Torque Calculation Method of a Permanent Magnet Spherical Motor

Hyung-Woo Lee†, Dong-Woo Kang* and Ju Lee*

Abstract - This paper presents the torque calculation method of a permanent magnet spherical motor. To calculate using the finite element method (FEM), three-dimensional (3D) FEM must be used. However, since the method requires excessive time and memory, an easier torque calculation method is hereby presented. In the proposed method, it is very important to obtain the approximation function of the torque profile curve. We present the approximation method of the torque profile curve and show that the torque calculation result can approximate the torque obtained by 3-D FEM.

Keywords: Permanent Magnet, Motor, Field Computation, Numerical Method

1. Introduction

Generally, several electromagnetic motors are used in order to construct a multi-degree-of-freedom (DOF) motion system. However, in such systems, the number of motors must be equal to or larger than the number of DOFs of the motion unit because general electromagnetic motors generate only a single DOF rotation. Hence, the system must be very large and heavy, necessitating a complex control mechanism. As a solution, the concept of a spherical motor that can generate multi-DOFs was suggested in the 1950s. Since then, studies on the spherical motor have been advanced at the international level.

To analyze the characteristics of the spherical motors, the three-dimensional finite element method (3D FEM) must be used. Although computer systems have developed rapidly, the use of 3D FEM requires a large amount of time and memory. For this reason, the easier torque calculation method has been presented [2]-[4]. In this method, obtaining the approximation function of the torque profile curve is important. To approximate more correctly, we use sigmoid functions instead of the generally used exponential functions. The results of the torque calculation are compared with those using 3D FEM.

2. Torque Calculation Method

Fig. 1 shows the permanent magnet spherical motor used in the study. The torque, which is generated by n (the number of rotor poles) and m (the number of stator coils), can be evaluated [1] by:

$$\hat{\mathbf{T}} = \sum_{k=1}^{n}\sum_{j=1}^{m} \hat{f}\left(\varphi_{jk}\right)\frac{\mathbf{s}_j \times \mathbf{r}_k}{\left\|\mathbf{s}_j \times \mathbf{r}_k\right\|} Ni \qquad (1)$$

† Corresponding Author: Department of Vehicle Dynamics & Propulsion System Research, Korea Railroad Research Institute, Korea. (krhwlee@krri.re.kr)

* Department of Electric Engineering, Hanyang University, Korea. (dwkang1222@gmail.com, julee@hanyang.ac.kr)

Received: July 13, 2009; Accepted: May 11, 2010

where $\mathbf{r}_k$ is the magnetization axis of the k-th rotor pole in the rotor frame, and $\mathbf{s}_j$ is the direction of the j-th stator coil in stator frame (Fig. 2.). In addition, N is the number of coil turns, i is the current in the coil, and $\hat{f}\left(\varphi_{jk}\right)$ is the torque profile approximation function of the angle between the stator coil and the rotor pole. The torque profile curve can be obtained by analyzing the simple model using 3D FEM (Fig. 3(a)). Given that the torque calculation is a numerical method using approximation of the torque profile, precision of approximation is extremely important.

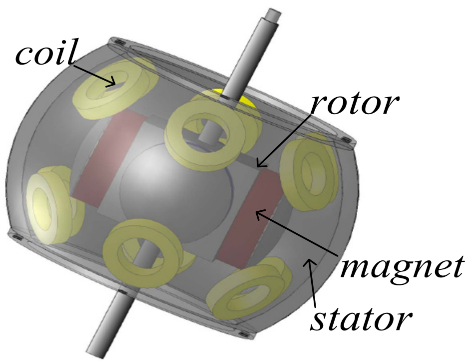


Fig. 1. Permanent magnet spherical motor.

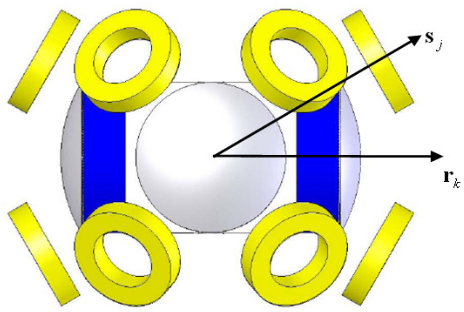
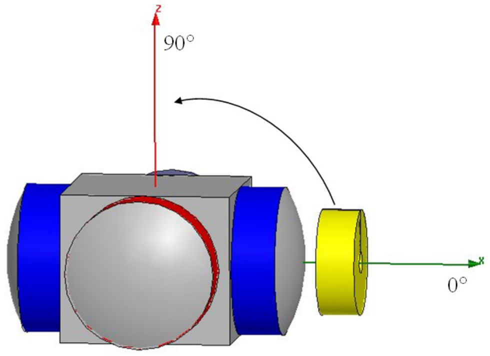
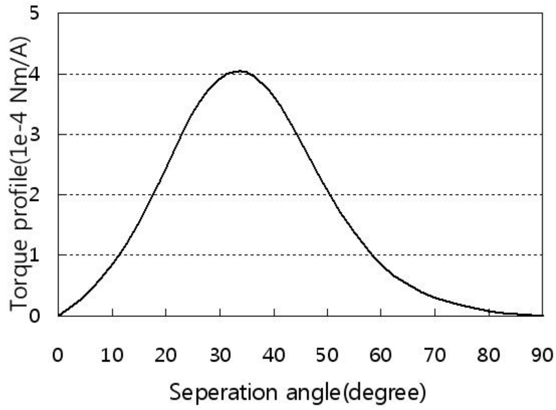


Fig. 2. Direction of a stator coil and the magnetization axis of a rotor pole.



(a) Model for torque profile



(b) Torque profile

Fig. 3. Model for torque profile and torque profile

3. The Approximation Method

To approximate the torque profile, exponential functions can be used [2]-[4]. However, we noticed that using sigmoid function has some advantages. Fig. 4 shows a sigmoid function and its symmetric transposition function.

The proposed approximation method uses the function presented by seven unknowns as shown in the equation. In this work, as a matter of convenience, the angle between the stator coil and the rotor pole φ_{jk} was replaced by the variable x, thus arriving at:

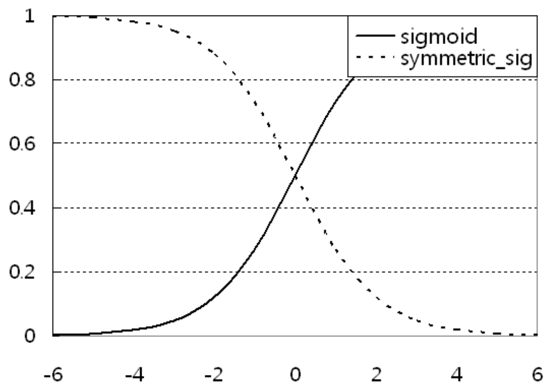


Fig. 4. Sigmoid curve and symmetric transposition curve.

$$\hat{f}(x)=\frac{k_1}{1+c_1e^{-b_1x}}+\frac{k_2}{1+c_2e^{-b_2(x-\pi/2)}}+k_3 \tag{2}$$

For finding the unknowns in (2), the numerical method was used [5]. If y is the real value of the torque profile, and the remainder between the approximation value and the real value is $g(x)$, then $g(x)$ has to come closer to 0, as demonstrated by:

$$g(x)=\hat{f}(x)-y=\frac{k_1}{1+c_1e^{-b_1x}}+\frac{k_2}{1+c_2e^{-b_2(x-\pi/2)}}+k_3-y=0 \tag{3}$$

From 0 to 90 degrees, at 5-degree intervals, there are 19 degree-torque profile data tables. Substituting these (x is the angle between the coil and the magnet, and y is the torque profile value in the angle) into (3) results in 19 equations with 7 unknowns as given by:

$$g_n(k_1+\delta k_1,k_2+\delta k_2,k_3+\delta k_3,c_1+\delta c_1,c_2+\delta c_2,b_1+\delta b_1,b_2+\delta b_2)=0 \quad n=1\sim 19 \tag{4}$$

where δ s are undecided corrections of each coefficients. Suppose that their values are very small, the first order Taylor expansion is derived as follows:

$$\frac{\partial f_n}{\partial k_1}\delta k_1+\frac{\partial f_n}{\partial k_2}\delta k_2+\frac{\partial f_n}{\partial k_3}\delta k_3+\frac{\partial f_n}{\partial c_1}\delta c_1+\frac{\partial f_n}{\partial c_2}\delta c_2+\frac{\partial f_n}{\partial b_1}\delta b_1+\frac{\partial f_n}{\partial b_2}\delta b_2 =y_n-\hat{f}(k_1,k_2,k_3,c_1,c_2,b_1,b_2), \quad n=1\sim 19 \tag{5}$$

These equations can be simply described using the following matrix algebra:

$$\begin{bmatrix} \frac{\partial f_1}{\partial k_1} & \frac{\partial f_1}{\partial k_2} & \frac{\partial f_1}{\partial k_3} & \frac{\partial f_1}{\partial c_1} & \frac{\partial f_1}{\partial c_2} & \frac{\partial f_1}{\partial b_1} & \frac{\partial f_1}{\partial b_2} \\ \frac{\partial f_2}{\partial k_1} & \frac{\partial f_2}{\partial k_2} & \frac{\partial f_2}{\partial k_3} & \frac{\partial f_2}{\partial c_1} & \frac{\partial f_2}{\partial c_2} & \frac{\partial f_2}{\partial b_1} & \frac{\partial f_2}{\partial b_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_{19}}{\partial k_1} & \frac{\partial f_{19}}{\partial k_1} & \frac{\partial f_{19}}{\partial k_3} & \frac{\partial f_{19}}{\partial c_1} & \frac{\partial f_{19}}{\partial c_2} & \frac{\partial f_{19}}{\partial b_1} & \frac{\partial f_{19}}{\partial b_2} \end{bmatrix} \begin{bmatrix} \delta k_1 \\ \delta k_2 \\ \delta k_3 \\ \delta c_1 \\ \delta c_2 \\ \delta b_1 \\ \delta b_2 \end{bmatrix} = \begin{bmatrix} y-\hat{f}(x_1) \\ y-\hat{f}(x_2) \\ \vdots \\ y-\hat{f}(x_{19}) \end{bmatrix} \tag{6}$$

Values of the corrections can be obtained by using iterations until the variation rate is reduced under 0.01%. The final approximation function is:

$$\hat{f}(x)=\frac{0.001}{1+16.86e^{-6.65x}}+\frac{0.00096}{1+219.36e^{6.3(x-\pi/2)}}-0.001 \tag{7}$$

Fig. 5 shows the comparison of the real torque profile, approximation using sigmoid function, and approximation using exponential function. As can be seen, the real torque profile and approximation using sigmoid function are almost the same.

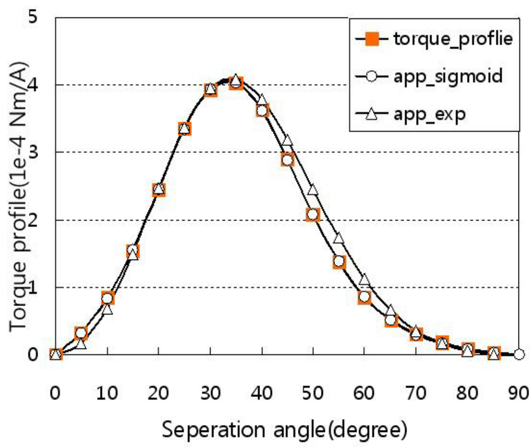


Fig. 5. Comparison of torque profile and the approximations.

4. Comparison of 3-FEM Result and the Torque Calculation Method

Fig. 6 shows the model used in this paper for 3D FEM. Supposing that there is no tilting, then 1/2 of the model can be used. Here, the generated torque, divided into about 114,000 elements using the transient solution, was calculated when the rotor was rotating. Fig. 7 shows the comparison of the torque obtained using 3D FEM and the torque obtained by the calculation method using approximation of torque profile at load angle 25°. The calculated torque by approximation using exponential function was also compared.

Although there is an error, we can obtain the torque comparatively closer to the torque by 3D FEM. The calculation using sigmoid function in this paper was closer on the torque by 3D FEM than that using exponential function.

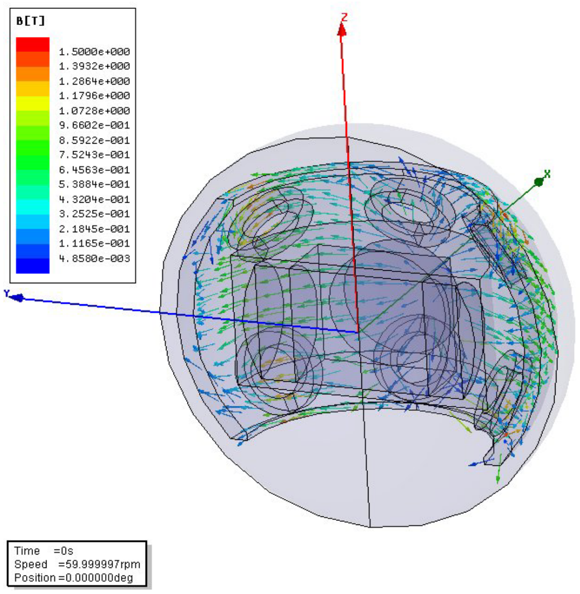


Fig. 6. Model for 3D FEM.

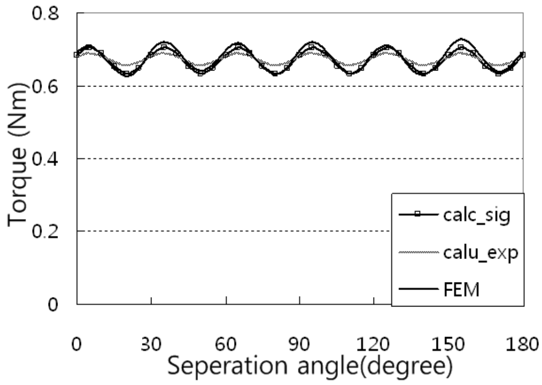


Fig. 7. Comparison of torques obtained by 3D FEM and by using the calculation method.

5. Conclusion

Using 3D FEM is inevitable when the aim is to obtain analytically the torque of a spherical motor. However, torque calculation using 3-D FEM has numerous limitations. The proposed approximation method using sigmoid function can reduce the approximation error better, compared with those using other functions. Moreover, this method is very suitable as a substitute for 3D FEM. This is proven by the comparison of the torques calculated using 3D FEM with the proposed approximation method.

Acknowledgements

This work was supported by the National Research Laboratory (NRL) through a grant (Contract Number R0A-2008-000-20048-0) provided by the Ministry of Education, Science, and Technology (MEST).

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Hyung-Woo Lee (S'98-M'04) received his B.S. and M.S. degrees from Hanyang University, Seoul, Korea in 1998 and 2000, respectively. He obtained his Ph.D. degree from Texas A&M University, College Station, in 2003. All these degrees are in the field Electrical Engineering. In 2004, he was a Post-doctoral Research Assistant in the Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, NY. In 2005, he held the position of Professor at the BK division of Hanyang University, Seoul, Korea. Since 2006, he has been a Senior Researcher at the Korea Railroad Research Institute in Uiwang, Korea. His research interests include design, analysis and control of motor/generators; power conversion systems; and applications of motor drives, such as Maglev trains, robots, and modern renewable energy systems.

Dong-Woo Kang (S'06) received his B.S. and M.S. degrees in Electrical Engineering from Hanyang University, Seoul, Korea in 2006 and 2008, respectively. Since 2008, he has been pursuing the Ph.D. degree ar the Department of Electrical Engineering, Hanyang University.
His research interests include design, analysis, testing and control of motor/generator; power conversion systems; and applications of spherical motor for robots.

Ju Lee (M'96-SM'04) obtained his B.S. and M.S. degrees from Hanyang University, Seoul, Korea in 1986 and 1988, respectively. He obtained his Ph.D. from Kyusyu University, Fukuoka, Japan, in 1997. He worked as an Assistant Researcher at the Agency for Defense Development from 1989 to 1993 and with the Korea Railroad Research Institute in 1997 as Chief of the Division on Light Subway Systems. He joined Hanyang University as Professor of the Department of Electrical Engineering in September 1997. His main research interests include electrical machinery and drives; electromagnetic field analysis; new transformation systems, such as hybrid electric vehicles and high-speed electric trains; and standardization. Dr. Lee was recognized by the Japan Electric Society as the writer of the Best Paper in 1995. He is a member of the IEEE Industry Applications Society, Magnetics Society, and Power Electronics Society. He has been a member of the editorial staff of the Korean Institute of Electrical Engineers since 1998 and a member of the editorial board of the International Journal of Electrical Engineering since 2000. In addition, he has been the Korea National Committee Secretary of the IEC/TC2 since 1999. He is also the General Manager of the Human Resource Development Center for Electric Machine and Devices, Seoul, Korea.