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A Sub-Wavelength Focusing Lens Composed of a Dual-Plate Metamaterial Providing a Negative Refractive Index

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Abstract

We have proposed a metamaterial lens that enables sub-wavelength focusing, which is shorter than an operating wavelength. Our lens is a two dimensional array of a unit cell consisting of a metallic dual-plate printed on a dielectric substrate. The unique dual-plate structure provides negativeness both in permittivity and permeability, with no help from conventional additional structures, which are normally printed on the opposite of metallic patterns. Therefore, we can focus a source (or an image) in a tiny distance shorter than the free space wavelength (λ) at the frequencies of interest. Furthermore, since the proposed geometry does not need separate supplementary structures to acquire negative permittivity or permeability, our lens is much simpler than conventional metamaterial lenses, which is a strong point in practical applications. We have validated sub-wavelength focusing ability in a 6 GHz frequency band through an experiment of near field scanning, which provided the width of about 0.19 λ at a half maximum of a peak value of an measured image. The width of the focused image through the lens is more than 4 times shorter than that without the lens, which confirms the validity of our design approach.

Key words: Metamaterial, Lens, Sub-Wavelength Focusing, Negative Refractive Index.

Ⅰ. Introduction

We can find both theoretically valuable analysis and practical applications in the literature [1] for an unusual material that has negative values for permittivity, permeability, and an index of refraction. This unusual material is referred to by various names, such as a metamaterial, backward wave material, single of a double-negative material, etc. In 1999, the discovery that negative permeability can also be artificially engineered by rolling up a metallic sheet or making a gap in an annular ring was announced by the group of Pendry for the first time [2]. Since then, research on metamaterials has progressed by work from many research groups [3]～[8].

One of numerous interesting applications of metamaterials is the capacity for sub-wavelength focusing of a source, a so-called 'superlens' [9]. The sub-wavelength focusing means that a lens consisting of metamaterials can focus the source in a very short distance (or a small area) shorter than a free space wavelength (λ) on the opposite side, which is impossible in classical optics. In other words, we can resolve two individual sources located very close together in a free space wavelength using metamaterials [10], [11]. In [10] and [11], a split-ring and a rod were combined to make lenses, and resolutions of 0.23 λ and 0.13 λ were obtained.

We can find a range of usability of metamaterial lenses in many applications. In [12], a planar silver layer was used in optical lithography to produce fine gratings on a photoresist layer. The metamaterial lens was also successfully used to detect a very narrow hole embedded in a dielectric material [13]. Bi- or plano-concave composition of a left-handed extraordinary transmission lens was used to concentrate a transmitted beam into a receiver [14].

In this paper, we have proposed another type of sub-wavelength focusing lens that uses a simple metallic pattern and gives a negative index of refraction in a frequency region of several GHz. Our lens consists of an array of a metallic dual-plate etched onto a dielectric substrate. The two plates are sufficient to obtain simultaneously negative values in permittivity and permeability without additional structure, which was inevitably needed in conventional metamaterials.

First, we show the negative behavior of the unit cell through a prediction of constitutive parameters using an effective material parameter retrieval algorithm reported in [15]. Next, we compare the validity of the retrieved parameters with results of an eigen-mode analysis and a direct transmission using the commercial simulation tool of CSTMWS. We then present an experimental result of sub-wavelength focusing, which results in an image-wid-

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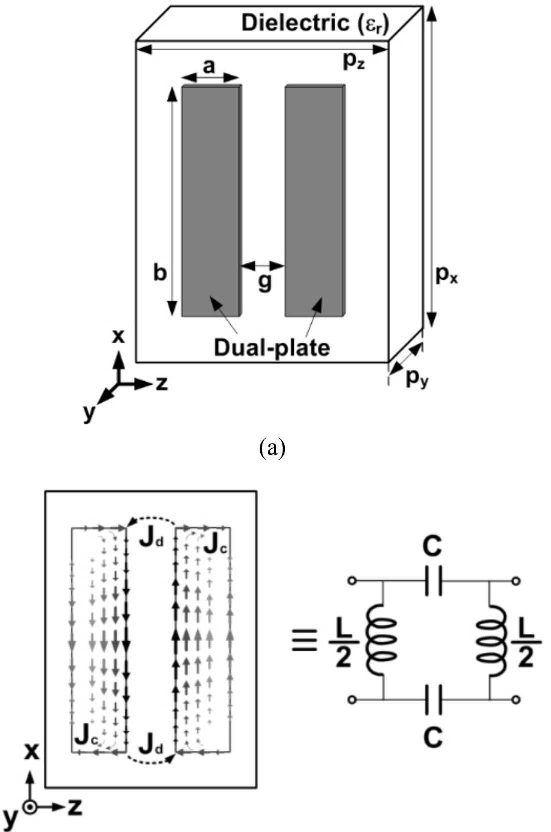
th of about 0.19 λ at 6.38 GHz.

Ⅱ. Design of a Metamaterial Unit Cell

The unit cell geometry used for fabrication of a proposed metamaterial lens is shown in Fig. 1. It consists of two individual metallic plates etched onto a commercial dielectric substrate of Taconic TRF-45 with a relative permittivity of 4.5.

The proposed metamaterial composition provides simultaneously negative values of permittivity (ε) and permeability (μ) in an overlapped frequency band of interest. Therefore, it presents a negative index of refraction (*n*) in that frequency region. This mechanism for obtaining a negative *n* is differentiated from many types of conventional left-handed metamaterials, which need independent structures to get negative ε and μ, separately [8].



(a)

(b)

Fig. 1. (a) Geometry of a proposed metamaterial unit cell with a=1 mm, b=9.6 mm, g=1.5 mm, p_x=10 mm, p_y=3.05 mm, p_z=5.5 mm, and ε_r=4.5, and (b) Induced surface current density distribution on the dual-patch and its equivalent resonant circuit representation at near 5.3 GHz.

For better understanding, the principle of acquiring negative μ is revisited in Fig. 1(b), which portrays distribution of induced surface current density. For an x-polarized incident plain wave propagating toward the positive z-direction, the induced surface current density forms a closed resonant loop that is composed of the conduction and displacement current density, which are denoted J_c and J_d, respectively. This resonance generates a magnetic dipole moment resisting against an applied external field, which results in a negative values of μ [2]~[4].

We show graphically the principle of generating negative permeability by computing the magnetic field intensity distribution at two different modes, which is given in Fig. 2. The distribution was obtained at each pass band edge of the two individual modes. The first mode ranges from 0 to 4.9 GHz. The second mode starts at 5.3 GHz and ends at 6.44 GHz (See Figs. 4 and 5). Therefore, the distribution corresponds to normalized wave vectors of $\beta p_z/2\pi$=0.5.

In Fig. 2, directions of the fields are exactly opposite to each other. In the first mode, the H-field is strong around the two edge region, but it vanishes at the center of the cell. In contrast to the first mode, in the second mode, the H-field is concentrated around the center region, which implies that a magnetic dipole moment will have different sign for the two cases. If the magnetic dipole moment vectors are heading toward the direction that resists an applied external field, then permeability will be negative.

To get physical insight into the negative constitutive parameters, we computed electric (**p**) and magnetic (**m**) dipole moments in Fig. 3, which are given by,

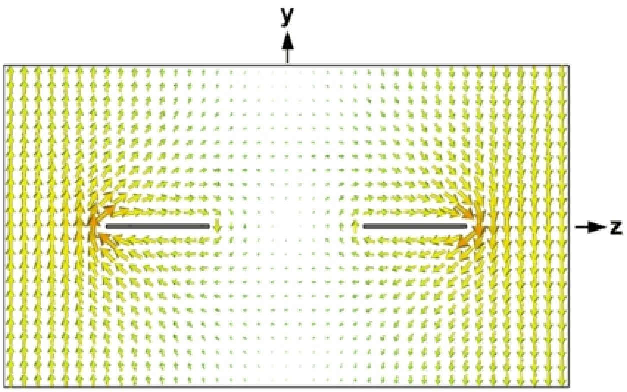
$$\mathbf{m} = \int_v \mathbf{r} \times \mathbf{J}(\mathbf{r})\, dv \tag{1}$$

$$\mathbf{p} = \int_v \mathbf{r}\, \rho(\mathbf{r})\, dv, \tag{2}$$

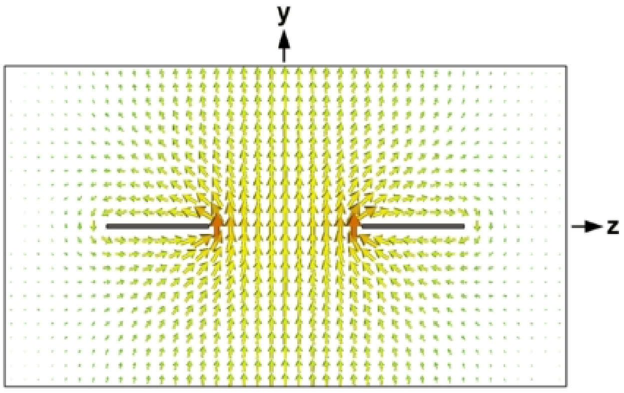
where **r** is a position vector, and ρ and **J** denote charge and current density, respectively [16].

Each dipole moment can be a measure of negative responses in ε and μ. We see that both the magnetic and electric dipole moments are negative from about 4.8 GHz to 6.7 GHz. From the result of Fig. 3, we can expect that ε and μ are both negative around 5 and 6 GHz bands, which will result in a negative value of *n*.

We examined the negative properties of the proposed left-handed metamaterial more closely by retrieving the



(a) The first mode (f=4.9 GHz)



(b) The second mode (f=5.3 GHz)

Fig. 2. Distribution of magnetic field intensity on any z-plain intersecting the origin at each edge.

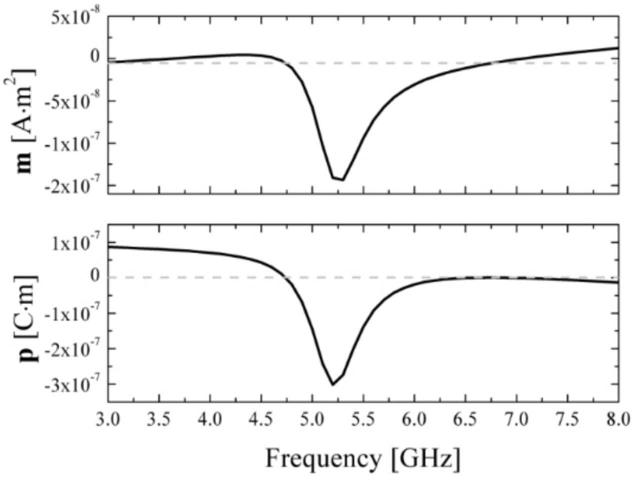


Fig. 3. Magnetic (**m**) and electric (**p**) dipole moments.

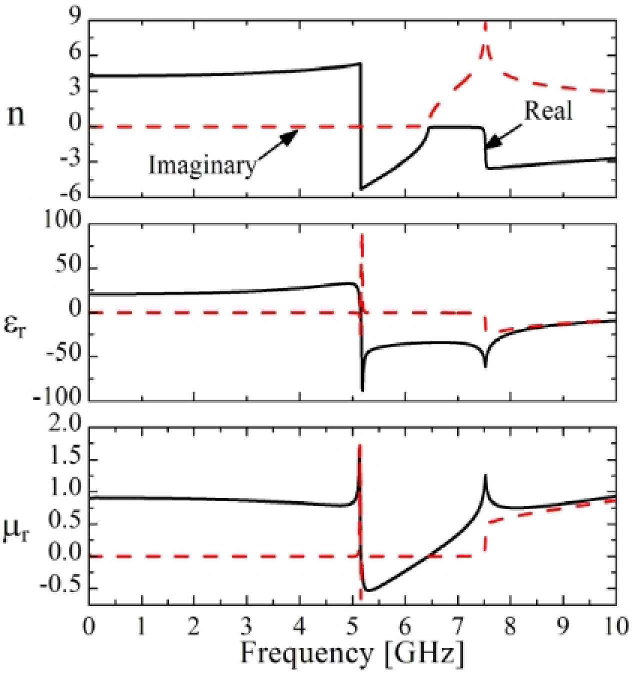


Fig. 4. Retrieved effective constitutive parameters.

effective constitutive medium parameters according to the method reported in [15], which is rewritten as follows:

$$n_{eff} = \frac{1}{k_0 p_z}\left[\ln\left(e^{ink_0 p_z}\right)'' + 2m\pi - i\left\{\ln\left(e^{ink_0 p_z}\right)\right\}'\right] \tag{3}$$

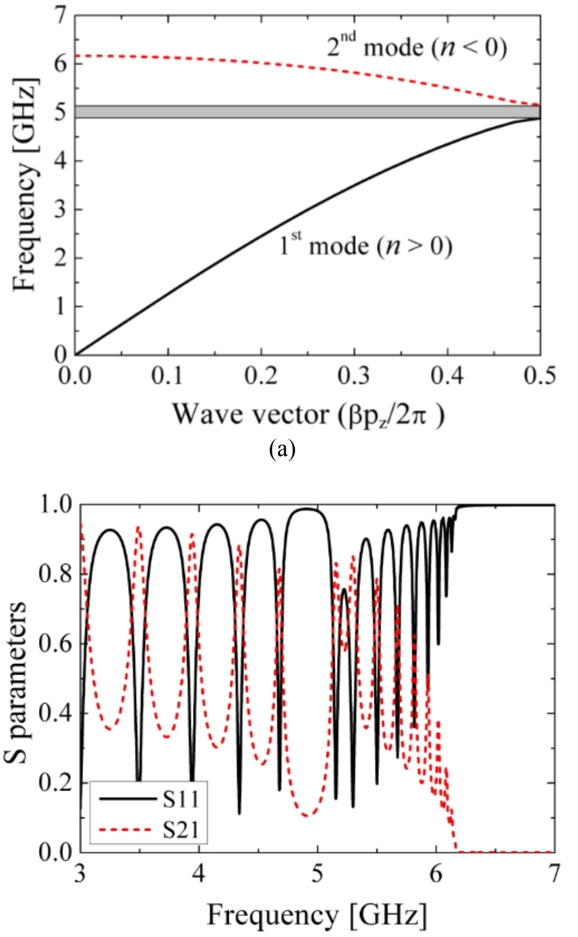
where

$$e^{ink_0 p_z} = \frac{S_{21}}{1 - S_{11}\left(\dfrac{Z_n - 1}{Z_n + 1}\right)} \tag{4}$$

$$Z_n = \pm\sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}} \tag{5}$$

with the constraint of $Z_n' \geq 0$ and $n'' \geq 0$. In the equations, $(\cdot)'$ and $(\cdot)''$ denote real and imaginary parts of $(\cdot)$, respectively. The retrieved effective medium parameters are given in Fig. 4. The frequency band of negative values of n ranges from about 5.3 GHz to 6.44 GHz. The n starts from about -5.3 at 5.2 GHz and continuously increases to 0 at 6.44 GHz. In the frequency region, the imaginary part of n is nearly zero. Thus, we can say that the region is a lossless pass band allowing a negative phase velocity.

Apart from the retrieval approach using (3) to (5), there is another way to check possible modes with signs of n. Fig. 5(a) shows an eigen-mode analysis result describing two pass bands with one stop band between them. The first mode starts from D.C. to about 4.9 GHz, which has positive values of n in the band. The second



(a)

(b)

Fig. 5. (a) Dispersion property of the proposed metamaterial structure, and (b) reflection and transmission characteristics of an array of the metamaterial composed of 10 cells in a wave propagation direction.

mode spans from 5.3 GHz to 6.44 GHz with negative values of n. We can easily obtain the mode properties shown in Fig. 5(a) with a simple calculation of a dispersion relation.

We double checked the existence of the pass and stop bands by computing reflection and transmission behaviors of the proposed metamaterial structure, which is shown in Fig. 5(b). For simulation, we stacked a total of 10 unit cells in the wave propagating z-direction, and measured scattering parameters. We can find the positive pass band under about 4.8 GHz, and the negative pass band over about 5.3 GHz, and the stop band between them. Consequently, we confirm that there are two separate modes with different signs of n.

Ⅲ. Fabrication and Experiment

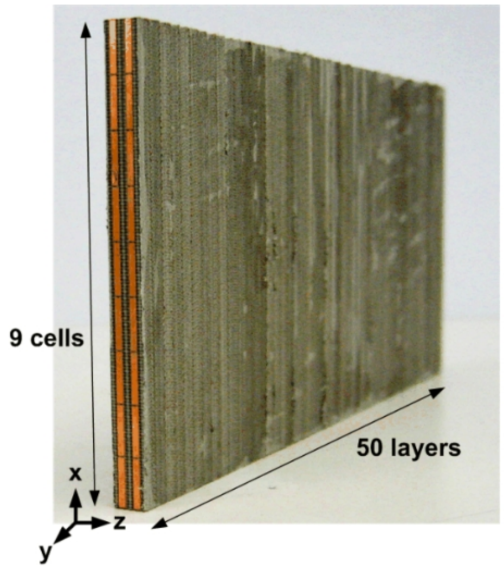


Fig. 6. Fabricated metamaterial lens consisting of 9 cells×18 layers, which measures 90 mm×152.4 mm in the x- and y-directions, respectively.

According to the analysis done in the previous section, we tried to investigate the wave focusing ability of our lens. First, we fabricated the lens with a two-dimensional array of the unit cell shown in Fig. 1, which is given in Fig. 6. The lens is composed of 9 cells and 50 layers in the x- and y-direction, respectively. Overall dimensions of the cell are 90 mm×152.4 mm×5.5 mm.

The experimental setup for determining a width of an image is depicted in Fig. 7. A dipole antenna was used as a signal source, and a coaxial probe was located on the opposite side of the source with a distance of 2.5 mm from the lens. The distance between the source and the lens was also set to 2.5 mm. While the dipole was fixed at the center position (y=0) of the lens, we moved the receiving probe from the end position of the lens to the other end part. We measured the signal that penetrated the lens, using the probe at every 1 mm on the scanning route. During the experiment, each movement step between each measurement position was automatically controlled by a micro-step-motor controller using a developed program, which also saved the measured data into a network analyzer. The measured data were then transformed to have every scanned position on an abscissa and a magnitude of the transmitted signal on an ordinate at each measured frequency point.

The measured results are given in Fig. 8. For ease of comparison, a focusing ability of the suggested lens is compared with that without the lens, which is shown as

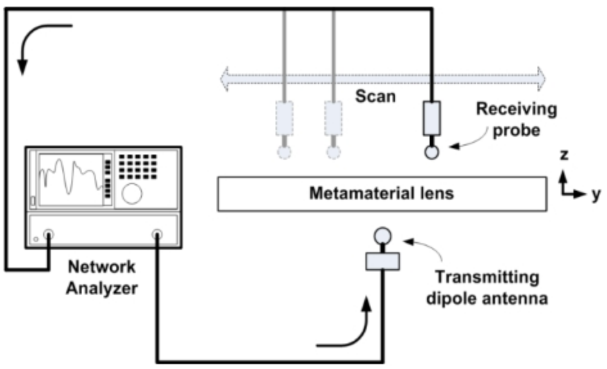


Fig. 7. Measurement setup to obtain a sub-wavelength image of a dipole source.

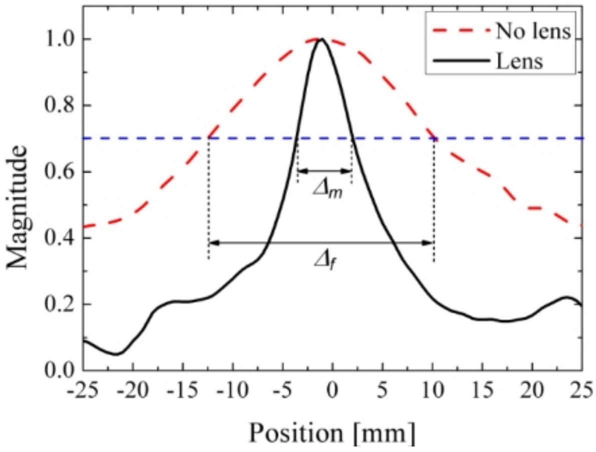


Fig. 8. Measured focused image in a sub-wavelength with Δ_m =9.2 mm, and Δ_f=37.6 mm.

the red broken line. That is to say, the red broken curve in Fig. 8 represents a measured signal magnitude when there is nothing between the source and the probe. For ease of comparison, the two curves are normalized to 1 by dividing them by their peak values, where the peak value with the lens is about 78 % of that without the lens. Here, resolution of the lens is defined as a width at half of a peak amplitude [10]~[12]. We interpolated additional data among the measured data for better assessment of the resolution. The resolutions with and without the lens were measured as Δ_m=9.2 mm and Δ_f =37.6 mm, which correspond to about 0.19 λ and 0.8 λ at 6.38 GHz, respectively. Therefore, we can say that our lens shows about 4 times higher resolution than that in free space. At 6.38 GHz, $n=-1$, $\varepsilon_r=-34.22$, and $\mu_r=-0.03$.

Ⅳ. Conclusion

We have proposed a sub-wavelength focusing lens consisting of a negative refractive index metamaterial. First, we showed that a negative response in permittivity and permeability is simultaneously obtainable in an overlapped frequency region with no help of additional structures, which were unavoidably necessary in conventional metamaterials to get separate negative values in permittivity and permeability. Next, we presented detailed analysis result of the proposed structure including the retrieval of effective constitutive parameters and the computation of eigen modes with sign of an index of refraction.

Based on these analyses, we provided measurement results explaining that a sub-wavelength focusing is possible, which corroborates the validity of our design and experimental approach.

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