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A Frozen Time Receding Horizon Control for a Linear Discrete Time-Varying System

오 명 환*, 오 준 호
(Myung-Hwan Oh and Jun-Ho Oh)

Abstract: In the case of a linear time-varying system, it is difficult to apply the conventional stability conditions of RHC (Receding Horizon Control) to real physical systems because of computational complexity comes from time-varying system and backward Riccati equation. Therefore, in this study, a frozen time RHC for a linear discrete time-varying system is proposed. Since the proposed control law is obtained by time-invariant Riccati equation solved by forward iterations at each control time, its stability can be ensured by matrix inequality condition and the stability condition based on horizon for a time-invariant system, and they can be applied to real physical systems effectively in comparison with the conventional RHC.

Keywords: receding horizon control, frozen time approach, horizon size, stability

I. INTRODUCTION

It is well known that the condition of the final state weighting matrix is important for the stability of receding horizon control (RHC) and the terminal equality condition [1,2] and the matrix inequality condition [3,4] were proposed. The matrix inequality condition is more flexible and realizable than the terminal equality condition but has some problems resulting from their complexity and the high value of the final state weighting matrix.

To search the easier and more flexible stability conditions, the stability conditions based on horizon size, which has been considered as an alternative important design factor of RHC, were proposed [5-7]. Since the horizon based stability conditions extend the possible range of the final state weighting matrix by increasing the horizon size, they solved some problems of the matrix inequality condition resulting from high value of the final state weighting matrix.

In the case of a time-invariant system, the horizon based conditions in [5,6] can guarantee the stability of any physical system controlled by RHC easily by solving forward Riccati equation while increasing horizon size until the control law satisfies the specific condition during control. However, in the case of a time-varying system, both the matrix inequality condition and the horizon based condition need many control efforts and re-calculations comes from complexities of a time-varying system and backward Riccati equation. Specially, in the case of the horizon based condition [6], if a selected horizon size does not satisfy the stability condition, we must change the horizon size and the control law must be re-calculated from initial stage because of backward Riccati equation. And thus, if the control law can be obtained by solving forward Riccati equation for a time-varying system, the horizon based stability condition can be applied easily like the case of RHC for a time-invariant system.

Therefore, in this study, a frozen time RHC for a linear discrete time-varying system is proposed. Since the proposed control law can be obtained by time-invariant Riccati equation solved by forward iterations, its stability can be ensured by the matrix inequality condition and the horizon based stability condition for a linear time-invariant system, and they can be applied to real physical systems effectively in comparison with the conventional RHC.

A frozen time RHC for a discrete time-varying system is introduced in chapter II and the newly proposed method to check the stability of the frozen time RHC are described in chapter III. And the proposed methods are verified by a simple numerical simulation in chapter IV. Finally, conclusion and proposals for future works are discussed in the final chapter.

* 책임저자(Corresponding Author)
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오명환: 삼성코닝정밀유리(kmuffin@chol.com)
오준호: 한국과학기술원 기계공학과(jhoh@kaist.ac.kr)

II. A FROZEN TIME RHC FOR A LINEAR DISCRETE SYSTEM

Consider a linear discrete time-varying system as given below.

$$x(t+1) = A(t)x(t) + B(t)u(t) \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and a cost function

$$\begin{aligned} J(x(t),t,t+T) = \sum_{i=t}^{t+T-1} [x^T(i)Q(i)x(i) + u^T(i)R(i)u(i)] \\ + x^T(t+T)Q_f(t+T)x(t+T) \end{aligned} \tag{2}$$

where $Q(\cdot) = C^T(\cdot)C(\cdot) \geq 0$, $R(\cdot) > 0$, and $Q_f(\cdot) \geq 0$. In this study, matrices $A(\cdot)$, $B(\cdot)$, $Q(\cdot)$, $R(\cdot)$, and $Q_f(\cdot)$ are assumed to be bounded.

RHC law at the current time t can be calculated by minimizing the cost function (2) about the input $u(t)$ and is given by

$$\begin{aligned} u^*(t) = -R^{-1}(t)B^T(t)[I + K(t+1,t+T) \\ \times B(t)R^{-1}(t)B^T(t)]^{-1} K(t+1,t+T)A(t)x(t) \end{aligned} \tag{3}$$

where $K(\tau,\sigma)$ satisfies

$$\begin{aligned} K(\tau,\sigma) = A^T(\tau)K(\tau+1,\sigma)[I + B(\tau)R^{-1}(\tau)B^T(\tau) \\ \times K(\tau+1,\sigma)]^{-1} A(\tau) + Q(\tau), \quad \tau \leq \sigma \end{aligned} \tag{4}$$

with the boundary condition

$$K(t+T,t+T) = Q_f(t+T). \tag{5}$$

The stability of the system (1) with RHC law (3) depends on the final state weighting matrix $Q_f(\sigma)\ge 0$ and if it satisfies non-increasing monotonicity $K(\sigma,\sigma+1)-K(\sigma,\sigma)\le 0$, the stability of the time-varying system (1) with RHC law (3) can be ensured by the matrix inequality condition regardless of the horizon T. However, if $Q_f(\sigma)\ge 0$ satisfies non-decreasing monotonicity $K(\sigma,\sigma+1)-K(\sigma,\sigma)\ge 0$, its stability can not be guaranteed by the matrix inequality condition and depends on the horizon. It is explained in Theorem 1 [6].

Theorem 1 [6]: Assume that the pairs $(A(\cdot),B(\cdot))$ and $(A(\cdot),C(\cdot))$ in (1) are uniformly completely controllable and observable respectively. If $Q_f(\sigma)\ge 0$ satisfies $K(\sigma,\sigma+1)-K(\sigma,\sigma)\ge 0$ and $K(\sigma,\sigma+2)-2K(\sigma,\sigma+1)+K(\sigma,\sigma)\le 0$ for all $\sigma\ge 0$, and $K(t,t+T)$ for $T\ge l+2$ ($l=\max(l_c,l_o)$ which are needed in uniform complete controllability and observability) satisfies the following inequality

$$K(t,t+T)-K(t,t+T-1)\le Q(t) \tag{6}$$

or its sufficient condition

$$K(t,t+T)\le Q_f(t)+TQ(t)\,, \tag{7}$$

then the system (1) with control law (3) is uniformly asymptotically stable. ■

Here, l_c and l_o are positive values and they satisfies $\alpha_1 I\le W(t,t+l_c)\le\alpha_2 I$ and $\alpha_3 I\le G(t,t+l_o)\le\alpha_4 I$ (α_1, α_2, α_3 and α_4 are positive constants) where $W(t,t+l_c)$ and $G(t,t+l_o)$ are controllability and observability grammian respectively.

To find T globally satisfying (6) for all time, many control efforts are needed in a time-varying system because the control law (3) is obtained by backward Riccati equation (4) and two other Riccati equations must be solved at the same time to obtain $K(t,t+T)$ and $K(t,t+T-1)$ in (6). That is, to obtain $K(t,t+T)$ and $K(t,t+T-1)$, they are solved from initial conditions $K(t+T,t+T)$ and $K(t+T-1,t+T-1)$ respectively, in each Riccati equation, and $K(t,t+T)$ can not be obtained from $K(t,t+T-1)$ continuously in the same Riccati equation. Also, if a selected horizon T does not satisfy (6), we must change the horizon size and all solving process is repeated from initial stage in another Riccati equation.

For example, if a horizon size T_1 is selected, $K(t,t+T_1)$ is obtained by backward Riccati equation (4) with the initial condition $K(t+T_1,t+T_1)$ and it is solved by the following sequence $K(t+T_1-1,t+T_1)$, $K(t+T_1-2,t+T_1)$, $\ldots$, $K(t+1,t+T_1)$, and finally, $K(t,t+T_1)$ is obtained and it is checked by (6). If $K(t,t+T_1)$ does not satisfy (6), we must repeat this process for other increased horizon $T_2(T_2>T_1)$. Since $K(t,t+T_2)$ can not be obtained from previous result $K(t,t+T_1)$, $K(t,t+T_2)$ is calculated from new initial condition $K(t+T_2,t+T_2)$ and solving process is repeated like the above process $K(t+T_2-1,t+T_2)$, $K(t+T_2-2,t+T_2)$, $\ldots$, $K(t+1,t+T_2)$, and $K(t,t+T_2)$.

Therefore, if backward Riccati equation (4) can be solved by forward iterations, T satisfying (6) is obtained conveniently while increasing T until $K(t,t+T)$ and $K(t,t+T-1)$ satisfies (6) like the case of RHC for a time-invariant system [5,6], and all problems of the inequality (6) will be solved. That is, if $K(t,t+T)$ can be obtained from the initial condition $K(t,t)$, $K(t,t+T)$ is obtained from $K(t,t+T-1)$ continuously in the same Riccati equation and it does not need to repeat all calculation process from initial stage when the horizon size is changed.

To meet with this problem, the frozen time RHC for a discrete time-varying system is proposed and it is applied for guaranteeing the stability of a linear discrete time-varying system. The frozen time RHC law is explained as follows.

If system and input matrices $A(t)$ and $B(t)$ are fixed at time t, the system (1) at time t can be expressed as the time-invariant system

$$x(\tau+1)=A_t x(\tau)+B_t u(\tau) \tag{8}$$

where A_t and B_t are $A(t)$ and $B(t)$ respectively. If the future states of the time-invariant system (8) are considered from t to $t+T$ in the cost function (2), the cost function (2) changes to

$$\begin{aligned}J(x(t),t,t+T)=&\sum_{i=t}^{t+T-1}[x^T(i)Q_t x(i)+u^T(i)R_t u(i)]\\&+x^T(t+T)Q_{ft}x(t+T)\end{aligned} \tag{9}$$

where $Q_t=Q(t)$, $R_t=R(t)$, and $Q_{ft}=Q_f(t)$, and it is the same as the cost function of RHC for a time-invariant system. The frozen time RHC law at the current time t can be calculated by minimizing the cost function (9) about the input $u(t)$ and is given by

$$\begin{aligned}u^*(t)=&-R_t^{-1}B_t^T[I+K_t(T-1)B_tR_t^{-1}B_t^T]^{-1}\\&\times K_t(T-1)A_t x(t)\end{aligned} \tag{10}$$

where $K_t(\tau)$ satisfies

$$K_t(\tau+1)=A_tK_t(\tau)[I+B_tR_t^{-1}B_t^TK_t(\tau)]^{-1}A_t+Q_t \tag{11}$$

with the boundary condition

$$K_t(0)=Q_{ft}. \tag{12}$$

Then the closed-loop system of the system (1) with the frozen time RHC law (10) is given below.

$$\begin{aligned}x(t+1)=&[A_t-B_tR_t^{-1}B_t^T[I+K_t(T-1)B_tR_t^{-1}B_t^T]^{-1}\\&\times K_t(T-1)A_t]x(t)\end{aligned} \tag{13}$$

The frozen time RHC law at each control time is obtained by assuming that the time-varying system is fixed from the current control time t to the prediction time or horizon size $t+T$, and the complex RHC law for a time-varying system is replaced by the simple RHC law for a time-invariant system.

The control law (10) is the frozen time RHC law for the system (1) at control time t and it is equivalent to RHC for the time-invariant system (8), which is the fixed system of the system (1) at time t. Since the above process is repeated at each control time, the frozen time RHC laws for the system (1) are composed of time-invariant

RHC law at each control time and its control law $K_t(T)$ is obtained by solving forward Riccati equation (11).

III. THE STABILITY OF THE FROZEN TIME RHC

Since the frozen time RHC law is obtained by the time-invariant Riccati equation at every control time, its stability can be also guaranteed by the stability conditions of time-invariant RHC with additional conditions of design matrices $Q(t)$, $R(t)$, and $Q_f(t)$. It is summarized in Theorem 2.

Theorem 2: Assume that the pairs $(A(\cdot),B(\cdot))$ and $(A(\cdot),C(\cdot))$ in (1) are uniformly completely controllable and observable, and $Q_f(t)\geq 0$ satisfies $K_t(1)-K_t(0)\geq 0$ and $K_t(2)-2K_t(1)+K_t(0)\leq 0$ for all $t\geq 0$. If $K_t(T)$ for $T\geq l+2$ satisfies $K_t(T)-K_t(T-1)\leq Q(t)$ or equivalently,

$$\begin{aligned}&A^T(t)K_t(T-1)[I+B(t)R^{-1}(t)B^T(t)K_t(T-1)]^{-1}\\&\times A(t)-K_t(T-1)\leq 0\end{aligned} \tag{14}$$

and the following inequalities

$$\begin{aligned}&H(t)-H(t-1)\\&=\begin{bmatrix}Q(t) & A^T(t)\\ A(t) & -S(t)\end{bmatrix}-\begin{bmatrix}Q(t-1) & A^T(t-1)\\ A(t-1) & -S(t-1)\end{bmatrix}\leq 0\end{aligned} \tag{15}$$

where $S(t)=B(t)R^{-1}(t)B^T(t)$ and

$$Q_f(t)-Q_f(t-1)\leq 0 \tag{16}$$

are satisfied for all time $t\geq 1$, then the system (1) with control law (10) is uniformly asymptotically stable.

Proof: This theorem can be proved by Lyapunov stability theorem. Consider the closed-loop system (13). The Lyapunov function is defined as $V(t,x(t))=x^T(t)K_{t-1}(T-1)x(t)$. Since $Q(\cdot)$ and $R(\cdot)$ are bounded, $K_t(T)$ is also bounded.

$$\begin{aligned}&V(t+1,x(t+1))-V(t,x(t))\\&=x^T(t+1)K_t(T-1)x(t+1)-x^T(t)K_{t-1}(T-1)x(t)\\&=x^T(t)[A^T(t)K_t(T-1)A(t)-A^T(t)K_t(T-1)(I\\&\quad+B(t)R^{-1}(t)B^T(t)K_t(T-1))^{-1}B(t)R^{-1}(t)B^T(t)K_t(T-1)\\&\quad-M(t)-K_{t-1}(T-1)]x(t)\\&=x^T(t)[K_t(T)-K_{t-1}(T-1)-Q(t)-M(t)]x(t)\\&=x^T(t)[K_t(T)-K_t(T-1)+K_t(T-1)-K_{t-1}(T-1)\\&\quad-Q(t)-M(t)]x(t)\end{aligned}$$

where $M(t)=A^T(t)K_t(T-1)B(t)[R(t)+B^T(t)K_t(T-1)B(t)]^{-1}R(t)\times[R(t)+B^T(t)K_t(T-1)B(t)]^{-1}B^T(t)K_t(T-1)A(t)$ and $M(t)>0$. In order to guarantee the Lyapunov stability theorem, $V(t+1,x(t+1))-V(t,x(t))\leq -\alpha I$ (α is a positive constant) must be satisfied and it can be expressed conservatively via the inequalities,

$$K_t(T)-K_t(T-1)\leq Q(t)\,,\quad K_t(T-1)-K_{t-1}(T-1)\leq 0$$

The former inequality, $K_t(T)-K_t(T-1)\leq Q(t)$ is satisfied because it is equivalent to the inequality (14). The other inequality $K_t(T-1)-K_{t-1}(T-1)\leq 0$ can be satisfied by the comparison theorem for matrix Riccati difference equation in [8] if $K_t(0)-K_{t-1}(0)\leq 0$ and the inequality (15) are satisfied. Since $K_t(0)=Q_f(t)$, if the inequalities (15) and (16) are satisfied, $K_t(T-1)-K_{t-1}(T-1)\leq 0$ is also satisfied and the closed-loop system (13) is uniformly asymptotically stable. ■

The stability of the closed-loop system (13) is guaranteed by the time-invariant RHC stability condition (14) at every control time t with the additional conditions (15) and (16). If T satisfies (14) for all time $t\geq 0$, all of the eigenvalues of the closed-loop system (13) are placed in the unit circle and it is pointwise stable. Also, since (15) and (16) are the condition for design matrices, it is needed to select $Q(t)$, $R(t)$, and $Q_f(t)$ relevantly. Therefore, the stability of frozen time RHC for a linear discrete time-varying system can be ensured by the time-invariant RHC stability condition and relevant selection of the design matrices.

The stability condition (14) is the time-invariant version of (6) and T satisfying (14) at each time t can be obtained easily by solving forward Riccati equation (11) while increasing T until $K_t(T)$ satisfies (14) differing from the conventional RHC for a time-varying system, that is, calculation is proceeded by the following sequence $K_t(0)$, $K_t(1)$, $K_t(2)$, …, $K_t(T-1)$, and finally $K_t(T)$ is obtained. Therefore, the inequality (14) is checked at each step and if (14) is not satisfied at a horizon T_1, forward iterations are continued from $K_t(T_1)$ in the same Riccati equation without repeating the calculation process from initial condition $K_t(0)$.

Theorem 2 is somewhat simple condition than Theorem 1 but it is not easy to select $Q_f(t)$ satisfying the assumptions in Theorem 2. If the horizon size is increased sufficiently, it means that the solution of Riccati equation $K_t(T)$ converges to the optimal LQ solution $\bar{K}_t$, that is, $K_t(T+1)-K_t(T)$ closes to zero. Therefore, although a final state weighting matrix $Q_f(t)$ satisfies non-decreasing monotonicity $K_t(1)-K_t(0)\geq 0$ at initial time and it remains for all time, there exists a horizon size T_f satisfying $K_t(T_f+2)-2K_t(T_f+1)+K_t(T_f)\leq 0$. This relation is explained in Lemma 1 and 2.

Lemma 1 [6]: If an arbitrary $Q_f(t)\geq 0$ satisfies $K_t(1)-K_t(0)\geq 0$, then there exists a horizon size $T_f\geq 0$ which satisfies

$$K_t(T_f+2)-2K_t(T_f+1)+K_t(T_f)\leq 0.$$

■

The inequality in Lemma 1 also has monotonicity and it is showed in Lemma 2

Lemma 2 [9]: If a horizon T_f satisfies the following monotonicity, $K_t(T_f+2)-2K_t(T_f+1)+K_t(T_f)\leq 0$. then the inequality $K_t(T+2)-2K_t(T+1)+K_t(T)\leq 0$ is also satisfied for all $T\geq T_f$. ■

Therefore, $Q_f(t)$ in the inequality (14) and its assumptions in Theorem 2 can be replaced by $K_t(T_f)$ and it explained in Remark 1.

Remark 1: Assume that the pairs $(A(\cdot),B(\cdot))$ and $(A(\cdot),C(\cdot))$

in (1) are uniformly completely controllable and observable respectively, and $K_t(T_f)$ which $T \ge T_f \ge 0$ satisfies $K_t(T_f+1) - K_t(T_f) \ge 0$ and $K_t(T_f+2) - 2K_t(T_f+1) + K_t(T_f) \le 0$ for all $t \ge 0$. If $K_t(T)$ for $T \ge l+2$ satisfies the inequalities (14) and (15) for all time $t \ge 1$, then the system (1) with control law (10) is uniformly asymptotically stable for an arbitrary $Q_f(\cdot) \ge 0$ satisfying (16). ■

T_f and T satisfying the assumptions in Remark 1 can be obtained simultaneously while increasing horizon until it satisfies the assumption in the middle of finding the control law.

On the other hand, if $Q_f(t)$ does not satisfy non-decreasing monotonicity $K_t(1) - K_t(0) \ge 0$ but satisfies non-increasing monotonicity $K_t(1) - K_t(0) \le 0$, the time-invariant matrix inequality condition [10] for RHC can be applied to ensure the pointwise stability of the closed-loop system (13) instead of the time-varying matrix inequality condition. It is summarized in Theorem 3.

Theorem 3: Assume that the pairs $(A(\cdot), B(\cdot))$ and $(A(\cdot), C(\cdot))$ in (1) are uniformly completely controllable and observable respectively. If $Q_f(t) \ge 0$ satisfies

$$\begin{aligned} &A^T(t)Q_f(t)[I + B(t)R^{-1}(t)B^T(t)Q_f(t)]^{-1} \\ &\times A(t) + Q(t) - Q_f(t) \le 0 \end{aligned} \tag{17}$$

and inequalities (15) and (16) are satisfied, then the system (1) with control law (10) is uniformly asymptotically stable for all $T \ge l+2$. ■

The inequality (17) is the matrix inequality condition for a linear discrete time-invariant system [10] and it can replace the horizon based pointwise stability condition (14) because the matrix inequality condition (17) satisfies $K_t(T) - K_t(T-1) \le Q(t)$ in the proof of Theorem 2 sufficiently. Since the matrix inequality condition for a time-invariant system (17) is simpler than the time-varying matrix inequality condition, it can be applied to real physical system easily.

IV. NUMERICAL SIMULATION

A numerical example for the stability of frozen time RHC is executed using the following system

$$A(t) = \begin{bmatrix} 5 & 3 \\ 2+2e^{-t} & 5 \end{bmatrix} \; B(t) = \begin{bmatrix} 0 \\ 4t \end{bmatrix} \; Q(t) = \begin{bmatrix} 1+e^{-t} & 0 \\ 0 & 3 \end{bmatrix} \; R = 1 \tag{18}$$

Then the inequality (15)

$$H(t) - H(t-1) = \begin{bmatrix} e^{-t-1} - e^{-t} & 0 & 0 & 2(e^{-t-1} - e^{-t}) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2(e^{-t-1} - e^{-t}) & 0 & 0 & -32t - 16 \end{bmatrix} \le 0 \tag{19}$$

is satisfied for all $t \ge 1$ and if $Q_f(t) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, it also satisfies the inequality (16). T_f and T satisfying Lemma 1 and the inequality (14) at every control time is obtained easily by increasing the horizon from zero, and $T_f \ge 2$ and $T \ge 6$ satisfy Lemma 1 and the inequality (14) for all t. Therefore, the minimum horizon size which

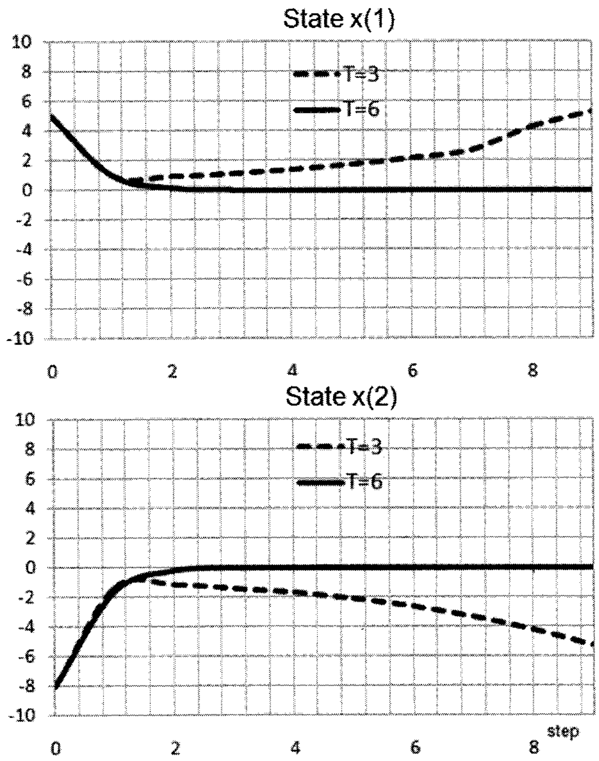

그림 1. 구간크기에 따른 제어 결과.

Fig. 1. Regulation results according to horizon size.

can guarantee the stability of the closed loop system (13) is $T = 6$. $Q(t)$ and $R(t)$ satisfying the inequality (15) is selected at each control time.

To prove this result, the regulation control is executed when the horizon size $T = 3$ and $T = 6$ for the system (18) with the frozen time control law. Initial state $x(0) = [5 \ \ -8]$ and the results are showed in Fig. 1. Since the stability of system (18) with the frozen time control law is ensured when $T \ge 6$ by the stability condition (14), the closed-loop system is stable when $T = 6$ but it is not stable when the horizon size $T = 3$ because its stability is not guaranteed when $T < 6$.

In the case of the conventional RHC for a time-varying system, since Riccati equation is solved by backward iterations and horizon size can not be changed during calculating control law, more control efforts are needed than frozen time RHC, that is, $K(t, t+T)$ is obtained from $K(t+T, t+T)$, and also, both $K(t, t+T)$ and $K(t, t+T-1)$ are must be calculated simultaneously.

V. CONCLUSION

As a new control strategy, the frozen time RHC and its new stability conditions are introduced in this study. It has been difficult to apply the conventional stability conditions of RHC for a time-varying system to a real control problem because of time-varying complexity and backward Riccati equation. Since the proposed control law is obtained by time-invariant Riccati equation solved by forward iterations at each control time, its stability can be ensured by the matrix inequality condition and the stability condition based on horizon for a linear time-invariant system, and they can be applied to real physical systems effectively in comparison with the conventional RHC.

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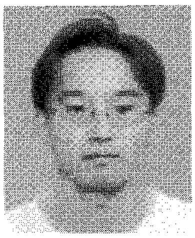
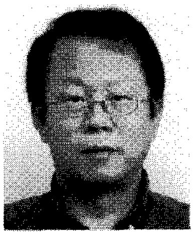


오 명 환

1996년 KAIST 기계공학과(공학사). 1998년 KAIST 기계공학과(공학석사). 2004년 KAIST 기계공학과(공학박사). 2007년~현재 삼성코닝정밀유리. 관심분야는 로보틱스, 최적제어.



오 준 호

1977년 연세대학교 기계공학과(공학사). 1979년 연세대학교 기계공학과(공학석사). 1985년 University of California, Berkeley(공학박사). 1979년~1981년 한국원자력연구소. 1985년~ 현재 KAIST 기계공학과 교수. 관심분야는 Humanoid Robot, 적응제어, 지능제어, 비선형제어.