An Improvement of Closed-Form Formula for Mutual Impedance Computation

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Abstract

In this paper, we present an improvement of a closed-form formula for mutual impedance computation. Depending on the center-to-center spacing between two rectangular microstrip patch antennas, the mutual impedance formula is separated into two parts. The formula based on synthetic asymptote and variable separation is utilized for spacings of more than 0.5 λ_0. When the spacing is less than 0.5 λ_0, an approximate formula is proposed to improve the computation for closely spaced elements. Simulation results are compared to computational results of mutual impedances and mutual coupling coefficients as functions of normalized center-to-center spacing in both E- and H-plane coupling configurations. A good agreement between simulation and computation is achieved.

Key Words: Array Antenna, Closed-Form Formula, Microstrip Patch, Mutual Coupling, Mutual Impedance.

I. INTRODUCTION

The design of a finite array requires an accurate determination of the mutual impedance between two elements and the mutual impedance matrix of whole array. An accurate approach using the moment method has been proposed for mutual impedance computation [1]. However, the moment method requires that each element be segmented into many basis functions; therefore, this method becomes tedious and time consuming as the number of elements in array increases. Several methods have been proposed that deal with the mutual impedance computation based on simplified models such as the transmission line model [2], and the magnetic current approximation [3]. These methods are much faster but may be inaccurate. Recently, a closed-form mutual impedance formula has been proposed, which is based on synthetic asymptote and variable separation [4−6]. In this

formula, only 12 unknown coefficients are determined by matching with the simulated data or measured data. Therefore, this method is very fast and accurate due to its use of a synthetic asymptote form of the separated variables of the center-to-center spacing and the azimuth angle between two elements. However, when the center-to-center spacing is less than 0.5 λ_0 (λ_0 is the free-space wavelength), the computational result for very closely spaced elements is incorrect if this formula is used.

In this paper, we propose a method to improve the closed-form formula for the mutual impedance computation between two very closely spaced elements. The mutual impedance formula is separated into two parts depending on the center-to-center spacing between two elements. When the spacing is more than 0.5 λ_0, the mutual impedance is computed by utilizing the synthetic asymptote formula [4]. An approximate formula is proposed when the spacing is less
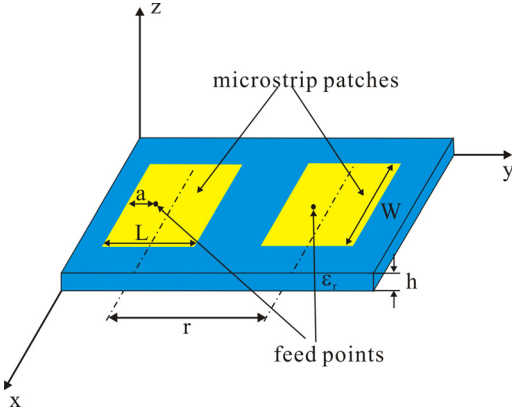

Fig. 1. Geometry of two coupled rectangular microstrip patch antennas.

than 0.5 λ_0. The simulated and computational results agree well in terms of mutual impedances and mutual coupling coefficients between two closely spaced microstrip patches in E-plane and H-plane.

II. FORMULATIONS

Fig. 1 shows the geometry of two coupled microstrip patch antennas. These antennas are designed to operate at 5 GHz on the dielectric substrate with $\varepsilon_r = 2.55$ and thickness $h = 1.57$ mm. The dimension of patch is determined as $W \times L = 22.6$ mm $\times$ 17.52 mm. The feed point is located at the center of W with the distance $a = 5$ mm. In this work, we only focus on the mutual coupling between two patches in E- and H-plane coupling configurations. The mutual impedance formula is separated into two parts corresponding to the spacing r less than 0.5 λ_0 and more than 0.5 λ_0.

1. The Mutual Impedance Formula Based on the Synthetic Asymptote and Variable Separation

When the center-to-center spacing r between two patches is more than 0.5 λ_0, the closed-form mutual impedance formula based on the synthetic asymptote and variable separation is utilized. In this method, the mutual impedance can be written as a function of spacing r and the azimuth angle φ, as shown in Fig. 2(a). The use of a synthetic asymptote form of separated variables of spacing r and angle φ, gives the following as the mutual impedance between the two elements [4]

$$Z_{ab} = \eta_0 \frac{e^{-jk_0 r}}{4\pi} \sum_{n=-1/2,0,1,2} \left\{ \left[1 \middle/ (k_0 r)^{n+1} \right] \right.$$
$$\left. \times \left[C_{n,0} + C_{n,2} \cos(2\varphi) + C_{n,4} \cos(4\varphi) \right] \right\}\Bigg|_{\frac{r}{\lambda_0} \geq 0.5} \quad (1)$$

where η_0 is the intrinsis impedance of free space, k_0 is the free space wave number, and the unknown complex coefficients $C_{n,m}$ ($n = -1/2$, 0, 1, 2 and $m = 0$, 2, 4) must be-
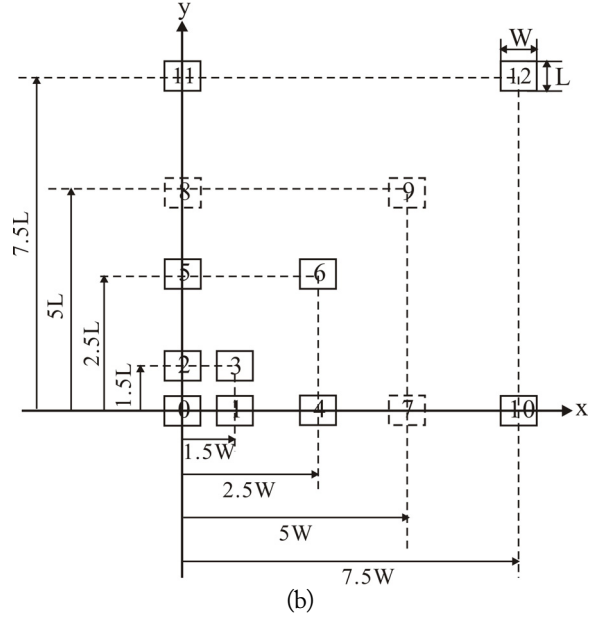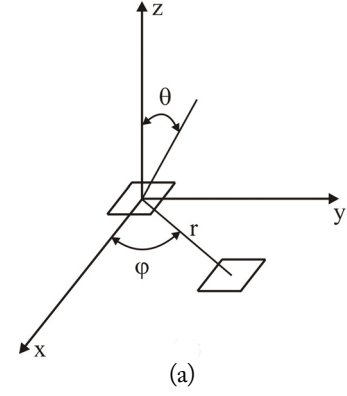

Fig. 2. (a) Coordinates of two coupled patches and (b) skeleton array configuration.

determined. These 12 coefficients can be found by matching with the simulated results of mutual impedance between center patch "0" and 12 coupled patches in a skeleton array, as shown in Fig. 2(b). The 12 models consisting of the center patch "0" and each of 12 coupled patches with the respective spacing set in skeleton array are simulated to obtain the mutual impedances at resonant frequency. From 12 values of the simulated mutual impedances, the Eq. (1) can be used to establish a set of 12 independent equations to be solved for the 12 coefficients $C_{n,m}$ by matrix inversion.

2. The Approximate Mutual Impedance Formula for Closely Spaced Elements

As mentioned above, when the spacing r is less than 0.5 λ_0, the computation of mutual impedance for closely spaced elements obtained using Eq. (1) is incorrect. Therefore, we propose an approximate formula to improve the computation for two very closely spaced elements. It is worth noting that

the mutual impedance between two elements in E- or H-plane coupling configuration only depends on the spacing r. Several sampling values of the spacing r are chosen and the simulated results of mutual impedances are obtained through simulation. By fitting some curves to the simulated data, the mutual impedance formulas for E- and H-plane can be expressed as

$$Z_{ab} = \frac{-1.76r^2 + 99.57r - 1261}{r - 16.13} \\ + j\frac{0.0468r^2 - 25.78r + 635.5}{r - 16.31}\Bigg|_{\frac{r}{\lambda_0} \leq 0.5} \quad (2)$$

for E-plane coupling configuration, and

$$Z_{ab} = \frac{12.17r - 291.4}{r - 21.11} + j\frac{-6.479r + 361.8}{r - 19.65}\Bigg|_{\frac{r}{\lambda_0} \leq 0.5} \quad (3)$$

for H-plane coupling configuration. By combining Eq. (2) or Eq. (3) with Eq. (1), the mutual impedance between two elements in the E- and H-plane, respectively, can be accurately computed even through the center-to-center spacing r is less than 0.5 λ_0.

Ⅲ. Results and Discussion

Fig. 3 plots the simulated reflection coefficients versus frequency of two microstrip patch antennas at the center-to-center spacing of 0.5 λ_0. The simulation was conducted by using Ansys High-Frequency Structure Simulator (HFSS) based on the three-dimensional finite element method. The two patches operate at the same resonant frequency of 5 GHz.

First, the 12 unknown coefficients $C_{n,m}$ in Eq. (1) must be determined by matching with the simulated data. The 12
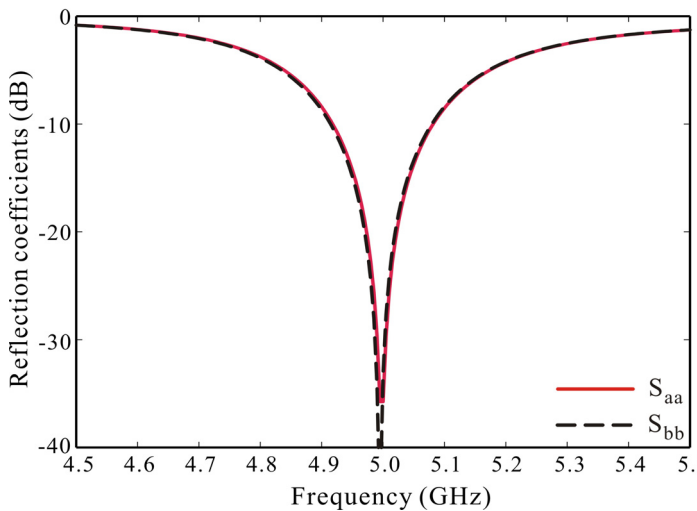

Fig. 3. Simulated reflection coefficients of two microstrip patch antennas.

Table 1. The 12 complex coefficients $C_{n,m}$

$C_{n,m}$	Value	$C_{n,m}$	Value
$C_{-1/2,0}$	$1.9744 - j0.6211$	$C_{1,2}$	$9.5801 - j17.5902$
$C_{0,0}$	$-11.9351 + j4.3854$	$C_{2,2}$	$-24.7769 + j15.5993$
$C_{1,0}$	$67.8895 - j29.3104$	$C_{-1/2,4}$	$-1.5619 + j1.4258$
$C_{2,0}$	$-161.33 + j69.561$	$C_{0,4}$	$9.9945 - j8.5942$
$C_{-1/2,2}$	$-0.0881 - j0.8532$	$C_{1,4}$	$-62.7809 + j50.3547$
$C_{0,2}$	$-0.1876 + j4.3003$	$C_{2,4}$	$147.11 - j112.41$

coupled patches on the skeleton array are arranged with the fixed sampling points as shown in Fig. 2(b). The 12 coefficients $C_{n,m}$ are computed and listed in Table 1.

Once the 12 coefficients $C_{n,m}$ of Eq. (1) have been obtained, the mutual impedance between the two patch antennas is calculated by combining Eq. (1) with Eq. (2) for the E-plane coupling configuration or with Eq. (3) for the H-plane coupling configuration. Figs. 4 and 5 show the mutual impedances versus normalized center-to-center spacing r between two patches in the E- and H-plane from simulation, and from computation using synthetic asymptote formula, and our proposed formula. Clearly, when the spacing r is more than 0.5 λ_0, the computation results agree well with the simulation results. When the spacing r is less than 0.5 λ_0, the results of mutual impedance using the synthetic asymptote formula are very different compared to the simulation results, especially the imaginary part of the mutual impedance. However, the use of our proposed formula to enhance the computation with the closely spaced elements achieves a good agreement between the simulation and computation.

Overlapping is avoided by choosing the minimum values of center-to-center spacing r as 0.3 λ_0 and 0.38 λ_0 for the E- and H-plane couping configurations, respectively. Table 2 shows the comparison between the simulation and computation results of the mutual impedance corresponding to the minimum spacing r. Good agreement between our formula and the simulation is observed.

The mutual coupling S_{ab} between two patches expressed in decibels can be defined as [7]

$$|S_{ab}|^{dB} = 20\log_{10}\left|\frac{2Z_0 Z_{ab}}{(Z_{aa} + Z_0)^2 - Z_{ab}^2}\right| \quad (4)$$

Table 2. Comparison of mutual impedance with the minimum spacing

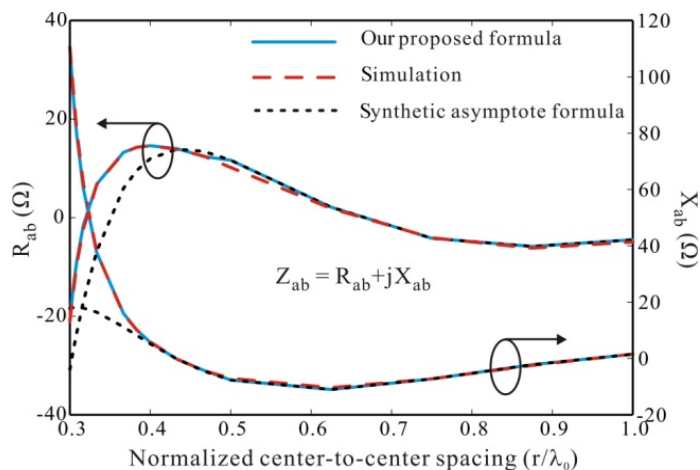|	E-plane	H-plane
Our formula	$-20.85 + j110.43$	$-8.24 + j67.96$
Simulation	$-21.02 + j110.63$	$-10.39 + j74.65$
Synthetic asymptote formula [4]	$-30.82 + j17.80$	$-7.33 + j50.29$

Fig. 4. Mutual impedance versus normalized center-to-center spacing between two patches in E-plane coupled configuration.
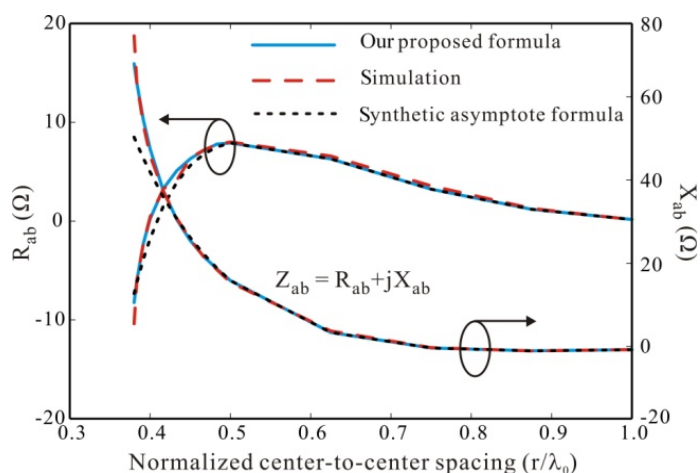

Fig. 5. Mutual impedance versus normalized center-to-center spacing between two patches in H-plane coupled configuration.
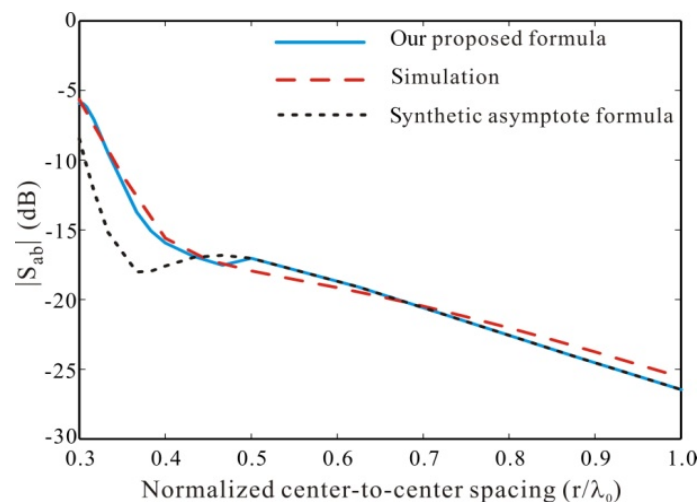

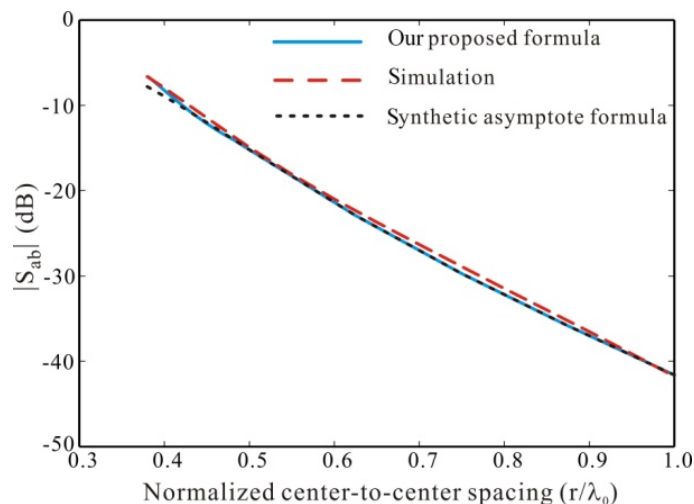Fig. 6. Comparison of mutual coupling between two patches in E-plane coupled configuration.

Fig. 7. Comparison of mutual coupling between two patches in H-plane coupled configuration.

where Z_{aa} is the self-impedance of the patch, Z_{ab} is the mutual impedance between the two patches, and Z_0 is the feed line impedance. We typically assume $Z_{aa} = Z_0 = 50$. Figs. 6 and 7 show the results of the mutual coupling versus normalized spacing r in the E-plane and H-plane coupled configurations, respectively. Clearly, the computation results using our formula and asymptote formula agree well with the simulation results, except for the greater difference in the E-plane coupling configuration from the asymptote formula when the spacing r is less than the half-wavelength.

IV. CONCLUSION

An improvement in the closed-form formula for mutual impedance computation between two very closely spaced microstrip antennas in E- and H-plane has been presented. A good agreement was achieved between the simulation and computation. The computational results show that the proposed approach is feasible for application to the design of linear microstrip patch arrays with arbitrary element spacing.

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