Arbitrary Sampling Method for Nonlinearity Identification of Frequency Multipliers

Youngcheol Park[1] · Hoijin Yoon[2]

Abstract

It is presented that sampling rates for behavioral modeling of quasi-memoryless nonlinear devices can be far less than the Nyquist rate of the input signal. Although it has been believed that the sampling rate of nonlinear device modeling should be at least the Nyquist rate of the output signal, this paper suggests that far less than the Nyquist rate of the input signal can be applied to the modeling of quasi-memoryless nonlinear devices, such as frequency multipliers. To verify, a QPSK signal at 820 MHz were applied to a frequency tripler, whereby the device can be utilized as an up-converting mixer into 2.46 GHz with the aid of digital predistortion. AM-AM, AM-PM and PM-PM can be successfully measured regardless of sampling rates.

Key words : Frequency Multipliers, Sampling Rate, Nonlinearity Identification, Pre-Distortion.

I . Introduction

Frequency multiplication is a well-known technique for the generation of high frequency local oscillator signals[1],[2]. Also, if properly done, pre-distortion linearization may be performed to linearize strong baseband nonlinearities to achieve the equivalence of the frequency translation of a modulated carrier into a harmonic zone of the device.

In [3], we showed that frequency multipliers can be characterized and pre-distorted using digital baseband techniques so that complex-modulated signals can be transmitted with minimal distortion. The method involves modifying the device input signal to counteract the signal distortion that arises from gain compression(AM-AM distortion), and phase deviation(AM-PM distortion). In addition, in order to compensate for the time-varying characteristics of devices, adaptive architectures are employed to maintain optimum correction. This requires feedback circuits that samples output signal to compare with the original input, and generate the pre-distortion signal[4]. Regarding the requirements for A/D converters in feedback paths of a pre-distorter, it is often assumed that full Nyquist-rate sampling of the output signal must be used to properly reconstruct waveforms to extract distortion characteristics[4]. As such, the output bandwidth of a frequency multiplier should be n times the input spectrum, which makes it a challenge to sample the output signal with full Nyquist rate for wideband signals. As a result, this sampling requirement burdens the sampling devices and thus the system would be costly for wideband signals such as orthogonal frequency division multiplexing(OFDM) modulation employed in next generation transmitters.

So far, it is generally assumed that at least full Nyquist-rate sampling of the input signal(input Nyquist rate sampling) must be used to properly model distortion characteristics[3],[4]. Therefore, the sampling rate of the input signal burdens the sampling devices and thus the system would be costly for wideband signals. Although some researches have been performed that do not depend on sampling frequencies mostly utilizing statistical distribution of signals[5],[6], they were unable to fully identify the nonlinearities such as AM-PM distortion. Therefore, in this paper, we present a sampling method to characterize AM-AM and AM-PM distortion whereby behavioral modeling can be performed without the limitation of Nyquist sampling requirement. This sampling method makes the system more affordable in design perspective of A/D converters, especially when the signal is wideband and devices have flat frequency response over the instantaneous signal bandwidth. In Section 2, we investigate harmonic(or zonal) transfer-function of frequency multipliers so that it can be utilized to understand complex-envelop signals through frequency multipliers. This harmonic transfer function is investigated whether the sampling interval necessarily affect the performance of the nonlinearity characterization. In Section 3, identification method of AM-AM and AM-PM distortions with the given sampled data at any sampling rate is suggested. Si-

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[1]Dept. of Electronics Engineering, Hankuk University of Foreign Studies, Yongin, Korea.
[2]Dept. of Computer Engineering, Hyupsung University, Hwasung, Korea.

mulation and measured results are presented in Section 4, followed by the conclusions in Section 5.

Ⅱ. Distortion in Frequency Multipliers

A bandpass RF input signal of a frequency multiplier can be represented by

$$x(t) = \mathrm{Re}\{x_L(t)\, e^{jw_c t}\} \tag{1}$$

where $x_L(t)$ is the complex baseband signal, and the l^{th}-zone output can be represented in the Volterra form as below[7],[8].

$$y(t) = \sum_{k=(l-1)/2}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_{2k+1}(\tau_1, \cdots, \tau_{2k+1}) x(t-\tau_1) \cdots x(t-\tau_{2k+1}) d\tau_1 \cdots d\tau_{2k+1} \tag{2}$$

When the device is quasi-memoryless, then the Volterra kernel in (2) can be replaced with the impulse function,

$$h_{2k+1}(\tau_1, \cdots, \tau_{2k+1}) = a_{2k+1}\delta(\tau_1, \cdots, \tau_{2k+1}), \tag{3}$$

resulting in memoryless power series with complex coefficients.

Then, after sampling with a period of T, its discrete sampled input and output signals, respectively, are as below.

$$x_{discrete}(t) = \mathrm{Re}\left[\sum_{k=-\infty}^{\infty} x_L(t)\, e^{jw_c t}\delta(t-kT)\right] \tag{4}$$

$$y_{l^{th}-zone,\, discrete}(t) = \mathrm{Re}\left\{\sum_{m=(l-1)/2}^{\infty}\left[\frac{a_{2m+1}}{2^{2m}}\binom{2m+1}{m+\frac{l+1}{2}} \sum_{k=-\infty}^{\infty} Z_{RF}(t)\,\delta(t-kT)\right]\right\},$$
$$= \mathrm{Re}\left\{\sum_{k=-\infty}^{\infty}\left[F(|x_L(t)|)\, e^{j(lw_c t + l\angle x_L(t) + G(|x_L(t)|))}\delta(t-kT)\right]\right\} \tag{5}$$

where

$$Z_{RF}(t) = |x_L(t)|^{2(m-1)} x_L(t)^l e^{j(lw_c t)}$$
$$F(|x_L(t)|) = \left[\sum_{k=0}^{\infty}\sum_{m=0}^{k} a_{2m+1}a^*_{2k+1-2m}|x_L(t)|^{2(m+1)}\right]^{0.5}, \tag{6}$$

and

$$G(|x_L(t)|) = \tan^{-1}\left[\frac{\sum_{m=0}^{\infty}\mathrm{Im}\{a_{2m+1}|x_L(t)|^{2m+1}\}}{\sum_{m=0}^{\infty}\mathrm{Re}\{a_{2m+1}|x_L(t)|^{2m+1}\}}\right] \tag{7}$$

Nonlinear functions, $F(|x_L(t)|)$, $G(|x_L(t)|)$ are respectively referred as l^{th}-zone AM-AM, and AM-PM functions while the PM-PM function from frequency multiplication is implicated in the exponential term in (5). It is also noticeable that although these functions as well as $\delta(t-kT)$ defines the system output, $F(|x_L(t)|)$ and $G(|x_L(t)|)$ are independent on the sampling instant. Even further, other than the discrete property of the original bandpass signal, the nonlinear coefficients are not dependent on the sampling period, T. Therefore, the information on quasi-memoryless nonlinearity, as in (5) is preserved among samples regardless of sampling periods

Ⅲ. Characterization of Distortions from Sub-Nyquist Rate Sampling-Frequency

When equations (4) and (5) are Fourier transformed, the corresponding signals are expressed by $X(w)$ and $Y(w)$ in the frequency domain.

$$X_{discrete}(w) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(w-kw_s-w_c) + \frac{1}{T}\sum_{k=-\infty}^{\infty} X(-w+kw_s-w_c) \tag{8}$$

$$Y_{discrete}(w) = \frac{1}{T}\sum_{i=0}^{\infty}\sum_{k=-\infty}^{\infty}\left[\begin{array}{l} a_{2i+1}X_{2i+1}(f-kw_s-w_c) \\ +\, a_{2i+1}X_{2i+1}(-w+kw_s-w_c)\end{array}\right] \tag{9}$$

where $X_{2i+1}(w)$ is the $2i+1^{th}$ order signal of $X(w)$

$$a_{2i+1}X_{2i+1}(w) = a_{2i+1}\prod^{2i+1} X(w) \tag{10}$$

When (9) is going through an anti-aliasing filter, the signal would be somewhat deviated: Assuming that the sampling rate is above the Nyquist rate of the input signal, then the input signal $X(w)$ in (9) is just multiplied by a constant whereas some higher order signals are attenuated by the filter due to their wider bandwidths. Thus, the filtered signal of (9) can be shown as below.

$$Y_{discrete}(w) = \frac{1}{T}\sum_{i=0}^{\infty}\left[\begin{array}{l} a_1X(w-w_c) + a_1X(-w-w_c) \\ +\, a'_{2i+1}\sum_{|k|\geq 2}^{\infty} X'_{2i+1}(w-kw_s-w_c) \\ +\, a'_{2i+1}\sum_{|k|\geq 2}^{\infty} X'_{2i+1}(-w+kw_s-w_c)\end{array}\right] \tag{11}$$

Therefore, as the sampling rate is further reduced, the information loss from the anti-aliasing filter gets significant, preventing successful restoration of distortion information. Consequently, the identification process of the distortion is suggested to be purely in the time domain as expressed in (4) and (5). In this case, the identification of a nonlinear system is to extract relations of $|x_L(t)|$ versus $F(|x_L(t)|)$, and $|x_L(t)|$ versus $G(|x_L(t)|)$, whereby the information can be utilized to generate the inverse functions for the transmission $x_L(t)$.

The amplitude function $F(|x_L(t)|)$ can be found by comparing the magnitude of the input and output signals [9]. For the extraction, envelope signals at the input and output terminals are directly captured by synchronized envelope detectors of minimal sampling jitter so that they are compared sample by sample. This method differs from conventional ways such as down-converting with lowpass filtering in the frequency domain in that the whole process is taking place in time-domain. The reason of this method is that since the suggested sampling is to sample signals at sub-Nyquist rates, where the sampled signal is aliased in frequency domain, the sampling of the down-converted envelope signal does not always transfer the full signal information into the digital domain. Similarly, $G(|x_L(t)|)$ should be restored through the comparison of the phases of the input and output signal, while the full restoration of the phase information is not possible by conventional down-conversion followed by a filter when the sampling is below the Nyquist rate. However, the absence of adequate information can be supplemented by capturing quadrature signal so that the relative phase offset at the output can be identified by constituting complex signals for the input and output as below.

$$x_{complex}(t) = x_{in-phase}(t) + j x_{quad-phase}(t),$$
$$\text{and } y_{complex}(t) = y_{in-phase}(t) + j y_{quad-phase}(t) \quad (12)$$

$$\angle x_{complex}(n) = \tan^{-1}\frac{\mathrm{Im}\{x_{complex}(n)\}}{\mathrm{Re}\{x_{complex}(n)\}} \text{ and}$$
$$\angle y_{complex}(n) = \tan^{-1}\frac{\mathrm{Im}\{y_{complex}(n)\}}{\mathrm{Re}\{y_{complex}(n)\}} \quad (13)$$

Also, it is worth to note that the phase in $y_{complex}(t)$ of frequency multipliers has the phase multiplication, which should be considered for the characterization.

After all, even in severe aliasing from the 'sub' Nyquist rate of the signals, the magnitude and phase can successfully restored, and no information loss is involved throughout the characterization, nor any restriction in sampling frequency is involved in the process.

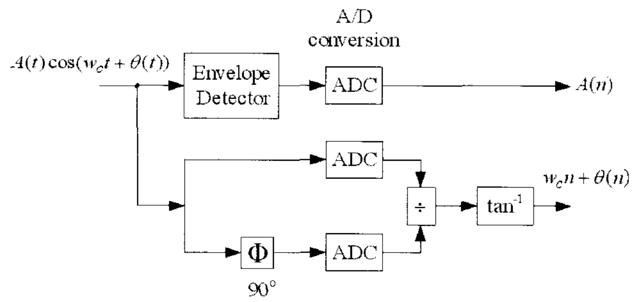


Fig. 2. Detailed diagram of the sub-sampling network in Fig. 1.

Fig. 1 shows the overall block diagram of this idea with sub-Nyquist samplers and nonlinearity identifier in a digital pre-distortion system, and Fig. 2 represents the detailed diagram of the sampling network in Fig. 1.

The key component in this architecture is the sampler that utilizes a narrow-aperture sampling window that effectively aliases the high frequency signal components down into the limited passband of the receiver[10]. In the time domain, this process will preserve the high peaks of the input signal envelope, thus enabling the frequency multiplier to be adequately characterized by comparing the individual pairs of input and output samples.

Ⅳ. Simulation and Measured Results

Simulations on nonlinear characterization of a Schottky-diode frequency tripler at 2.46 GHz with a QPSK signal of 1.25 MHz bandwidth were performed to verify the simplified sampling method.

As shown in Fig. 1, the system identification block identifies the AM/AM, AM/PM characteristics based on the samples from the input and output of the device.

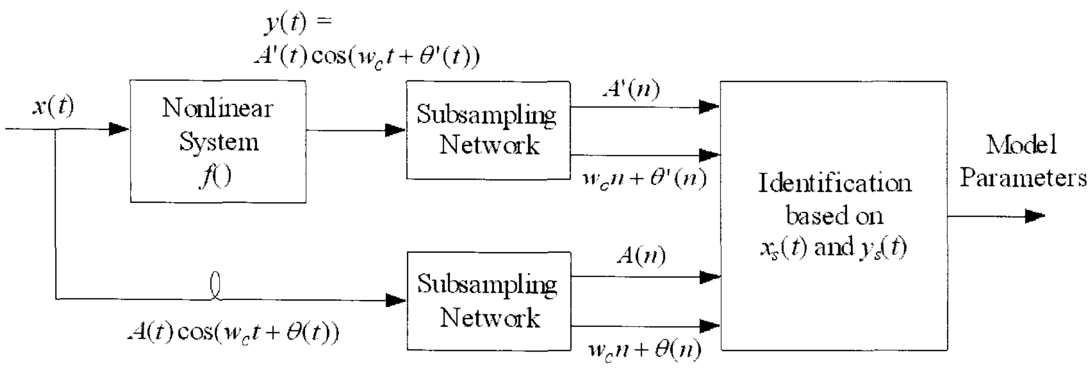


Fig. 1. Block diagram of the simplified sampling-system for the nonlinear device characterization in a digital-predistorter.

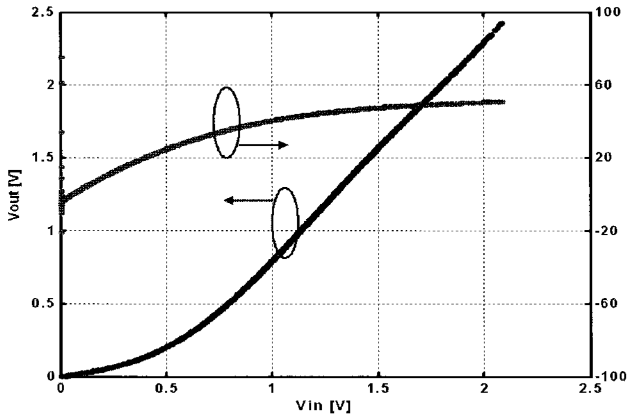


Fig. 3. Simulated AM/AM, AM/PM responses when sampling frequency is ten-times the Nyquist rate.

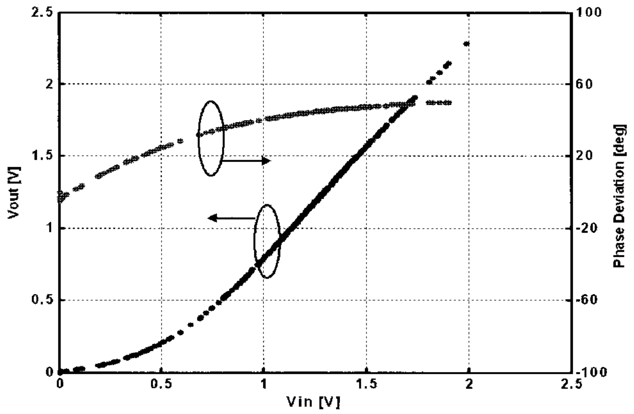
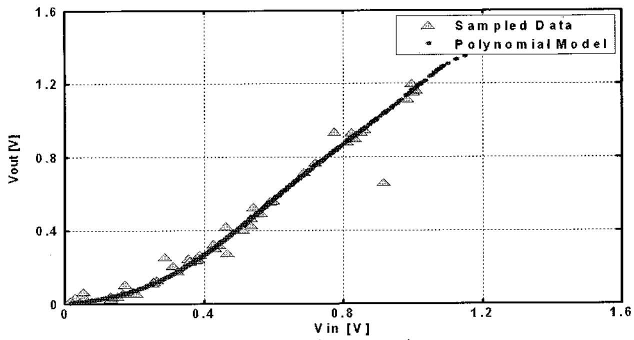


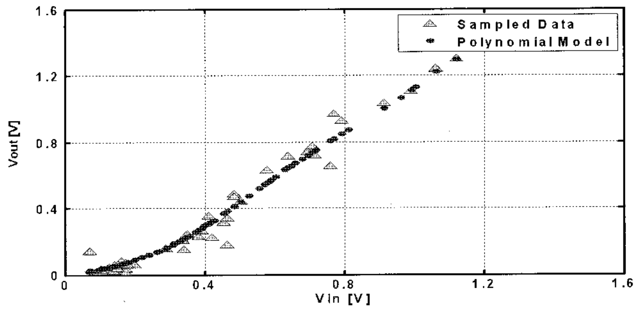
Fig. 4. Simulated AM-AM, AM-PM responses when sampling frequency is one-half the Nyquist rate.

The characterization result with 10-times the Nyquist rate sampling is shown in Fig. 3, whilst the result with 1/2 of the Nyquist rate sampling is shown in Fig. 4. Notice that the asymptotic lines of these results are identical except that Fig. 4 shows sparse dots because less number of samples was collected to represent the difference between two sampling frequencies. However, given the same number of samples were used, the characterization results would be indistinguishable at any sampling rates. Therefore, in both cases, memoryless frequency tripler can be equally identified whereby the transmission of complex modulated signal can be made.

The simulated frequency tripler was also implemented to transmit the signal to 2.46 GHz with the input at 820 MHz. A complex modulated signal was generated using MATLAB, and loaded into an Agilent E4432B arbitrary waveform signal generator. The up-converted RF signal was then applied to the tripler. The output signal from the device is down-converted to IF and digitized through a digital oscilloscope. Then, the nonlinear characterization is processed so that AM-AM and AM-PM polynomial models of the frequency tripler are extracted in a PC. By delicately changing the sampling period of a digi-

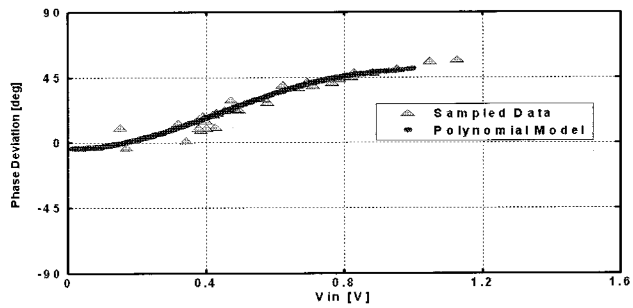


(a) Ten-times the Nyquist rate

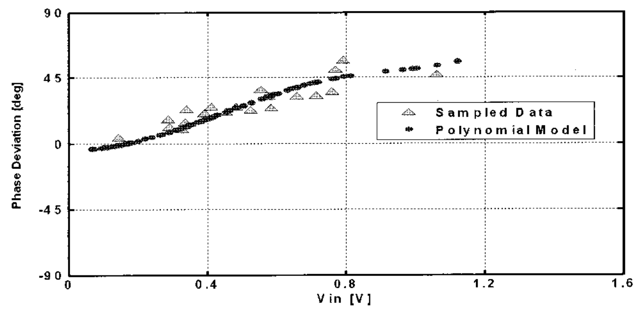


(b) One half the Nyquist rate

Fig. 5. AM-AM responses of measured, and modeled data when sampling frequencies.



(a) Ten-times the Nyquist rate



(b) One half the Nyquist rate

Fig. 6. AM-PM responses of measured, and modeled data when sampling frequencies.

tizer, we can control the amount of aliasing in frequency domain at lower sampling rates.

AM-AM characteristics measured with two different sampling rates were shown in Fig. 5. The first graph shows the results with samples taken with 10-times higher than the Nyquist rate, where none in the output spectrum is aliased, while the second graph shows the results with one half the Nyquist rate. From these figures, we can see that polynomial models can be successfully extracted with a reasonable accuracy regardless of the sampling frequency. Note that the sparse dots in one half the Nyquist rate were intentionally shown to express a fewer number of samples. But it can be compensated when it is sampled for longer period of time. Consequently, models of both cases can be equally used in digital pre-distorters. Similarly, AM-PM characteristics were measured in the same way, and successful characterization results of two sampling rates as shown in Fig. 6.

V. Conclusion

With the aid of pre-distortion techniques, a frequency multiplier can act as an up-converting mixer for complex-modulated signal via frequency multiplication. In the pre-distorter, in case of wideband signals such as OFDM, sampling signals above the Nyquist rate may be unrealistic with conventional samplers which normally have bandwidth of tens of megahertz. Therefore, a simplified sampling-architecture for pre-distorters of quasi-memoryless frequency multipliers has been developed, and has also shown that sampling below the Nyquist rate of the output bandwidth did not adversely affect the characterization of the frequency tripler. Although sampling below the Nyquist rate of the output signal causes aliasing of the signal spectrum, this method showed the possibility of being used for the characterization of a quasi-memoryless frequency tripler without any degradation of performance. Obviously, based on this characterization, the AM-AM, AM-PM, and PM-PM distortions might be compensated for, using a method previously reported by the authors[2].

After an envelope detector followed by a 90-degree phase shifter, three A/D converters collect samples of in-phase and quadrature-phase RF signals at the input and output of frequency multiplier. Consequently, time-domain nonlinearity characterization is processed without any information loss regardless of severe aliasing in frequency domain. Therefore, it has been proven that in a memoryless nonlinear system, if the equivalent number of samples were provided, the characterization results at any sampling rates are identical although longer collection time is required with lower sampling rate to get the same number of samples.

For the realization of the system, narrow aperture sampler is the key element to minimize the ambiguity in sampling timing due to the aperture jitter in three A/D converters. Otherwise, down-converters from RF to IF may be required to reduce the sampling uncertainty. However, benefits of arbitrary-rate sampling are still effective.

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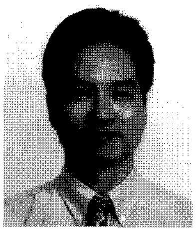
Youngcheol Park



Dr. Park completed his B.S. in 1992 and M.S. in 1994, both from Yonsei University. In 2004, he completed his Ph.D. in electrical engineering at Georgia Tech. In March 2007, Dr. Park joined the faculty at Hankuk University of Foreign Studies, where he is Assistant Professor in Dept. of Electronics Engineering. Dr. Park has over 12 years of industrial experience of such as CDMA, mWiMAX mobiles as well as RF circuits and systems.

Hoijin Yoon

received B.S. and M.S. degrees in computer science and engineering from Ewha Womans University in 1993 and 1998, respectively. She also received her Ph.D. about software component testing from Ewha Womans University. Afterwards, she was at Georgia Institute of Technology and Ewha Womans University before she joined the faculty at Hyupsung University in 2007.