한국산학기술학회논문지
Vol. 11, No. 6 pp. 1949-1954, 2010

The Mechanical Dither Design of Navigation Guide Structure

Jeong-Ick Lee[1*]
[1]**Department of Mechnical Design, Inha Technical College**

네비게이션 가이드 구조물의 기계적 진동설계

이정익[1*]
[1]인하공업전문대학 기계공학부 기계설계과

Abstract The gyroscopes have been used as a suitable inertial instrument for the navigation guidance and attitude controls. The accuracy as very sensitive sensor is limited by the lock-in region(dead band) due to the frequency coupling between two counter-propagating waves at low rotation rates. This frequency coupling gives no phase difference, and an angular increment is not detected. This problem can be overcome by mechanically dithering the gyroscope.
This paper presents the design method of mechanical dither by the theoretical considerations and the verification of the theoretical equations through FEM(Finite Element Method) applications. As a result, the maximum prediction error of resonant frequency and peak dither rate was under 5 percent. The theoretical equations for the mechanical performances of dither can be said to be feasible.

요 약 자이로스콥은 네비게이션을 가이드하거나 특성을 제어하는데 적절한 관성 측정도구로 사용되었다. 매우 민감한 센서로서의 정확성은 저회전율 역전파 사이의 진동수 커플링으로 인한 폐쇄영역(데드 밴드)으로 결정된다. 이 진동 커플링은 위상차가 없으며, 각증분 값은 검출되지 않는다. 이 문제는 자이로스콥의 기계적인 진동으로 해결될 수 있다. 본 논문은 FEM을 통해 이론적 식들의 이론적 고려사항과 증명의 방법으로 기계 진동의 설계방법을 제시한 것이다. 결과적으로, 공명 진동수와 최대 진동률의 최대 예측 오차는 5 %이하였다. 진동의 기계적 성능을 위한 이론식들은 타당하다고 할 수 있다.

Key Words : Ring Laser Gyroscope, Lock-in Region, Dead Band, Mechanical Dither, Resonant Frequency, Torsional Stiffness, Peak Angular Amplitude, Peak Dither Rate, FEM(Finite Element Method)

Nomenclature

P	Shear force
M	Bending moment
E	Modulus of elasticity
I	Moment of inertia of area
J	Polar moment of inertia of mass
δ	Circumferential deflection of spoke
f_n	Resonant frequency
σ	Bending stress of spoke
ε	Longitudinal strain of spoke
$\dot{\theta}_{\max}$	Peak dither rate

1. Introduction

When there is an applied rotation about the axis normal to the plane of rotation, the operation of gyroscope depends on the phase difference for beams traveling in opposite direction within a closed path. This difference gives rise to a phase difference. But the accuracy of gyroscopes have been limited by the lock-in region due to the fact that at low rotation rates

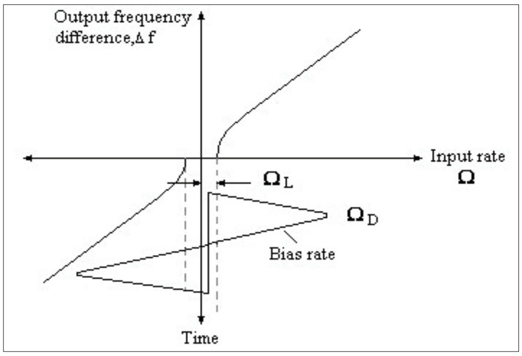
*Corresponding Author : Lee, Jeong-Ick(jilee@inhatc.ac.krr)
Received March 17, 2010 Revised April 21, 2010 Accepted June 18, 2010

of gyroscope the frequency coupling mechanism arises from back scattering of the mirrors. This effect gives no phase difference and hence no detect of angular increment. In other words, this reduces the angular sensitivities to the electrical and mechanical disturbances such as low frequency noise components, in the optical measurement. It is important to minimize the transition periods of lock-in region. The dithering motion is to operate the gyroscope under the lock-in region in order to be sensitive against input rotation rates which are below that certain region [Fig. 1]. The dithering method uses the electrical signal processing for the low back scatter in conjunction with a stable gas discharge, a suitable dither drive and a stabilized resonator cavity length. But the simplest dithering method is to use the mechanical dither.

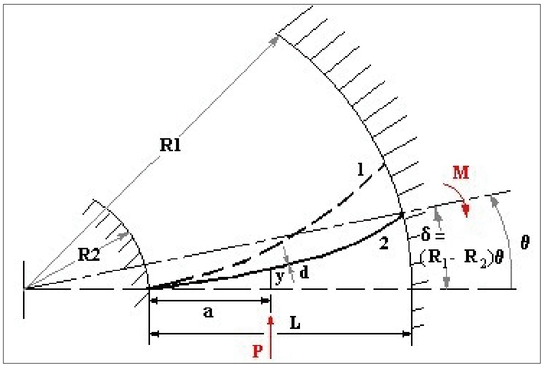
The purpose of the mechanical dithering is to suppress the dead band, oscillate the block about the rotation axis and add an external rotation rate. The design considerations of mechanical dither include the followings : structural resonant frequency, peak angular amplitude and dither rate, driving technique, and inertia of dithered components.

The formulation of mechanical dither was explained by Shackleton B. R. in 1987[5]. His paper presented the geometrical approach of spoke, but had no considerations of the loading condition and angular characteristics due to the piezo element deformation.

In this paper, the mechanical performances of dither are theoretically presented on the basis of any loading condition and angular characteristics (peak dither amplitude and rate) due to the piezo element deformation. And using the transverse vibration behavior of elastic spoke, the resonant frequency of mechanical dither is calculated. Through the finite element analysis, the structural performances of mechanical dither are compared.



[Fig. 1] Laser gyroscope alternating bias technique



[Fig. 2] Dither spoke geometry

2. Theoretical Consideration

To design the dither is to calculate the torsional stiffness of dither, bending profile, point of inflection and stress caused by the bending behavior of spokes. In Fig. 2, a dither spoke may be considered as a cantilever of length L with the inertia effect at the end R_1 and rotated with regard to the center R_2.

The angular rotation may be due to the combined effects of an end load P and a moment M, which must meet the condition that the spoke isn't free at the other end. An end load P gives curve 1 as the bending profile and a moment M give curve 2. From the above parameters, the structural deflection and slope of spoke due to P and M are given as follows ; deflection due to P and M,

$$\delta = \frac{Pa^2}{6EI}(3L-a) - \frac{ML^2}{2EI} \tag{1}$$

and bending slope of spoke due to P and M.

$$\theta = \frac{Pa^2}{2EI} - \frac{ML}{EI} \tag{2}$$

From Fig. 2, the resulting torque is given by P and M,

$$T = Pa - M \tag{3}$$

For the small elastic circumferential deflection of spoke, using Eqs. (1), (2) and $\delta = (R_1 - R_2)\theta$, it can be shown as the ratio of moment, which is due to the geometric parameters.

$$\frac{Pa}{M} = \frac{3(R_1-R_2)^2}{a^2} = G \tag{4}$$

where $L = R_1 - R_2$. And the torsional stiffness of a spoke may be defined using Eqs. (2), (3), and (4).

$$\frac{T}{\theta} = 2EI\frac{G-1}{Ga-2L} \quad (5)$$

and Eq. (5) can be rewritten in terms of the geometric parameters of a spoke,

$$K_\theta = \frac{2EI}{a}\frac{3(R_1-R_2)^2 - a^2}{3(R_1-R_2)^2 - 2a(R_1-R_2)} \quad (6)$$

where R_1 and R_2 are the fixed ends of spoke span(L) which depends on the deflection behavior of the spoke span.

The point of inflection concerned with the behavior of an elastic spoke is given by the differential equation of flexural beam. The differential equation of flexural beam states that the local curvature equals the local bending moment (M_b) divided by the local flexural stiffness of beam,

$$M_b = EI\frac{d^2y}{dx^2} \quad (7)$$

The point of inflection of spoke is a point given by,

$$\text{Since} \quad \frac{d^2y}{dx^2} = 0, \quad M_b = 0 \quad (8\text{-a})$$

and the bending moment in a spoke may be written for $0 < x < a$,

$$M_b = P(a-x) - M \quad (8\text{-b})$$

Using Eqs. (2), (8-a) and (8-b), the point of inflection, x, can be written as follows,

$$x = \frac{G-1}{G}a \quad (8\text{-c})$$

The bending stress induced by the local bending moment in the cantilever spoke, is given by,

$$\sigma_b = \frac{M_b t}{I} = \frac{P(a-x)-M}{I}t \quad (9)$$

where I is an area moment of inertia.

Setting $R_2 = \alpha R_1$ and $L = R_1 - R_2$, the point of inflection along the spoke length is given by Eqs. (4) and (8),

$$\frac{G-1}{G} = \frac{3R_1^2(1-\alpha)^2 - a^2}{3R_1^2(1-\alpha)^2} \quad (10)$$

From Eq. (10), as R_2 approaches zero and the applied load position approaches the end of spoke, α approaches zero. Hence $(G-1)/G$ approaches $2/3$. The point of inflection cannot occur beyond $2/3$ of the spoke length. This limits the area of spoke that may be used by piezo element for driving the spokes since they cannot induce two opposite forms of curvature in the spoke, that is, the piezo element cannot be extended beyond the point of inflection.

3. Mechanical Performance of Dither

The aforesaid mono block system is assumed to be mounted on the outer diameter R_1 of spoke. But if a spoke has the end mass m at a point R_1, the resonant frequency for the mounting location of mono block system at an arbitrary point L' on the spoke may be given by,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_\theta}{mL^2}} = \frac{1}{2\pi}\sqrt{\frac{nK_\theta}{J_{sys}(L/L')^2}} \quad (11)$$

where n is the number of elastic spokes, K_θ is the torsional stiffness of the dither from Eq. (6), J_{sys} is the polar mass moment of inertia of the dithering system, L is the swing arm of spoke assumed as $R_1 - R_2$. The relationship between m and J_{sys} is given by

$$m = \frac{J_{sys}}{nL'^2} \quad (12)$$

for a spoke.

Since both ends of spoke are respectively built at the central post and mono block, the bending stresses at the ends must to be verified within the allowable stress. Using Eq. (9) and setting $x = 0$, the bending stress at R_2 is given by,

$$\sigma_{R_2} = \frac{Pa-M}{I}t \quad (12\text{-a})$$

and can be rewritten, using Eq. (4), and $t = d/2$

$$\sigma_{R_2} = \frac{M(G-1)}{I}\frac{d}{2} \quad (12\text{-b})$$

Also, using Eqs. (2), (3), (4) and (5), the torque can be written in terms of the geometric parameters and mechanical properties,

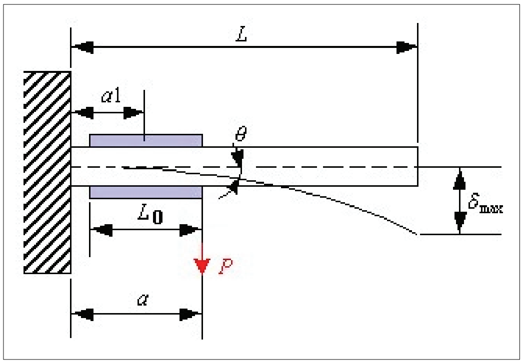
$$T = \frac{Ga-a}{a}M = 2EI\theta\frac{G-1}{Ga-2L} \quad (13\text{-a})$$

$$\frac{M}{I} = \frac{2E\theta}{(Ga-2L)} \quad (13\text{-b})$$

Hence the bending stress may be expressed in terms of the geometry of the spoke and the angular deflection.

4. Loading Condition

The mechanical dither is mounted via the beam-shaped spokes to the gyroscope mono block and to the inertial rigid reference frame. The deformation of spoke is under the alternate deformation of piezo element on both faces. Each opposite voltage signal on two faces is loaded and results in the compression and expansion deformations of spoke. The bending behavior of spoke due to the stretching and contracting deformations of piezo element is shown in Fig. 3.



[Fig. 3] Spoke deformation due to bending moment

The longitudinal strain ε_x of spoke and the bending moment are given by,

$$\varepsilon_x = \frac{1}{E}\left(-\frac{M_b y}{I}\right), \quad M_b = -\frac{EI}{y}\varepsilon_x \quad (14)$$

Setting the maximum strain $\varepsilon_x \approx \Delta L / L_0$ at the piezo element bonding face ($y = d/2$), the bending moment of spoke may be given by,

$$M_b = -\frac{2EI}{d}\left(\frac{\Delta L}{L_0}\right) \quad (15)$$

on the neutral axis, where L_0 is the length of piezo element, d is the thickness of spoke, ΔL is the maximum deformation of piezo element along the longitudinal direction.

Since the maximum deflection due to a moment M_b or a vertical load P must be equal at the end, the equivalent force is given as,

$$\delta_{\max} = \frac{Pa^2}{6EI}(3L-a) = \frac{M_b}{EI}L_0(L-a_1) \quad (16)$$

$$P = \frac{6M_b(L-a_1)L_0}{a^2(3L-a)} \quad (17)$$

The bending angle θ_a at the loading point a is approximated as the peak angular amplitude.

$$\theta_a = \frac{Pa^2}{2EI_{zz}} = \frac{3M_b(L-a_1)L_0}{EI_{zz}(3L-a)} = \frac{6}{d}\frac{L-a_1}{3L-a}\Delta L \quad (18\text{-a})$$

And from the condition that the maximum external work is equal to the maximum kinetic energy, the peak dither rate may be given by,

$$U_{\max} = T_{\max}$$

$$\frac{1}{2}P\delta_a = \frac{1}{2}\left(\frac{J_{sys}}{n}\right)\dot{\theta}_{\max}^2$$

$$\dot{\theta}_{\max} = \frac{(L-a_1)\Delta L}{(3L-a)d}\sqrt{\frac{48nEI}{aJ_{sys}}} \quad (18\text{-b})$$

where δ_a is the vertical deflection at the loading point a.

5. Simulation

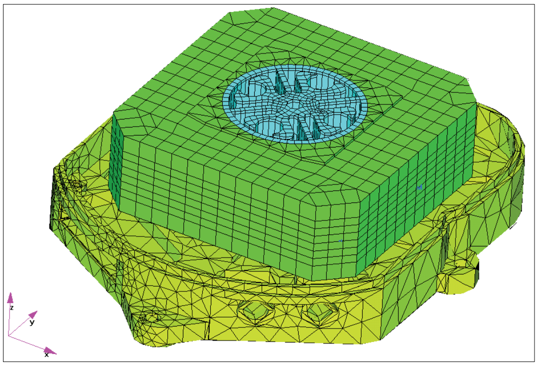
The primary goal of the finite element analysis is to guide the structural design of dither. A typical simulation of the dynamic behavior based on the structural stiffness of the dither system takes into account two aspects: the first is the mode at the detection space of gyroscope and the second is the frequency history of the angle in the desired direction referred to as the dither amplitude.

The radial dither in Fig. 4-(a) is mounting via four wedge-shaped spokes to the gyroscope solid block and to the inertial reference frame which is rigid. The bending moment of a spoke under the driving deformation of piezo element ($\Delta L = \pm 2\mu m$) may be calculated from Eq. (15). From Fig. 2 and 3, the shape parameters of a spoke in Fig. 4 are as follows: $n = 4$, $R_1 = 19.25mm$, $R_2 = 5.00mm$, $L' = 10.50mm$, $w = 30.00mm$, $a = 6.25mm$, $a_1 = 4.00mm$. From Eq. (19), the equivalent polar mass moment of inertia of system is 1.66 *ton-mm*^{2} except the grounded constituent components, based on the test resonant frequency ($f_{sys} = 382\ Hz$) and the inertial effect of system is taken into account as the scalar dummy inertia. This equivalent mass moment of inertia of system may be used for representing the inertial effect of the dithering structure. If the resonant frequency is basically given, the equivalent polar mass moment of inertia (J_{sys}) may be approximated as follows,

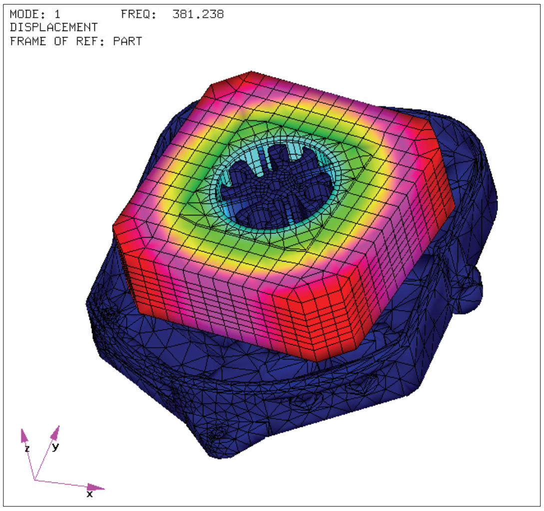
$$J_{sys} \approx \left(\frac{f_{dither}}{f_{sys}} \right)^2 J_{dither} \qquad (19)$$

where f_{dither} is the natural frequency of mechanical dither structure without the consideration of mono block system, which is 4897.12 *Hz* as shown in Fig. 4-(b) and J_{dither} is the mass moment of inertia of mechanical dither., which is 0.01095 *ton-mm*^{2} .

The structural damping factor is given as $Q-factor$ ($Q = 1/2\zeta$), which is 150. The material properties are given as *INVAR 36 ALLOY*($E = 144.9\ GPa$, $\nu = 0.29$, $\rho = 8.11 \times 10^{-6}\ kg/mm^3$). The four spokes are constrained for the connection with the rigid inertial reference frame. For 8-node hexahedron solid element model of mechanical dither, the peak dither rate and resonant frequency are simulated through the direct frequency response analysis, using *MSC/NASTRAN V70.5*. Table 2 shows the 1st resonant frequency for the rotational mode and peak dither rate.



(a) Finite element model of ring laser gyroscope



(b) Resonant frequency of typical mechanical dither

[Fig. 4] Schematic model of ring laser gyroscope

Table 1. Resonant frequencies of mechanical dithers to thickness variations

No	Thickness d (mm)	Test Freq. (Hz) Peak dither rate(deg/sec)	FEA Freq. (Hz) Peak dither rate(deg/sec)	Theoretical Eq. Freq. (Hz), Peak dither rate(deg/sec)
1	2.94	363.00, 134.15	359.54, 137.25	359.77, 137.19
2	3.08	382.00 145.10	381.23, 141.61	380.07, 144.24
3	3.22	400.00 150.00	400.44, 143.52	402.50, 147.03

In Table 1, the results of the finite element analyses and numerical solutions show that the maximum relative errors to test results are within 3 percent in the resonant frequency and 5 percent in the peak dither rate, and two

methods are relatively available for designing the mechanical dither. From the aforesaid equations, it can be known that the relative error between finite element analyses and theoretical solutions depend on the working elastic length, which is given as $L = R_1 - R_2$. This working elastic length is the effective bending length under the bending deformation of a spoke.

6. Conclusion

From the geometric parameters and material properties of the dither, the mechanical characteristics can be given as follows: torsional stiffness, bending profile, points of inflection, stress caused by bending of the spokes, resonant frequency and peak dither rate. And since the stress within the spoke and piezo element may further limit the angular amplitude and rate, the peak dither rate is usually limited to the angular amplitude which did not cause stresses in the spoke material to exceed the effective material stress. The effective bending length as the working elastic length is the important factor which has an influence on the angular deflection as the bending deformation of spoke. But the geometrical parameters of spoke can approximately be used. And the finite element analysis can be used for designing the concrete and complicated spoke within the given design range.

References

[1] Ansel C. Ugural, Saul K. Fenster, Advanced strength and applied elasticity, Prentice-Hall, 3rd edition, 1995.

[2] Robert J. Kline-Schoder, Michael J. Wright, Design of a dither mirror control system, Mechanics, Vol. 2, No. 2, pp. 115-127, 1992.

[3] Roy R. Craig, Structural dynamics An introduction to computer methods, John Wiley & Sons, 1981.

[4] Schleich W. , Dobiasch P., Noise analysis of ring laser gyroscope with arbitrary dither, Optics communications, Vol. 52, No. 1, pp. 63-68, 1984.

[5] Shackleton B. R., Mechanical design considerations for a ring laser gyro dither mechanism", Proceeding Instrument Mechanical Engineering, pp. 105-112, 1987.

[6] Stephen H. Crandall, Norman C. Dahl and Thomas J. Lardner, An introduction to the mechanics of solids, 2nd, McGraw-Hill, New York, 1978.

[7] Stepen P. Timoshenko, James M. Gere, Theory of elastic stability, McGraw-Hill, New York, 2nd edition, 1963.

[8] Thomas J. Hutchings, "Scale factor non-linearity of a body dither laser gyro", Proc. IEEE Nat'l Aerospace and Electronic conf., pp. 549-555, 1978.

Lee, Jeong-Ick [Regular Member]

- Feb. 1991 : Dept. of Mechanical Eng. College of Engineering, Hanyang Univ. (Bachelor of Eng.)
- Feb. 1993 : Dept. of Precision Mechanical Eng. College of Engineering, Hanyang Univ. (Master of Eng.)
- Aug. 1999 : Dept. of Precision Mechanical Eng. College of Engineering, Hanyang Univ. (Ph.D.)
- Jan. 1993 ~ Dec. 1999 : Daewoo Electronics Company, Research Center (Senior Researcher)
- Mar. 2000 ~ Feb. 2007 : Yong-In Songdam College, Dept. of Automotive Mechanical Design (Professor)
- Mar. 2007 ~ current : Dept. of Mechanical Design, Inha Techinical College (Professor)

<Research Interests>
CAD/CAM/CAE, Factory Automation, Manufacturing Automation, Mold Injection, Ubiquitous Eng., MEMS, Biomechanics