diag\left\{\begin{matrix} -\Omega_1^T (P_1 - P_2)\Omega_1, -\Omega_1^T P_2 \Omega_1, \\ -\Omega_1^T P \Omega_1 + \Omega_1^T P_1 \Omega_1 \end{matrix}\right\},$$
$$\bar{J} = \begin{bmatrix} \Omega_2^T \bar{A}_{1c}\Omega_1 & \Omega_2^T \bar{A}_{2c}\Omega_1 & \Omega_2^T \bar{A}_{0c}\Omega_1 \end{bmatrix},$$
$$\bar{V} = \begin{bmatrix} \Omega_2^T \bar{L}_{1c} & \Omega_2^T \bar{L}_{2c} & \Omega_2^T \bar{L}_{0c} \end{bmatrix},$$
$$\bar{Q} = \begin{bmatrix} \Omega_2^T \bar{M}_1 & \Omega_2^T \bar{M}_2 & \Omega_2^T \bar{M}_0 \end{bmatrix}, \bar{P} = \Omega_2^T P^{-1} \Omega_2,$$
$$\bar{E} = diag\{C_c\Omega_1, C_c\Omega_1, C_c\Omega_1\}, \bar{F} = diag\{D_c, D_c, D_c\},$$
$$\bar{U} = diag\{\bar{N}\Omega_1, \bar{N}\Omega_1, \bar{N}\Omega_1\},$$
$$\bar{K} = diag\{\bar{M}_c, \bar{M}_c, \bar{M}_c\}.$$

Now, pre- and post-multiplying (27) by $\text{diag}\left\{\Omega_1^{-T}, \Omega_1^{-T}, \Omega_1^{-T}, I, I, I, \Omega_2^{-T}, I, I, I, I, I, I, I, I, I\right\}$ and its transpose, respectively, and by the Schur complement formula and Lemma 1, the desired result follows immediately.

Remark 4: Theorem 2 provides us with a sufficient condition for the solvability of the robust H_∞ control of uncertain 2-D discrete-time systems described by General Model. Therefore, using the similar approach in [3], it is easy to solve (21) and (23).

5. NUMERICAL EXAMPLE

In this example, we consider the thermal processes in chemical reactors, heat exchangers and pipe furnaces, which can be described by the partial differential equation [3]:

$$\frac{\partial T(x,t)}{\partial x} = -\frac{\partial T(x,t)}{\partial t} - T(x,t) + U(t), \tag{28}$$

where $T(x,t)$ is usually the temperature at $x(space) \in [0, x_f]$ and $t(time) \in [0, \infty]$, and $U(t)$ is a given force function.

Define

$$x(i,j) = T(i,j),$$

where $T(i,j) = T(i\Delta x, j\Delta t)$. It is easy to see that the equation (28) can be converted into the following 2-D GM:

$$\begin{aligned} x(i+1,j+1) &= a_1 x(i+1,j) + a_0 x(i,j) \\ &\quad + b_1 u(i+1,j) + l_0 w(i,j), \end{aligned} \tag{29}$$

where

$$a_1 = 1 - \frac{\Delta t}{\Delta x} - \Delta t,.\ a_0 = \frac{\Delta t}{\Delta x}, b_1 = \Delta t.$$

Assume that the controlled output and the measurement output are given by

$$z(i,j) = 0.4x(i,j) + 0.1u(i,j)$$
$$y(i,j) = 10x(i,j) + 10w(i,j).$$

Let

$$a_1 = 0.4, a_0 = 0.5, \Delta x = 0.2,\ \Delta t = 0.1, b_1 = 0.1, l_0 = 0.1.$$

By Theorem 2, a desired output feedback controller can be constructed as

$$\begin{aligned} \hat{x}(i+1,j+1) &= 0.0695\hat{x}(i+1,j) + 0.1679\hat{x}(i,j) \\ &\quad -4.1849\times 10^{-6} y(i+1,j) - 4.2044\times 10^{-6} y(i,j), \\ u(i,j) &= 8.6721\hat{x}(i,j) - 0.3999 y(i,j). \end{aligned}$$

By the Matlab LMI Control Toolbox, the minimum γ is obtained as $\gamma^* = 0.4001$.

6. CONCLUSIONS

This paper has studied the problem of robust H_∞ control for uncertain discrete 2-D GM systems. A sufficient solvability condition has been proposed. This desired output feedback controllers can be designed by solving a certain LMI. Examples have shown that the proposed approach is effective.

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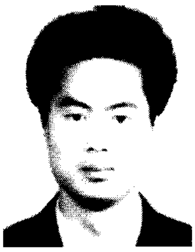

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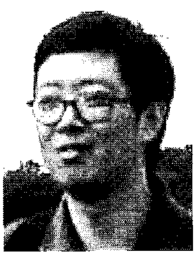

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