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SLIGHTLY FUZZY CONTINUOUS MAPPINGS

M. Sudha, E. Roja and M. K. Uma

Abstract. In this paper slightly fuzzy continuous mappings is introduced and some interesting properties and characterizations of slightly fuzzy continuous mappings are discussed.

1. Introduction

Fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [13]. Fuzzy sets have applications in many fields such as information [11] and control [12]. The theory of fuzzy topological spaces was introduced and developed by Chang [5] and since then various notions in classical topology have been extended to fuzzy topological spaces [1-4]. The concept of slightly continuous mappings was introduced by Singal and Jain [9].

This paper is devoted to the study of slightly fuzzy continuous mappings. Some interesting properties and characterizations of slightly fuzzy continuous mappings are discussed with necessary examples. In this paper, almost* fuzzy continuous mappings, θ*-fuzzy continuous mappings and weakly* fuzzy continuous mappings are introduced and studied. It is observed that slightly fuzzy continuous mappings preserve fuzzy connectedness and every slightly fuzzy continuous mapping into a fuzzy extremally disconnected space is almost* fuzzy continuous.

2. Preliminaries

Definition 2.1. ([10]) A mapping $f : X \to Y$ is said to be almost continuous at a point $x \in X$, if for every neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset \text{int cl}\, V$.

Definition 2.2. ([6]) A mapping $f : X \to Y$ is said to be θ-continuous at a point $x \in X$, if for every neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(\text{cl}\, U) \subset \text{cl}\, V$.

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Definition 2.3. ([7]) A mapping $f : X \to Y$ is said to be weakly continuous at a point $x \in X$, if for each point $x \in X$ and each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset \mathrm{cl}\, V$.

Definition 2.4. ([9]) A mapping $f : X \to Y$ is said to be slightly continuous if for each point $x \in X$ and each clopen neighbourhood V of $f(x)$, there exists a neighbourhood U of x such that $f(U) \subset V$.

Definition 2.5. ([8]) Let $(D, \geq)$ be a directed set. Let X be an ordinary set. Let $\mathcal{F}$ be the collection of all fuzzy points in X. The function $S : D \to \mathcal{F}$ is called a fuzzy net in X. In otherwords, a fuzzy net is a pair $(S, \geq)$ such that S is a function : $D \to \mathcal{F}$ and $\geq$ directs the domain of S. For $n \in D$, $S(n)$ is often denoted by S_n and hence a net S is often denoted by $\{S_n, n \in D\}$.

Definition 2.6. ([5]) A sequence of fuzzy sets, say $\{A_n; n = 1, 2, \cdots\}$, is eventually contained in a fuzzy set A iff there is an integer m such that, if $n \geq m$, then $A_n \subset A$.

Definition 2.7. ([1]) Let (X, T) and (Y, S) be any two fuzzy topological spaces. For a mapping $f : (X, T) \to (Y, S)$, the graph $g : X \to X \times Y$ of f is defined by $g(x) = (x, f(x))$, for each $x \in X$.

Definition 2.8. ([3]) A fuzzy topological space (X, T) is said to be fuzzy extremally disconnected if the closure of every fuzzy open set is fuzzy open.

3. Properties and characterization of slightly fuzzy continuous mappings

Definition 3.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be almost* fuzzy continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mathrm{int}\ \mathrm{cl}\, \mu$.

Definition 3.2. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be θ*-fuzzy continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\mathrm{cl}\, \sigma) \leq \mathrm{cl}\, \mu$.

Definition 3.3. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be weakly* fuzzy continuous if every fuzzy set $\alpha \in I^X$ and every fuzzy open set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mathrm{cl}\, \mu$.

Definition 3.4. Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f : (X, T) \to (Y, S)$ is said to be slightly fuzzy continuous, if for every fuzzy set $\alpha \in I^X$ and every fuzzy clopen set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mu$.

Remark 3.1. A weakly fuzzy continuous mapping is slightly fuzzy continuous obviously.

Hence **fuzzy continuity $\Rightarrow$ almost* fuzzy continuity $\Rightarrow$ θ*- fuzzy continuity $\Rightarrow$ weak* fuzzy continuity $\Rightarrow$ slight fuzzy continuity.**

But none is reversible and the following examples show the same.

Example 3.1. Let $X = \{a, b, c\}$. Define $T_1 = \{0, 1, \lambda\}$ and $T_2 = \{0, 1, \mu, \gamma, \delta_1, \delta_2\}$ where λ, μ, γ, $\delta_1, \delta_2 : X \to [0, 1]$ are such that $\lambda(a) = 0.5$, $\lambda(b) = 0.5$, $\lambda(c) = 0.5$, $\mu(a) = 0.5$, $\mu(b) = 0.5$, $\mu(c) = 0.5$, $\gamma(a) = 1$, $\gamma(b) = 0.5$, $\gamma(c) = 0.5$, $\delta_1(a) = 0$, $\delta_1(b) = 0$, $\delta_1(c) = 0.3$, $\delta_2(a) = 1$, $\delta_2(b) = 0$, $\delta_2(c) = 0.3$. Clearly, (X, T_1) and (X, T_2) are fuzzy topological spaces. Define $f : (X, T_1) \to (X, T_2)$ as $f(a) = b$, $f(b) = a$ and $f(c) = c$. Let $\alpha : X \to [0, 1]$ be any fuzzy set such that $\alpha(a) = 0$, $\alpha(b) = 0$, and $\alpha(c) = 1$. For the fuzzy clopen set μ in (X, T_2), $f(\alpha) \leq \mu$. Now, λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\lambda) \leq \mu$. **Hence f is slightly fuzzy continuous.**

$f(\alpha) \leq \delta_1$, where δ_1 is a fuzzy open set in (X, T_2). But λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\lambda) \not\leq \operatorname{cl} \delta_1$. **Hence f is not weakly * fuzzy continuous.**

Example 3.2. Let $X = \{a, b, c\}$. Define $T_1 = \{0, 1, \lambda\}$ and $T_2 = \{0, 1, \mu, \gamma\}$ where λ, μ, $\gamma : X \to [0, 1]$ are such that $\lambda(a) = 0.3$, $\lambda(b) = 0$, $\lambda(c) = 0.5$, $\mu(a) = 0.5$, $\mu(b) = 0.5$, $\mu(c) = 0.5$, $\gamma(a) = 1$, $\gamma(b) = 0.5$, $\gamma(c) = 0.5$. Clearly, (X, T_1) and (X, T_2) are fuzzy topological spaces. Define $f : (X, T_1) \to (X, T_2)$ as $f(a) = b$, $f(b) = a$, $f(c) = c$. Let $\alpha : X \to [0, 1]$ be any fuzzy set such that $\alpha(a) = 0$, $\alpha(b) = 0$, $\alpha(c) = 0.1$. For every fuzzy open set ρ in (X, T_2) with $f(\alpha) \leq \rho$, λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\lambda) \leq \operatorname{cl} \rho$. **Hence f is weakly* fuzzy continuous.**

$f(\alpha) \leq \mu$, where μ is a fuzzy open set in (X, T_2). But λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\operatorname{cl} \lambda) \not\leq \operatorname{cl} \mu$. **Hence f is not θ*-fuzzy continuous.**

Example 3.3. Let $X = \{a, b, c\}$. Define $T_1 = \{0, 1, \lambda\}$ and $T_2 = \{0, 1, \mu, \gamma\}$ where λ, μ, $\gamma : X \to [0, 1]$ are such that $\lambda(a) = 0.7$, $\lambda(b) = 1$, $\lambda(c) = 0.5$, $\mu(a) = 0.5$, $\mu(b) = 0.5$, $\mu(c) = 0.5$, $\gamma(a) = 1$, $\gamma(b) = 0.5$, $\gamma(c) = 0.5$. Clearly, (X, T_1) and (X, T_2) are fuzzy topological spaces. Define $f : (X, T_1) \to (X, T_2)$ as $f(a) = b$, $f(b) = a$, $f(c) = c$. Let α be any fuzzy set such that $\alpha(a) = 0$, $\alpha(b) = 0$, $\alpha(c) = 0.2$. For every fuzzy open set ρ in (X, T_2) with $f(\alpha) \leq \rho$, λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\operatorname{cl} \lambda) \leq \operatorname{cl} \rho$. **Therefore f is θ*-continuous.**

$f(\alpha) \leq \delta_1$, where δ_1 is a fuzzy open set in (X, T_2). But λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\lambda) \not\leq \operatorname{int} \operatorname{cl} \delta_1$. **Therefore f is not almost* fuzzy continuous.**

Example 3.4. Let $X = \{a, b, c\}$. Define $T_1 = \{0, 1, \lambda\}$ and $T_2 = \{0, 1, \mu, \gamma\}$ where λ, μ, $\gamma : X \to [0, 1]$ are such that $\lambda(a) = 0.3$, $\lambda(b) = 0.3$, $\lambda(c) = 0.5$, $\mu(a) = 0.5$, $\mu(b) = 0.5$, $\mu(c) = 0.5$, $\gamma(a) = 1$, $\gamma(b) = 0.5$, $\gamma(c) = 0.5$. Clearly,

(X, T_1) and (X, T_2) are fuzzy topological spaces. Define $f : (X, T_1) \to (X, T_2)$ as $f(a) = b$, $f(b) = a$, $f(c) = c$. Let $\alpha : X \to [0, 1]$ be such that $\alpha(a) = 0$, $\alpha(b) = 0$, $\alpha(c) = 0.1$. For the fuzzy clopen set μ in (X, T_1) with $f(\alpha) \leq \mu$, λ is a fuzzy open set in (X, T_1) with $\alpha \leq \lambda$ such that $f(\lambda) \leq \text{int cl}\,\mu$. **Hence f is almost* fuzzy continuous.**

For the fuzzy open sets μ, γ in (X, T_2), $f^{-1}(\mu)$ and $f^{-1}(\gamma)$ are not fuzzy open in (X, T_1). **Hence f is not fuzzy continuous.**

Proposition 3.1. Let (X, T_1) and (Y, T_2) be any two fuzzy topological spaces. For a mapping $f : (X, T_1) \to (X, T_2)$ the following conditions are equivalent:

(a) f is slightly fuzzy continuous.

(b) Inverse image of every fuzzy clopen set of (Y, T_2) is a fuzzy open set of (X, T_1).

(c) Inverse image of every fuzzy clopen set of (Y, T_2) is a fuzzy clopen set of (X, T_1).

(d) For each fuzzy set $\alpha \in I^X$ and for every fuzzy net $\{S_n, n \in D\}$ which converges to α, the fuzzy net $\{f(S_n), n \in D\}$ is eventually in each fuzzy clopen set λ with $f(\alpha) \leq \lambda$.

Proof. $(a) \Rightarrow (b)$. Let σ be a fuzzy clopen set of (Y, T_2). Let $\lambda \in I^X$ be any fuzzy set such that $\lambda \leq f^{-1}(\sigma)$. Then $f(\lambda) \leq \sigma$. Now σ is a fuzzy clopen set with $f(\lambda) \leq \sigma$. Hence by (a), there exists a fuzzy open set μ of (X, T_1) with $\lambda \leq \mu$ such that $f(\mu) \leq \sigma$. Hence $f^{-1}(\sigma)$ is a fuzzy open set of (X, T_1).

$(b) \Rightarrow (c)$. Let β be a fuzzy clopen set of (Y, T_2). Now, $1 - \beta$ is a fuzzy clopen set of (Y, T_2). Therefore by (b), $f^{-1}(1-\beta) = 1 - f^{-1}(\beta)$ is a fuzzy open set of (X, T_1). That is, $f^{-1}(\beta)$ is a fuzzy closed set of (X, T_1). By (b), $f^{-1}(\beta)$ is a fuzzy open set of (X, T_1). Thus $f^{-1}(\beta)$ is a fuzzy clopen set of (X, T_1).

$(c) \Rightarrow (d)$. Let $\{S_n, n \in D\}$ be a fuzzy net converging to a fuzzy set α and let μ be a fuzzy clopen set with $f(\alpha) \leq \mu$. By (c), there exists a fuzzy open set λ with $\alpha \leq \lambda$ such that $f(\lambda) \leq \mu$. Since the net $\{S_n, n \in D\}$ converges to α implies $S_n \leq \alpha$. Now, $S_n \leq \alpha \leq \lambda$. Thus $f(S_n) \leq f(\lambda) \leq \mu$. Hence $\{f(S_n), n \in D\}$ is eventually in μ.

$(d) \Rightarrow (a)$. Suppose that f is not slightly fuzzy continuous. Then there does not exist a fuzzy open set λ with $\alpha \leq \lambda$, such that $f(\lambda) \leq \mu$ and hence $f(S_n) \leq \mu$. That is the fuzzy net $\{f(S_n), n \in D\}$ is not eventually in a clopen set μ with $f(\alpha) \leq \mu$, which is a contradiction. Hence f is slightly fuzzy continuous.

□

Proposition 3.2. Let (X, T) be any fuzzy topological space. Let A be a subspace of X. Then the inclusion map $j : (A, T/A) \to (X, T)$ is slightly fuzzy continuous.

Proof. The inclusion map is fuzzy continuous and hence is slightly fuzzy continuous. □

Proposition 3.3. Let (X,T), (Y,S) and (Z,R) be any three fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ are slightly fuzzy continuous mappings then their composition $g\circ f$ is slightly fuzzy continuous.

Proof. Let λ be a fuzzy clopen set of (Z,R). By Proposition 3.1, $g^{-1}(\lambda)$ is a fuzzy clopen set of (Y,S). Since f is slightly fuzzy continuous $f^{-1}(g^{-1}(\lambda))$ is a fuzzy open set of (X,T). But $f^{-1}(g^{-1}(\lambda))=(g\circ f)^{-1}(\lambda)$. Hence $g\circ f$ is slightly fuzzy continuous. □

Proposition 3.4. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be fuzzy open and let $g:(Y,S)\to(Z,R)$ be any function. Then $g\circ f:(X,T)\to(Z,R)$ is slightly fuzzy continuous iff g is slightly fuzzy continuous.

Proof. Let λ be a fuzzy clopen set of (Z,R). By Proposition 3.1, $(g\circ f)^{-1}(\lambda)$ is fuzzy open in (X,T). That is $f^{-1}(g^{-1}(\lambda))$ is fuzzy open. Since f is fuzzy open, $f(f^{-1}(g^{-1}(\lambda)))$ is fuzzy open in (Y,S). That is $g^{-1}(\lambda)$ is fuzzy open in (Y,S). Therefore by Proposition 3.1, g is slightly fuzzy continuous. □

Proposition 3.5. Every restriction of a slightly fuzzy continuous mapping is slightly fuzzy continuous.

Proof. Let $f:(X,T)\to(Y,S)$ be a slightly fuzzy continuous mapping and let A be a subspace of X. Then the restriction map $f/A:(A,T/A)\to(X,T)$ is the composition of f and the inclusion map j. Hence the proof follows from Proposition 3.2 and Proposition 3.3. □

Proposition 3.6. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be a mapping. Then the graph of $f, g:X\to X\times Y$ is slightly fuzzy continuous iff f is slightly fuzzy continuous.

Proof. Suppose that $f:(X,T)\to(Y,S)$ is slightly fuzzy continuous and let $g:X\to X\times Y$ be the graph of f. Let $\lambda\times\mu$ be a fuzzy clopen set of $X\times Y$. Then

$$\begin{aligned} g^{-1}(\lambda\times\mu)(x) &= (\lambda\times\mu)g(x)\\ &= (\lambda\times\mu)(x,f(x))\\ &= (\lambda(x),\mu(f(x)))\\ &= \lambda\wedge f^{-1}(\mu(x)). \end{aligned}$$

Therefore $g^{-1}(\lambda\times\mu)=\lambda\wedge f^{-1}(\mu)$. Since $g^{-1}(\lambda\times\mu)$ is a fuzzy open set of (X,T), by Proposition 3.1, g is slightly fuzzy continuous.

Conversely, let λ be fuzzy clopen in (Y,S). Then $1\times\lambda$ is fuzzy clopen in $X\times Y$. Since g is slightly fuzzy continuous, $g^{-1}(1\times\lambda)$ is fuzzy open in (X,T). Also, $g^{-1}(1\times\lambda)=f^{-1}(\lambda)$. Thus $f^{-1}(\lambda)$ is fuzzy open in (X,T). Hence f is slightly fuzzy continuous.

□

Proposition 3.7. Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be slightly fuzzy continuous. Then the mapping $g : (X, T) \to (Z, R)$ where $R = S/Z$ is slightly fuzzy continuous.

Proof. Let μ be any fuzzy clopen set in (Z, R). Then $\mu = \lambda/Z$ for some clopen set λ of (Y, S). By hypothesis $f^{-1}(\lambda) = g^{-1}(\mu)$. By Proposition 3.1, $f^{-1}(\lambda)$ is fuzzy open in (X, T). Hence $g^{-1}(\mu)$ is fuzzy open in (X, T). Thus g is slightly fuzzy continuous. □

Proposition 3.8. Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces and let $Y \subset Z$ be a subspace of Z. Then the mapping $h : (X, T) \to (Z, R)$ obtained by expanding the range of the slightly fuzzy continuous mapping $f : (X, T) \to (Y, S)$ is slightly fuzzy continuous.

Proof. The mapping $h : (X, T) \to (Z, R)$ is the composition of $f : (X, T) \to (Y, S)$ and the inclusion map $j : (Y, S) \to (Z, R)$. That is $h = j \circ f$. By Proposition 3.3, h is slightly fuzzy continuous. □

Proposition 3.9. Let $h : X \to \prod_{\alpha \in \mathrm{I}} \mathrm{X}_\alpha$ be a slightly fuzzy continuous mapping. For each $\alpha \in I$, define $f_\alpha : X \to X_\alpha$ by setting $f_\alpha(\lambda) = (h(\lambda))_\alpha$. Then f_α is slightly fuzzy continuous, for every $\alpha \in I$.

Proof. Let $\delta \in I^X$ and let μ be any fuzzy clopen set in X_α. Let $h(\delta) \leq \mu$. Then $f_\alpha(\delta) = (h(\delta))_\alpha \leq \mu$. Since h is slightly fuzzy continuous, there exists an open set λ with $\delta \leq \lambda$ such that

$$\begin{aligned} & h(\lambda) \leq \mu \\ \Rightarrow \quad & (h(\lambda))_\alpha \leq \mu \\ \Rightarrow \quad & f_\alpha(\lambda) \leq \mu. \end{aligned}$$

Hence f_α is slightly fuzzy continuous. □

Proposition 3.10. Let (X, T), (X_1, T_1) and (X_2, T_2) be any three fuzzy topological spaces and let $p_i : (X_1 \times X_2) \to X_i (i = 1, 2)$ be the projections of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is a slightly fuzzy continuous mapping then $p_i \circ f$ is also slightly fuzzy continuous.

Proof. The projections $p_i (i = 1, 2)$ are fuzzy continuous and therefore are slightly fuzzy continuous. Hence the proof follows from Proposition 3.3. □

Proposition 3.11. Let (X, T) and (Y, S) be any two fuzzy topological spaces. Let $X = A \cup B$, where A and B are subsets of X such that ψ_A, $\psi_B \in T$. Let $f : (X, T) \to (Y, S)$ be such that f/A and f/B are slightly fuzzy continuous. Then f is slightly fuzzy continuous.

Proof. Let λ be a fuzzy open set of Y. Since f/A is slightly fuzzy continuous, $(f/A)^{-1}(\lambda)$ is fuzzy open in A and similarly $(f/B)^{-1}(\lambda)$ is fuzzy open in B. Therefore there exist $\mu/A \in T/A$ and $\mu/B \in T/B$ such that $\mu/A = (f/A)^{-1}(\lambda)$

and $\mu/B = (f/B)^{-1}(\lambda)$. Then $\mu = f^{-1}(\lambda)$ and $\mu \in T$. Hence by Proposition 3.1, f is slightly fuzzy continuous. □

Proposition 3.12. Let (X, T) and (Y, S) be any two fuzzy topological spaces such that elements of S are both fuzzy open and fuzzy closed. If $f : (X, T) \to (Y, S)$ is slightly fuzzy continuous, then f is fuzzy continuous.

Proof. Proof follows from $(a) \Rightarrow (b)$ of Proposition 3.1. □

Proposition 3.13. Every mapping from a fuzzy topological space to a fuzzy connected space is slightly fuzzy continuous.

Proof. Let (X, T) be a fuzzy topological space and (Y, S) be a fuzzy connected space. Let $f : (X, T) \to (Y, S)$ be a mapping. Since (Y, S) is fuzzy connected, it has no set which is both fuzzy open and fuzzy closed other than 0 and 1. $f^{-1}(0)$ and $f^{-1}(1)$ are both fuzzy open in X. Hence by Proposition (3.1), f is slightly fuzzy continuous. □

Proposition 3.14. A slightly fuzzy continuous image of a fuzzy connected space is fuzzy connected.

Proof. Let (X, T) be a fuzzy connected space and (Y, S) be a fuzzy topological space. Let $f : (X, T) \to (Y, S)$ be a slightly fuzzy continuous mapping. Suppose that (Y, S) is fuzzy disconnected. Let λ be a proper fuzzy clopen set of (Y, S). Since f is slightly fuzzy continuous, $f^{-1}(\lambda)$ is a proper fuzzy clopen set of (X, T) which is a contradiction. Hence Y is fuzzy connected. □

Proposition 3.15. Let (X, T) be a fuzzy topological space and (Y, S) be a fuzzy extremally disconnected space. If $f : (X, T) \to (Y, S)$ is a slightly fuzzy continuous mapping, then f is almost* fuzzy continuous.

Proof. Let μ be a fuzzy open set of (Y, S) with $f(\lambda) \leq \mu$. Since (Y, S) is fuzzy extremally disconnected, $\operatorname{cl} \mu$ is fuzzy open and therefore fuzzy clopen. Now, $f(\lambda) \leq \operatorname{cl} \mu$ and since f is slightly fuzzy continuous, there exists a fuzzy open set σ with $\lambda \leq \sigma$ such that $f(\sigma) \leq \operatorname{cl} \mu$. Since $\operatorname{cl} \mu$ is open, $f(\sigma) \leq \operatorname{int} \operatorname{cl} \mu$. Hence f is almost* fuzzy continuous. □

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Department of Mathematics
Sri Sarada College For Women
Salem - 636 016, Tamilnadu