Numerical Study on Frequency Up-conversion in USPR using MATLAB

Young-Su Roh†

Abstract - In this paper, the O-mode ultrashort-pulse reflectometry (USPR) millimeter-wave signals that propagate into the plasma and cover a frequency bandwidth of 33-158 GHz are examined numerically using MATLAB. Two important processes are involved in the computation: the propagation of the USPR impulse signal through a waveguide and the frequency up-conversion using millimeter-wave mixers. These mixers are limited to intermediate frequency signals that are less than 500 mV; thus, it is necessary to disperse the impulse signal into a chirped waveform using the waveguide. The stationary phase method is utilized to derive a closed-form formula for a chirped waveform under the assumption that the USPR impulse is Gaussian. In the process of frequency up-conversion, the chirped waveform is mixed with the mixer LO signal, and the lower frequency components of the RF signal are removed using high pass filters.

Keywords: Chirped waveform, Frequency bandwidth, Frequency up-conversion, Method of stationary phase, Ultrashort-pulse reflectometry

1. Introduction

An O-mode ultrashort-pulse reflectometry (USPR) system has been installed on a Sustained Spheromak Physics Experiment (SSPX) device to measure electron density profiles and fluctuations [1]-[5]. In the original SSPX design, the predicted electron density is in the range of 0.5-3×10^{14} cm^{-3} [6]. For the measurement of the maximum density layer, the frequency bandwidth of the USPR system should be 158 GHz. In other words, the pulse duration of the USPR impulse generator should be less than 5 psec. In addition, the pulse power has to be sufficiently high to maintain an acceptable measure of signal-to-noise ratio. Due to the practical difficulty of finding such ultrashort impulse generator, it is inevitable to use a 5 V, 65 psec duration full width half maximum (FWHM) impulse generator with a maximum frequency bandwidth of 18 GHz.

A technique for frequency conversion using millimeter-wave mixers has been applied to the USPR system to achieve the desired frequency bandwidth. In the process of frequency conversion, the lower frequency components of the USPR impulse generator are up-converted to higher frequency components. This technique allows the use of a commercially available impulse generator in which the pulse duration does not have to be extremely short as envisioned in the original USPR concept on SSPX.

In this paper, the up-converted millimeter-wave signals that propagate into the plasma are derived numerically using MATLAB. There are two important processes in the computation: the propagation of the USPR impulse signal through a waveguide and the frequency conversion using millimeter-wave mixers. These mixers are limited to intermediate frequency (IF) signals that are less than 500 mV; thus, it is necessary to disperse the impulse signal into a chirped waveform using the former process. The method of stationary phase is utilized to derive a closed-form formula for a chirped waveform under the assumption that the USPR impulse is Gaussian. In the latter process, the chirped waveform is mixed with a mixer LO signal, and the lower frequency component of the RF signal is removed using a high pass filter.

This paper is organized as follows. In the next section, a brief description of the USPR system on SSPX is presented. In Section 3, the chirped waveform transformed from a Gaussian pulse via waveguide is derived using the method of stationary phase. Some physical aspects of the chirped waveform are also mentioned. In Section 4, the process of frequency up-conversion is explained. Finally, the conclusion of the paper is presented in Section 5.

2. USPR Impulse Generator

This section contains quantitative descriptions of the required pulse duration of an impulse generator for the entire density profile measurement of the SSPX plasma. Before proceeding to the discussion of the details regarding the properties of the impulse generator, a brief description of the USPR system is presented to emphasize the topics of the paper.

2.1 Brief Description of the USPR system

The SSPX USPR system has been fully discussed in [5], along with the basic principle of USPR; hence, the subsys-

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tems directly associated with the topics of the paper are described briefly. Fig. 1 illustrates a schematic diagram of the USPR system on SSPX. The USPR input source can provide a 5 V, 65 psec duration FWHM pulse, which is dispersed through a waveguide to form a chirped waveform. Six broadband millimeter-wave mixer assemblies are utilized to up-convert the chirp frequency range of 6+18 GHz to the plasma frequency range of 33+158 GHz. These mixers are limited to IF signals that are less than 500 mV.

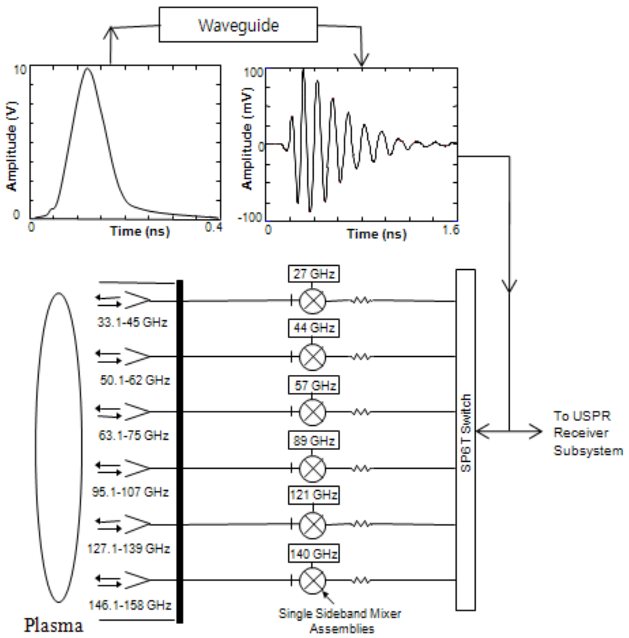

Fig. 1. Schematic diagram of the USPR system.

2.2 Frequency Bandwidth of a Gaussian Pulse

To examine the frequency bandwidth of the USPR impulse source, it is necessary to consider the relation between pulse duration and its frequency bandwidth. If the USPR impulse signal, $f(t)$, is assumed to be Gaussian as shown in Fig. 2, it can be expressed in terms of time t as given by:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}, \tag{1}$$

where σ is a parameter used to determine the pulse duration. Conventionally, the pulse duration is often expressed in terms of FWHM, which is defined as the time interval between the leading and trailing edges of the pulse at a time where the amplitude is 50 % of the peak value. In Fig. 2, FWHM is equal to $2\sigma\sqrt{2\ln 2}$.

The Fourier transform of $f(t)$, $F(\omega)$ is then given by:

$$F(\omega) = e^{-\frac{\sigma^2\omega^2}{2}}, \tag{2}$$

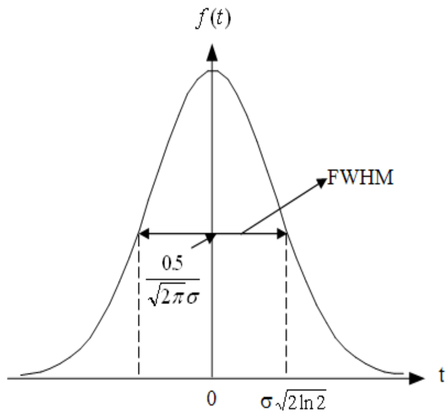

Fig. 2. Plot of a Gaussian pulse.

where $\omega = 2\pi f$, and f is frequency. In this paper, the frequency bandwidth of $f(t)$ is assumed to be the interval between the center frequency and a frequency where the amplitude is 10% of the peak value of $F(\omega)$. Therefore, the relation between the pulse duration, Δt, and frequency bandwidth, Δf, can be expressed as:

$$\Delta t \Delta f = 2\sqrt{\ln 2 \ln 10} / \pi \tag{3}$$

Fig. 3 shows the Fourier power spectra of the Gaussian pulses with respect to FWHM. Here, the frequency at which the y-axis value equals -20 dB is equivalent to the frequency bandwidth of the pulse. With the help of Eq. (3), it can be proved that the pulse duration is required to be at least 5 psec to provide a frequency bandwidth of 158 GHz. However, ultrashort pulse generators that satisfy the SSPX diagnostic requirements are not commercially available.

As can be seen in Fig. 3, the frequency bandwidth of 65 psec FWHM is approximately 12 GHz, which is too narrow to cover a frequency range of 6-18 GHz. As a matter of fact, three chirps and microwave amplifiers have been employed in the actual USPR system [5] to boost the power of the high frequency (>10 GHz) components.

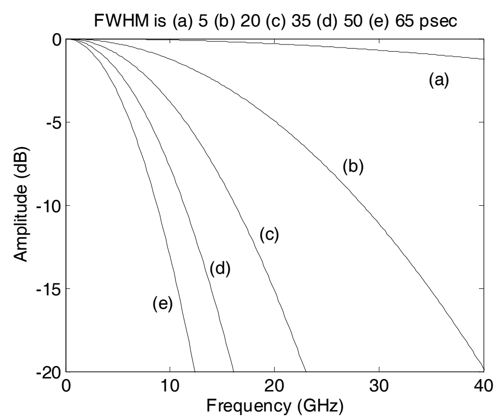

Fig. 3. Fourier power spectra of Gaussian pulses.

However, the transmitter subsystem becomes complicated compared with that in Fig. 1. For simplicity of computation, in this paper, only a single chirped waveform was derived and used as an IF signal of the mixer. It was necessary, however, that the pulse duration of the impulse generator be shorter than 65 psec to ensure a frequency bandwidth of 18 GHz. According to Eq. (3), the pulse duration of 44 psec was chosen for computation of a chirp with a frequency bandwidth of 18 GHz.

3. Propagation of a Gaussian Pulse through a Waveguide

In this section, the chirped waveform of a Gaussian pulse propagating through a waveguide is derived numerically. Assume that the Gaussian pulse is an electromagnetic plane wave propagating along the z-direction in a rectangular waveguide; if transverse spatial variation is negligible, the time dependent field can be expressed as:

$$E(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{E}(0,\omega)e^{j(\omega t - kz)}d\omega \quad (4)$$

where $\hat{E}(0,\omega)$ is the Fourier transform of the input signal of the waveguide, $E(0,t)$, as expressed by:

$$\hat{E}(0,\omega) = \int_{-\infty}^{\infty} E(0,t)e^{-j\omega t}dt = e^{\frac{-\sigma^2\omega^2}{2}} \quad (5)$$

The propagation characteristics of each frequency component of the wave along the waveguide are determined by the factor $e^{-jk(\omega)z}$. The wave number for a waveguide of cutoff frequency, ω_c, is $k(\omega) = \sqrt{\omega^2 - \omega_c^2}/c$, where c is the speed of light.

The stationary phase method was utilized to compute Eq. (4) numerically. The method of stationary phase is a procedure for evaluation of integrals expressed by [7]:

$$I = \int_{-\infty}^{\infty} F(\omega)e^{j\phi(\omega)}d\omega \quad (6)$$

where $F(\omega)$ is assumed to be varying slowly compared with the phase $\phi(\omega)$. The main contribution to the integral comes at the frequencies of stationary phase that are labeled ω_s and defined by $\phi'(\omega_s) = 0$. Using the waveguide dispersion relation, finding the stationary phase is as straightforward as:

$$\omega_s = \frac{\omega_c}{\sqrt{1 - [z/(ct)]^2}} \quad (7)$$

Expanding the phase in a Taylor series near ω_s gives:

$$\phi(\omega) \approx \phi(\omega_s) + \frac{1}{2}\phi''(\omega_s)(\omega - \omega_s)^2 \quad (8)$$

This expansion reduces I to a form that can be integrated analytically, thereby giving:

$$I = \left[\frac{j2\pi}{\phi''(\omega_s)}\right]^{1/2} F(\omega_s)e^{j\phi(\omega_s)} \quad (9)$$

Substituting $\exp(-\sigma^2\omega_s^2/2)$ into $F(\omega_s)$ and considering the causality condition, it is possible to obtain a complete expression of $E(z,t)$ for $t \ge L/c$, where L is the waveguide length as expressed by:

$$E(z,t) = A(t)\exp(-\frac{\sigma^2{\omega_s}^2}{2})\cos\left(\frac{\pi}{4} + \omega_c t\sqrt{1 - \left(\frac{z}{ct}\right)^2}\right) \quad (10)$$

where

$$A(t) = \left[\frac{z^2\omega_s^3}{2\pi c^2 t^3 \omega_c^2}\right]^{\frac{1}{2}} \quad (11)$$

Fig. 4 illustrates a typical chirped waveform of the Gaussian pulse that has been computed using Eq. (10) after the pulse propagated through the WRD 500→650 waveguide transition with cut-off frequency and length of 5.5 GHz and 0.6 m, respectively. Here, the peak value of the Gaussian pulse is 5 V, and the pulse duration is 44 psec (FWHM). The higher frequency component comes out of the waveguide earlier than the lower frequency component. This is due to the dependence of group velocity on frequency in the waveguide. In other words, the group velocity of the higher frequency was faster than that of the lower frequency. It can be also seen that the chirp amplitude was very small compared with the Gaussian pulse because the frequency components that are below the waveguide cutoff frequency cannot propagate through the waveguide and most of the power of the Gaussian pulse resides in these frequencies.

Chirp duration is one of important factor in determining the performance of the USPR system. In fact, an extremely long chirp, as shown in Fig. 4, cannot be used because it may give rise to spurious reflections inside the system. In order to avoid reflection problems, a high speed switch (2 ns switching time) has been used to gate out 2-3.5 ns duration chirps [2], [3]. Fig. 5 depicts a 3 ns duration chirp obtained by applying the following switch function $S(t)$ to Fig. 4:

$$S(t) = \begin{cases} 1 & \text{for } t < t_a \\ \dfrac{t_b - t}{t_b - t_a} & \text{for } t_a \le t \le t_b \\ 0 & \text{for } t > t_b \end{cases} \quad (12)$$

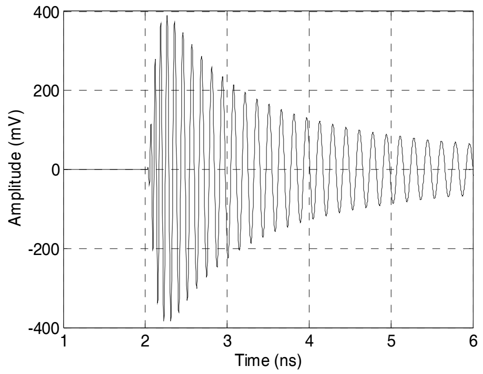

Fig. 4. The chirped waveform of a Gaussian pulse.

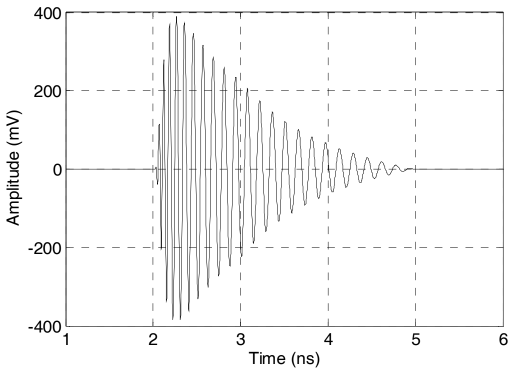

Fig. 5. Chirped waveform of 3 ns duration.

where t_a and t_b are set as 2 ns and 5 ns for a 3 ns chirp duration, respectively.

Due to the switching time, the chirp duration has a significant effect on the Fourier spectrum of the chirp as shown in Fig. 6. The y-axis denotes the ratio of the Fourier spectrum amplitude of a chirp and the maximum amplitude of the Fourier spectrum of a 3.5 ns duration chirp. In particular, the lower frequency (6-10 GHz) components were sensitive to the change in the chirp duration. This meant that the power of these components decreased as the chirp duration became shorter.

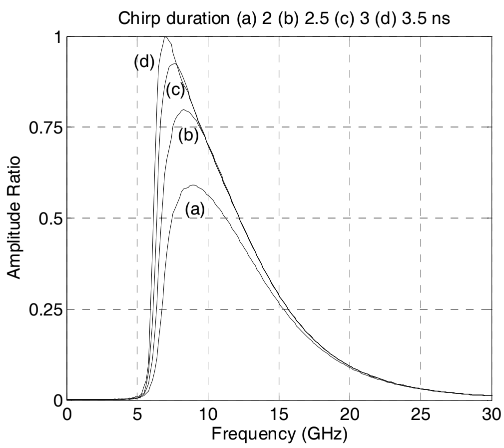

Fig. 6. Fourier spectrum of the chirped waveform in Fig. 4 with respect to the chirp duration.

4. Frequency Up-conversion using Mixers

In this section, the millimeter-wave signals with frequencies ranging from 33-158 GHz were derived numerically using MATLAB to launch into the plasma. To compute these signals, it was necessary to consider how frequency up-conversion took place in a mixer.

The chirped waveform in Fig. 5 was sent to the IF port of the mixer. The IF signal was then mixed with the LO signal. Consequently, the output frequency of the mixer became $f_{RF} = f_{LO} \pm f_{IF}$. The mixer was typically followed by a corresponding high-pass filter, the frequency after the filter was $f_{LO} + f_{IF}$. Table 1 enumerates the frequency specifications of the six mixers employed in the USPR system.

Table 1. Frequency specification of mixers

Mixer	IF Frequency (GHz)	LO Frequency (GHz)	RF Frequency (GHz)
1	6 - 18	27	33 - 45
2		44	50 - 62
3		57	63 – 75
4		89	95 - 107
5		121	127 - 139
6		140	146 – 158

The following MATLAB code was written based on the operational principle of the mixer to compute the frequency up-conversion process as explained above.

```
% Computation of up-converted signal via mixer and high-pass filter
% High-pass filter of cutoff frequency= W_cutoff.
h=fir1(fix(num/3),W_cutoff, 'high');
% Signal of the local oscillator.
signal_LO=cos(2*pi*f_LO*t); % f_LO=Frequency of the local oscillator.
% Output signal of mixer,
signal_out=Ez_Sw.*signal_LO;% Ez_Sw=IF signal
% Up-converted signal
signal_up=filtfilt(h,1,signal_out)
```

The code uses the function, "fir1," for a high-pass filter whose impulse response is shown in Fig. 7. In the code, "signal_up" represents the output of the high-pass filter or the up-converted frequency signal. Fig. 8 shows an example of up-converted frequency signals obtained by the code. In Figs. 8 (b) and (c), the frequency in the beginning part of the signal is around 60 GHz, and that in the ending part is approximately 45 GHz. Therefore, it may be considered that the MATLAB code works properly in the computation of up-converted frequency signals. In order to examine the frequency bandwidths of the RF signals precisely, the Fourier spectrum of the signals were hereby computed. Fig. 9 illustrates the Fourier spectrum of all RF signals. Here, the normalized amplitude can be obtained easily by dividing the Fourier spectrum using the maximum value of the spectrum. An amplitude value of 0.1, which is indicated by a dotted line, is equivalent to -20 dB. Therefore, the frequency bandwidth of the system can be defined as the frequency range where the amplitude is equal to or greater than the line. As can be seen, the low frequency (6-18 GHz) bandwidth shifts to high frequency bandwidth by the LO frequency as a result of frequency up-conversion. This confirms that the process of frequency up-conversion, as well as chirp transformation, has been computed accurately using the code.

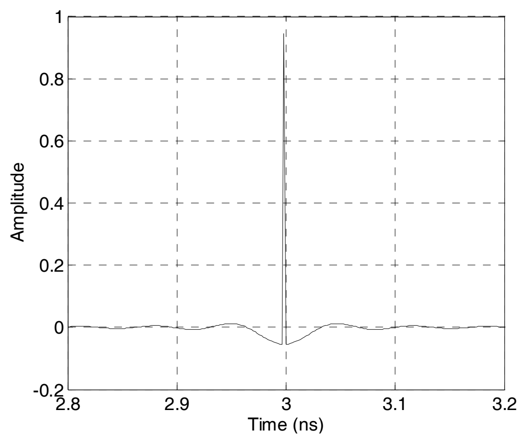

Fig. 7. Impulse response of a high-pass filter with cutoff frequency of 44 GHz.

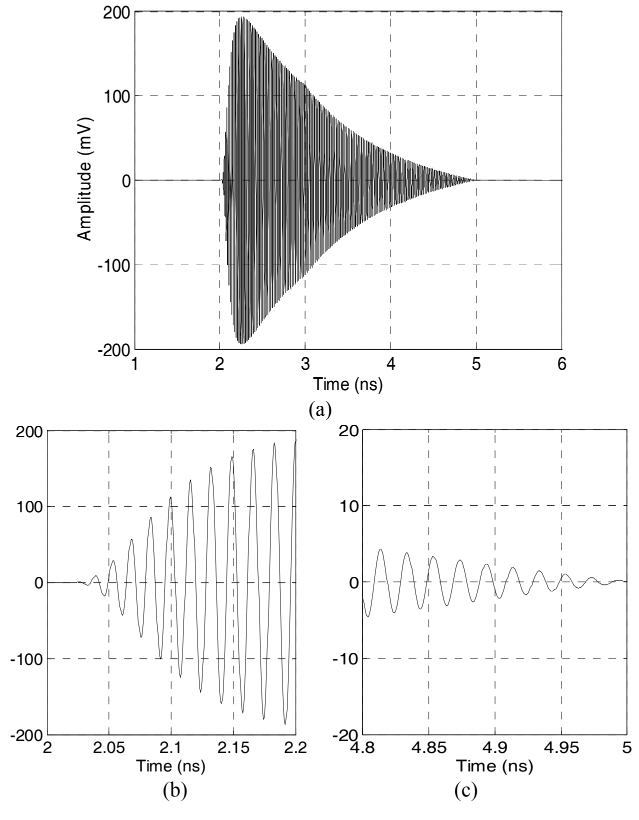

Fig. 8. Up-converted frequency chirped waveform with 44 GHz LO frequency in a time domain of (a) 1-6 ns (b) 2-2.2 ns (c) 4.8-5 ns.

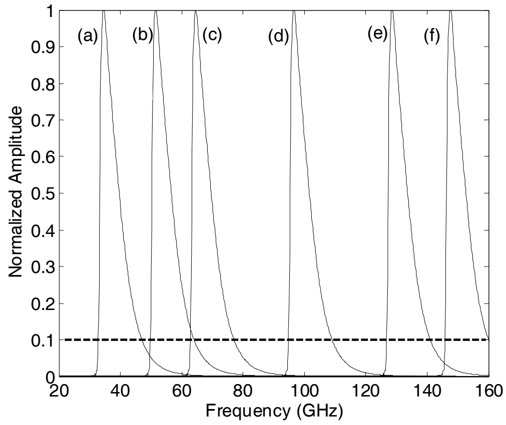

Fig. 9. Fourier spectrum of up-converted frequency chirps with a LO frequency of (a) 27 GHz, (b) 44 GHz, (c) 57 GHz, (d) 89 GHz, (e) 121 GHz, and (f) 140 GHz.

5. Conclusion

The numerical computation of frequency up-conversion and the transformation of a chirped waveform was conducted using MATLAB to derive the broadband USPR signals to propagate into the plasma. The method of stationary phase was employed successfully to transform a Gaussian pulse of 44 psec duration (FWHM) into a chirped waveform of 6—18 GHz. It was found that a trade-off between short and long chirp durations. The computed Fourier transforms of the mixer RF signals prove that the Matlab code is capable of generating accurately broadband USPR signals based on the frequency up-conversion.

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