Journal of Institute of Control, Robotics and Systems (2014) 20(8):849-853
http://dx.doi.org/10.5302/J.ICROS.2014.14.0046 ISSN:1976-5622 eISSN:2233-4335

초저전력 마이크로 서보시스템의 모델식별을 위한 계측 파라미터 선정 기법

Sensing Parameter Selection Strategy for Ultra-low-power Micro-servosystem Identification

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Abstract: In micro-scale electromechanical systems, the power to perform accurate position sensing often greatly exceeds the power needed to generate motion. This paper explores the implications of sampling rate and amplifier noise density selection on the performance of a system identification algorithm using a capacitive sensing circuit. Specific performance objectives are to minimize or limit convergence rate and power consumption to identify the dynamics of a rotary micro-stage. A rearrangement of the conventional recursive least-squares identification algorithm is performed to make operating cost an explicit function of sensor design parameters. It is observed that there is a strong dependence of convergence rate and error on the sampling rate, while energy dependence is driven by error that may be tolerated in the final identified parameters.

Keywords: recursive least square, sample rate, identification, ultra-low-power systems

I. INTRODUCTION

For micro-scale actuation systems operating with feedback, power consumption of sensors and sensing circuitry can rival or exceed that of the miniature actuators that they are used to control. These phenomena can be observed in the comparison of actuator and sensor power loads for examples of piezoelectric [1] or electrostatic [2] actuation. Nonetheless, despite the relatively high cost of sensing, feedback in some form is often necessary because of extremely precise movements may be desired, and because micro-systems themselves may contain significant variation from nominal models due to atmospheric effects, large microfabrication variations, or modeling error.

An alternative approach to control of such systems is to operate primarily in open-loop using a model that is precisely identified over a relatively short duration of sensor use, and only return to active use of feedback for critical situations. However, the energy consumed while identifying system parameters may depend significantly on how the sensor is designed and utilized. Two primary design parameters are fixed measurement sampling rate, for which a higher rate leads to higher power consumption, and sensor noise, for which lower variance requires higher power consumption. Previous studies of sampling rate selection for system identification have indicated that optimal sampling rates exist that minimize the total time required to identify a given set of model parameters [3-8] without power constraint.

This paper introduces an approach to evaluating those two parameters so as to minimize the expected energy needed to perform system identification of a linear system to a desired error tolerance using a recursive least-squares algorithm. The specific motivated problem is high-precision positioning of an electrostatic rotary micro-stage. For this application, feedforward control of rotation to a desired position requires significantly less power than active feedback control, provided that the system's model parameters are accurately known.

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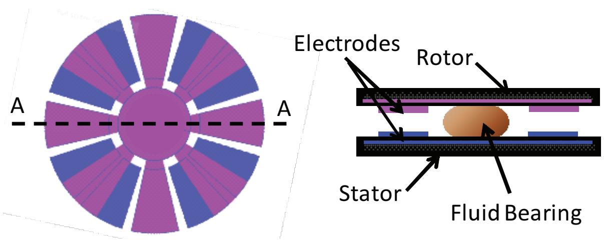
II. SYSTEM DESCRIPTION

The system to be considered is assumed to be a linear system subject only to measurement noise. In the motivating example as shown schematically in Fig. 1, this reflects a cylindrical rotor spinning on a fluid bearing, with constant inertia, damping, and input gain parameters. As a general form, it is treated as a continuous LTI system with the Gaussian white noise such that

$$\begin{aligned} \dot{\mathbf{x}}_c(t) &= \mathbf{A}\mathbf{x}_c(t) + \mathbf{B}u(t) \\ y_{c0}(t) &= \mathbf{C}x_c(t) \\ y_c(t) &= y_{c0}(t) + w_c(t) \\ w_c(t) &\sim \aleph(0, v_c) \end{aligned} \tag{1}$$

where the state vector, an input, an output, and the noise are $\mathbf{x}_c \in \mathbf{R}^m$, $u \in \mathbf{R}$, $y_c \in \mathbf{R}$, and $w_c \in R$, respectively.



(a) Top view. (b) A-A side view.

그림 1. 적용예: 마이크로 회전 구동기.
Fig. 1. The Motivating Example of a Rotary Actuator.

표 1. 노이즈 및 전력 모델 (3)과 (4)에 사용된 계수.
Table 1. Coefficient of noise and power models of (3) and (4).

Coefficient	Specific Values
v_1 [V^2_{ms}/Hz]	2.21×10^{-10}
v_2	3.16×10^{3}
p_1 [V]	90
p_2 [W/Hz]	4.2×10^{-4}
p_3 [W]	0.104

Alternatively, the corresponding input/output difference forms of (1) with a Gaussian white noise w and a sample time T_s are

$$\begin{aligned} &y(k+1)=\Phi^T(k)\theta(T_s)\\ &\theta^T(T_s)=[\alpha_1(T_s)\ \cdots\ \alpha_n(T_s)\ \ \beta_0(T_s)\ \cdots\ \beta_n(T_s)]\\ &\Phi^T(k)=[-y(k)\ \cdots\ -y(k-n+1)\ \ u(k)\ \cdots\ u(k-n)]\\ &y(k)=y_0(k)+w(k)\\ &w(k)\sim\aleph(0,v_n) \end{aligned} \tag{2}$$

where $\boldsymbol{\theta}$ is a vector of system parameters and $\boldsymbol{\Phi}$ is a vector of sensing measurements for input and output.

A common type of sensing technique for miniature or micro-scale systems is capacitive sensing with a differential capacitive sensing circuit. Under many circumstances, a noise model of v_n from (2) and power consumption, P_{tot}, can be approximately related to the selected sample time, T_s, and the voltage noise spectral density, e_n, and the supply current, I_s, of an operational-amplifier used for filtering and signal amplification as follows:

$$v_n^2=v_1\frac{1}{T_s}+v_2\frac{[e_n(I_s)]^2}{T_s} \tag{3}$$

$$P_{tot}=p_1I_s+p_2\frac{1}{T_s}+p_3 \tag{4}$$

Here, v_1 and v_2 are constants of noise generation and p_1, p_2, and p_3 are constants of power consumption dictated by resistors, capacitors, and other circuit components. The coefficients in (3) and (4) are shown in Table 1. For common commercially available op-amps, e_n is roughly inversely proportional to the supply current of the op-amp, I_s, as shown in [7] and [9]. Based on the data in [9], the estimated e_n with respect to I_s is roughly

$$e_n(I_s)=7.67\times10^{27}I^{15}_s-0.43\times10^{27}I_s^{14}+0.01\times10^{27}I_s^{13} \tag{5}$$

III. PARAMETER IDENTIFICATION ALGORITHM

Parameter adaptation of (2) is assumed to be performed using a standard recursive least squares (RLS) with forgetting factor.

$$\begin{aligned} &\hat{\boldsymbol{\theta}}(k+1)=\hat{\boldsymbol{\theta}}(k)+\mathbf{P}(k)\boldsymbol{\Phi}(k)[y(k+1)-\hat{y}(k+1)]\\ &\hat{y}(k+1)=\boldsymbol{\Phi}^T(k)\hat{\boldsymbol{\theta}}(k)\\ &\mathbf{P}(k+1)=\left[\mathbf{P}(k)-\frac{\mathbf{P}(k)\boldsymbol{\Phi}(k)\boldsymbol{\Phi}^T(k)\mathbf{P}(k)}{\lambda+\boldsymbol{\Phi}^T(k)\mathbf{P}(k)\boldsymbol{\Phi}(k)}\right]\frac{1}{\lambda} \end{aligned} \tag{6}$$

where $\hat{\boldsymbol{\theta}}$ is an estimation vector of unknown $\boldsymbol{\theta}$ from (2), $\hat{y}$ is the estimation of output, y, $\mathbf{P}$ is a adaptation gain, $\boldsymbol{\Phi}$ is a vector of sensing measurements for input and output of (2), and λ is forgetting factor.

Because (2)-(5) are functions of sampling rate, the parameter estimation of (2) by (6) also depends on sensing sample rate and OP-Amp's supply current, such that there may exist a trade-off between the accuracy of parameter estimation and the energy usage of a system.

IV. SENSING PARAMETER OPTIMIZATION

1. Modification of RLS to find optimal sample rate and noise properties

To perform a rapid evaluation of the effects of sampling rate and noise amplitude on convergence time and identification error, the standard RLS equation (6) is converted to a cumulative function of measurements up to an arbitrary number of samples when $\hat{\boldsymbol{\theta}}(1)=0$, as follows:

$$\hat{\boldsymbol{\theta}}(K)=\sum_{k=1+m}^{K}\boldsymbol{\Lambda}(k)+\sum_{k=1+m}^{K-1}\left\{\left[\mathop{\mathrm{X}}_{i=k}^{K-1}\boldsymbol{\Omega}(i+1)\right]\boldsymbol{\Lambda}(k)\right\} \tag{7}$$

Here, we define $\boldsymbol{\Lambda}(k)$=$\mathbf{P}(k)\boldsymbol{\Phi}(k$-1)[$y_0(k)$+$w(k)$], $\boldsymbol{\Omega}(k)$=-$\mathbf{P}(k)$ $\boldsymbol{\Phi}(k$-1)$\boldsymbol{\Phi}^T(k$-1), and the operator, $\mathrm{X}_{i=k}^{K}$, such that for instance a function $z(k)$ and i = 1,..,3,

$$\begin{aligned} \mathop{\mathrm{X}}_{i=1}^{3}z(i)=&\ z(3)+z(2)+z(1)+z(3)\big(z(2)\\ &+z(1)+z(2)z(1)\big)+z(2)z(1) \end{aligned} \tag{8}$$

Let $k=(n\text{-}1)T_s$, a given number of samples is N, and a converged parameter set at a given final time is $\boldsymbol{\theta}_f$, then, we can obtain the modified RLC function which explicitly depends on T_s and noise properties, ω, as follows in (9)

$$\boldsymbol{\theta}_f=\sum_{n=1+m}^{N}\boldsymbol{\Lambda}((n-1)T_s)+\sum_{n=1+m}^{N-1}\left\{\left[\mathop{\mathrm{X}}_{i=n}^{N-1}\boldsymbol{\Omega}(iT_s)\right]\boldsymbol{\Lambda}((n-1)T_s)\right\} \tag{9}$$

where

$$\mathbf{P}(nT_s)=\frac{1}{\lambda}\left[\mathbf{P}((n-1)T_s)-\frac{\mathbf{P}((n-1)T_s)\boldsymbol{\Phi}((n-1)T_s)\boldsymbol{\Phi}^T((n-1)T_s)\mathbf{P}((n-1)T_s)}{\lambda+\boldsymbol{\Phi}^T((n-1)T_s)\mathbf{P}((n-1)T_s)\boldsymbol{\Phi}((n-1)T_s)}\right]$$

By performing this conversion, the parameter estimate becomes a direct function of the sampling rate through the dependence of a final desired $\boldsymbol{\theta}_f$ on T_s under a given number of samples, which may then be optimized using standard numerical solvers.

2. Problem statement and optimization procedure

The next objective is to search for an optimal sample rate and selection of amplifier noise density using (9) based on balancing maximum available energy usage of the system against estimation performance. Therefore, assuming that there exist a desired solution, $\boldsymbol{\theta}^*$, for the unknown parameter vector over a given number of time step, N, the search for reasonable sample rate with a given choice of noise density can be presented by

$$\min_{T_s,e_n}E\left[\left\|\boldsymbol{\theta}_f-\boldsymbol{\theta}^*\right\|\right] \tag{10a}$$

subject to:

$$\begin{aligned} &1.\ \int_0^{(N-1)T_s}p_{tot}(t)dt<E_{\max}\\ &2.\ y_0\left(nT_s\right)=\mathbf{C}e^{\mathbf{A}(nT_s-t_0)}\mathbf{x}_c(t_0)+\mathbf{C}\int_{t_0}^{nT_s}e^{\mathbf{A}(t_0-\tau)}\mathbf{B}u(\tau)d\tau \end{aligned} \tag{10b}$$

where E_{max} is a maximum available energy for the system.

Alternatively, if there exists a desired performance of parameter estimation, ε_n, and a given scalar value, ε_1, such that $\varepsilon_1 \le \varepsilon_n$, then (10) can be modified by

$$\min_{T_s, e_n} \int_0^{(N-1)T_s} p_{tot}(t)dt \tag{11a}$$

subject to

$$E\left[\left\|\boldsymbol{\theta}(N) - \boldsymbol{\theta}^*\right\|\right] \le \varepsilon_1 \tag{11b}$$

Having made the conversion of the standard RLS problem to (9), the (10) or (11) becomes simply a numerical search problem to obtain the solution of a nonlinear equation, which can be solved by using common numerical techniques.

A sample optimization, in full, is carried out as follows:

(a) Select desired values of ε_1 and E_{max}.
(b) Assume a value of I_s and obtain a value of e_n using (5)
(c) Select an initial T_s, and N
(d) Solve (10a) by searching for the optimal T_s and N under the constraint (10b)-2 with a numerical solver.
(e) If no solution is found, return to (c). Or, if multiple local minima are found, evaluate (10b)-1 from the several solutions of T_s
(f) If the condition of (10b)-1 is satisfied and the smallest of candidate solutions, the solution of T_s is a minimum and verified to meet constraints. Otherwise return to (b).

To check results, the obtained T_s and N are utilized in the standard RLS (6) to verify estimation performance with the identified set of sensing parameters.

V. CASE STUDY

1. System description

Here, an example of sensing parameter selection based on the modified RLC function (9) and the optimizing problem (10) is presented for the MEMS rotary actuator introduced earlier. The corresponding input/output difference model of the system is

$$\begin{aligned}
&Y(k) = \boldsymbol{\Phi}^T(k-1)\boldsymbol{\theta}(T_s)\\
&\boldsymbol{\theta}^T(T_s) = [a_1 T_s \quad b_0 T_s^2]\\
&\boldsymbol{\Phi}^T(k-1) = [-y(k-1) + y(k-2) \quad y(k-2)]\\
&Y(k) = y(k) - 2y(k-1) + y(k-2)\\
&y(k) = y_0(k) + w(k)\\
&w(k) \sim \aleph(0, v_n)
\end{aligned} \tag{12}$$

where $\boldsymbol{\theta}^*$ = $[a_1\ b_0]$ = $[4896\ 2572]$ is the nominal continuous model parameters to be identified, the perturbing input is $u(t)$ = $\alpha\sin(2\pi f_d t)$, α = 50, f_d = 33.3 Hz, and the forgetting factor from (9) is λ = 0.9.

2. Selection of proper noise amplitude of an OP-Amp and initial values to perform (10)

For the optimization procedure described in the previous section, first we selected target ε_1 and E_{max} as 0.5 and 50 J respectively. The maximum power consumption, P_{max}, to satisfy (10b-1) is about 200 mW. For initial op-amp selection, 1 mA for I_s was selected based on (4), since the power consumption of the corresponding sensing circuit is 190 mW with assumptions of p_1 = 90, p_3 = 100 mW. Using the initial I_s, the starting e_n becomes 8 nV/rHz based on (5).

3. Performing (10) and obtaining an optimal sample time and the number of sample

Through careful selection of sampling rates, as shown in the Fig. 2, error in estimates of system operating parameters can be minimized. This allows a reduced number of samples to be taken to satisfy the constraint (10b-1). According to Fig. 2 and Table 2, the optimized sample time and the number of samples for the test system are 4 msec and 100, respectively. Fig. 3 shows the associated trends in energy and power consumption versus sampling rate for fixed numbers of samples.

Additionally, a trend between energy usage and estimation error are tested with respect to the several noise spectral densities of OP-Amps for the selected parameter, T_s = 4 msec and N = 100. As shown in Fig. 4, if very high amplitude noise were used the energy consumption could be dramatically reduced, while the estimation error could be increased extremely. Therefore, proper selection of noise amplitude of an OP-Amp is necessary.

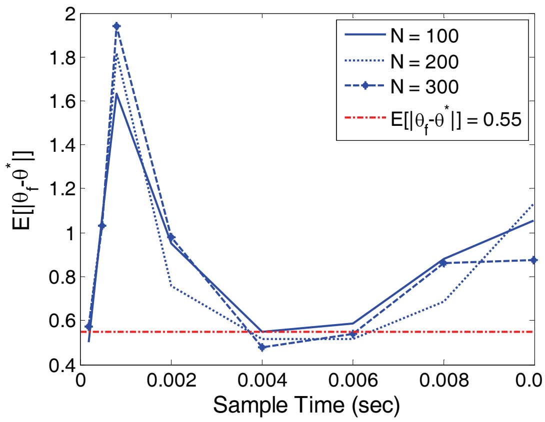


그림 2. 계측 센서의 사용 전력 최소화 조건을 위한 한정된 샘플 개수(N) 하에서, 시스템 파라미터($\boldsymbol{\theta}$)의 고정밀 모델 식별을 위한 샘플 시간 선정.

Fig. 2. Proper selection of sampling times for highly accurate identification of system parameter ($\boldsymbol{\theta}$) in a finite number of samples (N), under minimizing duration of the relatively high position sensor power load.

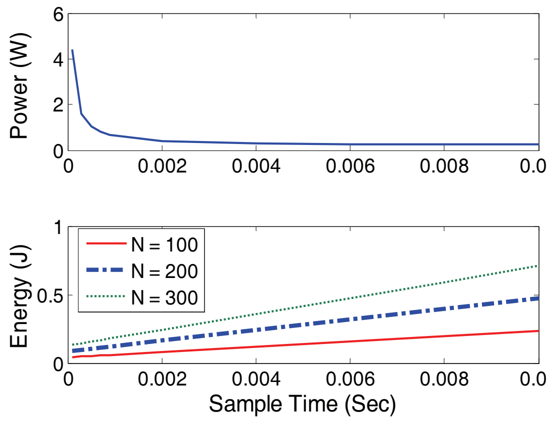


그림 3. 계측 샘플 시간과 샘플 개수 변화에 따른 전체 에너지 및 전력량 비교.

Fig. 3. Total Energy and Power Consumption vs. Sample Times with respect to the Number of Samples.

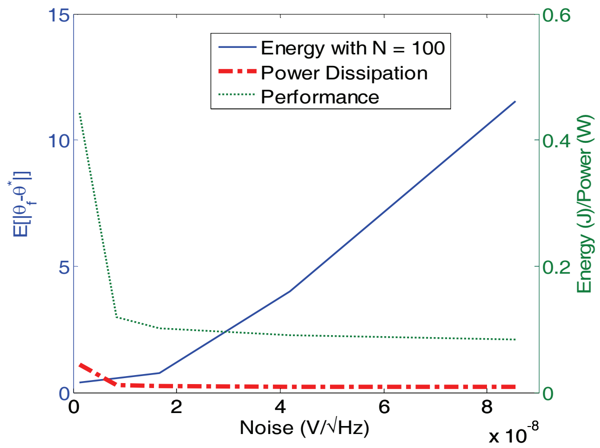


그림 4. 앰프의 노이즈 특성에 따른 에너지 및 전력 사용과 모델 식별 오차의 증감 경향.

Fig. 4. Energy and Power Usage Vs. Estimation Error Trend for Noise Spectral Densities of OP-Amps.

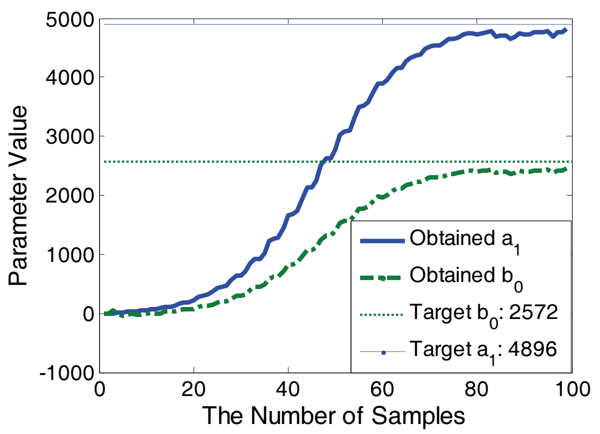


그림 5. $T_s = 4$ msec 이고 $N = 100$ 인 경우, 일반적인 RLS를 이용하여 식별된 시스템 파라미터 결과

Fig. 5. Estimation of System Parameters Using the Standard RLS, $T_s = 4$ msec, and $N = 100$.

4. Verifying the obtained sensing parameter with the standard RLS

The obtained sensing parameters are utilized on the standard RLS (6) to verify estimation performance of system parameter. As shown in Fig. 5, the estimated parameters are converged at 100 numbers of samples and norm of estimation error is similar to the obtained results on Table 2.

표 2. 샘플 개수와 속도에 따른 식별 성능과 에너지 비교.

Table 2. Obtained performance and Energy for the number of sample and sample rates.

Error	T_s (msec)	N	Power (W)	Energy (mJ)
0.55	0.22	100	2.1	46.3
	4	100	0.29	119
	3.7	200	0.3	227
	6.4	200	0.26	332
	3.7	300	0.3	341
	6	300	0.26	475

VI. CONCLUSION

The work in this paper sought to explore whether optimal rates may also exist. Unlike other papers on model identification, sampling rate and noise amplitude were weighted by sensor power consumption, for certain sensing situations. It was hypothesized that there would exist optimal selections of sampling rate and amplifier noise density that would minimize total energy to meet a desired maximum identification error, or to minimize error for a given energy usage. It was found that for the test case there was an optimal sampling rate to minimize error in a limited number of time steps, but that energy trade offs were primarily associated with selection of amplifier power consumption. To evaluate these trends, a rearrangement of the standard recursive-least squares algorithms was made that could be directly optimized by numerical optimization solvers, and was found to remain an accurate predictor of true RLS performance in simulation.

The current work is clearly limited in studying only a single, relatively simple candidate system, but the methods used are applicable to more general linear systems. More general results for linear system identification when costs are constrained or weighted by power consumption of sensing circuitry would be desirable. The existing conclusions drawn would be strengthened by future quantification of computational gains over simulation studies of RLS, and study of the effects of forgetting factor selection. Nonetheless, the approach provided in this paper at least provides a method for optimizing elements of capacitive sensing systems for micro-electromechanical systems when precise knowledge of system dynamics under power or energy constraints is desired.

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