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Synthesis of Bulk Medium with Negative Permeability Using Ring Resonators

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Abstract

This paper presents simple expressions for the effective permeability of bulk metamaterial consisting of ring resonators (RRs) or split ring resonators (SRRs) based on the convenient geometrical factors of the structure compared with wavelength. The resonant frequency dependence of the medium permeability, including loss effects, is analyzed in detail. Inverting the analysis equations, useful design (or synthesis) equations are derived for a systematic design process with some examples. This paper may particularly be useful for the design of a bulk metamaterial with a specific negative relative permeability at a desired frequency. The loss of metamaterials consisting of RRs (or SRRs) is also analyzed over a wide frequency band from 10 MHz to 10 THz.

Key Words: Design Equation, Effective Medium, Metamaterial, Negative Permeability, Ring Resonator.

I. INTRODUCTION

Since Pendry et al. [1] introduced the split ring resonator (SRR) to construct an effective medium with negative permeability, a variety of research activities has been conducted. In [2], the electromagnetic fields generated from a source can be focused to a point using a fat lens characterized by an effective $\mu_r = -1$ (realized by SRR) and $\varepsilon_r = -1$ (realized by thin wires [3]) medium. In the magnetostatic and electrostatic limits, the $\mu_r = -1$ slab and the $\varepsilon_r = -1$ slab can do the same, respectively. Specifically, quasi-static longitudinal magnetic fields from a magnetic point source can be focused by a $\mu_r = -1$ slab, and quasi-static longitudinal electric fields from an electric point source can be focused by an $\varepsilon_r = -1$ slab. The artificial periodic structure consisting of SRRs and thin wires was experimentally verified to have the property of negative refraction among others [4]. Isotropic bulk metamaterials were introduced [5] and developed [6]. A review paper [7] on bulk metamaterials made of resonant rings was also published. Although many applications have been made on transmission line-based metamaterials [8–10], bulk metamaterials have been only applied in cloaking [11], magnetic resonance imaging (MRI) [12–14], and wireless power transfer [15]. The usefulness of the metamaterial slab was demonstrated experimentally by comparing the MRI images with and without it [13]. Despite the many promising features of bulk metamaterials, their narrow band and relatively high loss inherent in SRRs prevent their rapid progress. As presented in [2], although considerable focusing is achieved, the imaginary part of the dielectric function prevents an ideal reconstruction.

In this work, we investigate the degree of these limitations in a systematic and quantitative manner. In particular, the capacitively loaded ring resonators (RRs) initially introduced by Schelkunoff and Friis [16] are analyzed in terms of their bandwidth and loss. First, the effective permeability is expressed as a func-

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tion of structural factors of the resonator relative to the used wavelength. Using this expression, the characteristics of the medium are examined over a wide frequency range, especially in terms of the newly defined mu-negative bandwidth and losses on RRs. Simple and convenient design equations are then derived by inverting the analysis equations. Some examples are also provided to demonstrate the effectiveness of the formulation.

II. ANALYSIS OF AN EFFECTIVE MEDIUM WITH RANGE OF RELATIVE PERMEABILITY

A natural magnetic material such as ferrite has long been modeled as a bulk structure consisting of many magnetic dipole moments (*ms*) [17], which are defined as a product of a spin current and a spin area on an atomic or a molecular scale. Magnetization (*M*) is then defined as their vector sum divided by a volume containing them. The effective medium concept, which replaces the particle behaviors on a microscopic scale with a simple effective permeability on a macroscopic scale, is a useful and reliable model when the wavelength is much larger than the particle dimensions. This conventional and familiar procedure can be extended to the problem of artificial effective medium consisting of magnetic particle-like RRs.

Fig. 1 shows the unit of the artificial effective medium composed of many RRs. The RR is chosen instead of the SRR for its simple modeling. Similar effects are expected for SRRs. The details of the RR and its orientations with respect to an incident TEM wave are depicted in Fig. 1. The wave travels in the *z* direction with the electric and magnetic fields oriented in the *x* and *y* directions, respectively. In this work, only the coupling of the magnetic field H_y leading to a relative effective permeability is considered. The side length of the unit cell is *a*. The radius of the RR is *r*, and *t* is the width of the planar-type RR and the diameter of the ring-type RR. The chip capacitor is represented by *C*. It is inserted for the resonance of the RR, which has an inductance (*L*) roughly proportional to *r*. The total resistance (*R*) of the RR is the sum of the ohmic resistance and the radiation resistance. The radiation resistance is negligible when *r* is much smaller than the wavelength. The modeling of RRs was already developed in [6, 17], but it is briefly outlined here. Let $H_y = H_0$ be the incident (or applied) magnetic field. Then, the induced phasor voltage to let the current (*I*) flow in the reference direction (Fig. 1) is given by

$$V = j\omega\mu_0 H_0 \pi r^2, \qquad (1)$$

where μ_0 is the free space permeability, and ω is the angular frequency of the incident fields. The induced current (*I*) in Fig. 1 is determined by

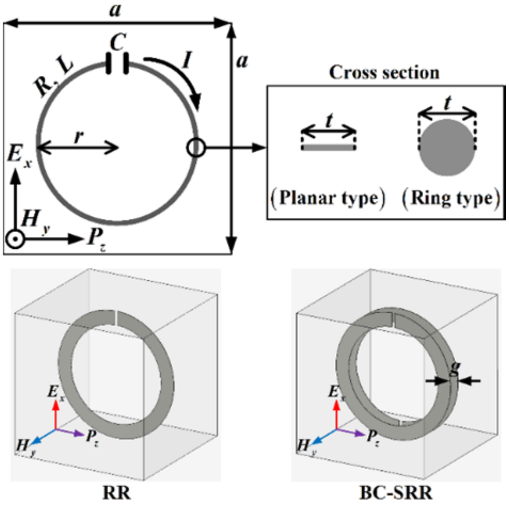


Fig. 1. Unit of resonant ring, its typical cross-section types, and various structures of resonant rings. RR = ring resonator, BC-SRR = broadside-coupled SRR.

$$I(\omega) = \frac{j\omega\mu_0 H_0 \pi r^2}{R + j\sqrt{\frac{L}{C}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}, \qquad (2)$$

where ω_0 is the resonant angular frequency given by $1/\sqrt{LC}$, and $\sqrt{L/C}$ is the reactance slope parameter related to the bandwidth of the induced magnetism. The definition of magnetization (*M*) [18], which has long been used for natural magnetic materials, can still be applicable to artificial bulk metamaterials made of RRs when the side length of the unit cell (*a*) is much smaller than the wavelength. It is expressed as

$$\overline{M} = -\overline{a}_y \frac{j\omega\mu_0 H_0 \left(\pi r^2\right)^2}{a^3\left[R + j\sqrt{\frac{L}{C}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]}. \qquad (3)$$

When $R = 0$, the magnetization (3) due to the current (2) is in the same direction (paramagnetism) as that of the incident magnetic field when $\omega < \omega_0$, and the opposite (diamagnetism) is true when $\omega > \omega_0$. A well-established constitutive relation is given by

$$\overline{B} = \mu_0\left(\overline{H} + \overline{M}\right) = \mu_0\left(1 + \chi_m\right)\overline{H} = \mu_0\mu_r\overline{H}, \qquad (4)$$

where ***B*** is the magnetic flux density and χ_m is the magnetic susceptibility. The relative effective permeability (μ_r) is obtained as

$$\mu_r(\omega) = 1 - \frac{j\omega\mu_0\left(\pi r^2\right)^2}{a^3\left[R + j\omega_0 L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]}, \qquad (5)$$

where $\omega_0 L$ is also the reactance slope parameter.

Fig. 2 shows the real and imaginary parts of the relative effective permeability for the RRs presented in Fig. 1 obtained using (5) and an extraction method [19] based on EM-simulated *S*-parameters. Two types (planar type and ring type shown in Fig. 1) of typical RRs with $a = 12$ cm, $r = 3.5$ cm, and $t = 1$ cm are considered. They have been designed to have $\mu' = -1$ at 13.56 MHz by a trial and error method. The EM-based extracted [19] effective permeability is shown to be in excellent agreement with (5), thus validating the derived expression (5). At the same frequency of 13.56 MHz, $\mu'' = 0.2$ for the planar type and $\mu'' = 0.04$ for the ring type.

Here, we intend to analyze the bulk metamaterials made of RRs in terms of the effective medium in a more systematic manner. The ring-type RR is considered first. When $a < 0.1\lambda_0$, the radiation resistance is much smaller than the ohmic resistance, and R can be approximated as [18].

$$R = R_s \frac{2\pi r}{\pi t} = \sqrt{\frac{\pi\mu_0 f}{\sigma}} \cdot \frac{2r}{t}, \tag{6}$$

where σ is the conductivity, R_s is the surface resistance per square, and $2r/t$ is the aspect ratio of the ring-type RR (Fig. 1). The only difference in the planar-type RR is that its aspect ratio is π times larger than that of the ring-type RR. The inductance of the ring-type RR is expresses as [18].

$$L = \mu_0 r\left(\ln\frac{16r}{t} - 1.75\right). \tag{7}$$

The implication here is that if t becomes relatively large for a fixed r, L and the slope parameter $\omega_0 L$ in (5) decrease and result in a larger bandwidth. To analyze (5) in detail, define

$$m_1 = \frac{a}{\lambda_0}, \tag{8}$$

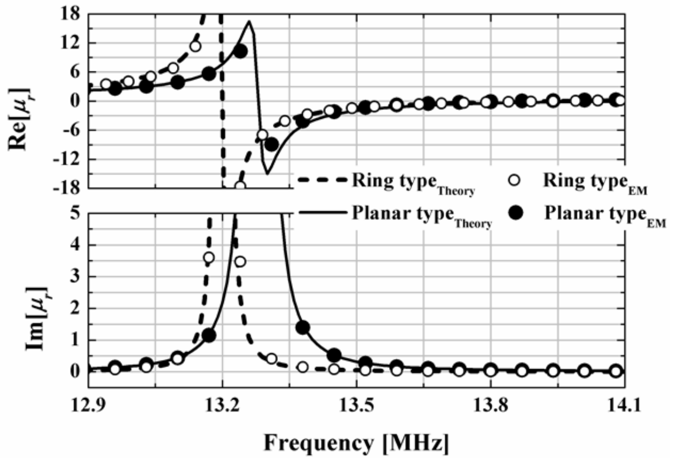


Fig. 2. Relative effective permeability of the ring resonator in Fig. 1 ($a = 12$ cm, $r = 3.5$ cm, and $t = 1$ cm).

where λ_0 is the free space wavelength at the resonant frequency (f_0),

$$m_2 = \frac{r}{a}, \tag{9}$$

which must be reasonably greater than 0 to have the effect of diamagnetism and less than 0.5 to prevent a contact with an adjacent RR unit, and

$$m_3 = \frac{t}{r}, \tag{10}$$

where t must be smaller than $2(a/2 - r)$ but is not recommended to be very small because it increases losses of the medium. The m_i's may be called structural reduction factors, and $m_1 m_2 m_3 = t/\lambda_0$. m_1 must be very small, i.e., less than 0.1, to apply an effective medium concept.

Using (6)–(10), (5) may be, as a function of a normalized frequency f/f_0, expressed as

$$\mu_r\left(\frac{f}{f_0}\right) = 1 - \frac{j\pi^2 m_2^3 \frac{f}{f_0}}{\sqrt{\frac{\mu_0 f_0}{\pi\sigma}\left(\frac{f}{f_0}\right)}\frac{1}{\eta_0 m_1 m_2 m_3} + j\ln\frac{2.78}{m_3}\cdot\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}, \tag{11}$$

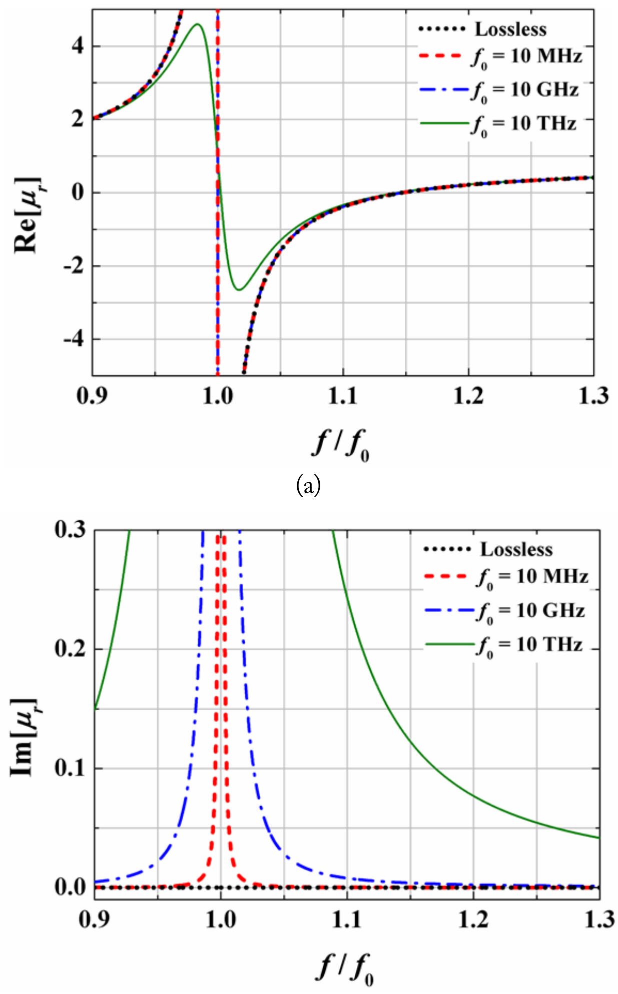
where η_0 is the intrinsic impedance in free space given by $\sqrt{\mu_0/\varepsilon_0}$ (= 377 Ω). The first term in the denominator may be understood as a loss-perturbing factor in a lossless effective medium. As an initial observation, for a medium with lossless RRs ($\sigma \to \infty$), the relative effective permeability μ_r is reduced to

$$\mu_r\left(\frac{f}{f_0}\right) = 1 - \frac{\pi^2 m_2^3 \frac{f}{f_0}}{\ln\frac{2.78}{m_3}\cdot\left(\frac{f}{f_0} - \frac{f_0}{f}\right)} = 1 - \frac{\pi^2 m_2^3}{\ln\frac{2.78}{m_3}\cdot\left[1 - \left(\frac{f_0}{f}\right)^2\right]}, \tag{12}$$

which is purely real and independent of m_1 ($= a/\lambda_0$). Moreover, the functional behavior of (12) is the same irrespective of f_0 once m_2 ($= r/a$) and m_3 ($= t/r$) are chosen.

In Fig. 3, we plot the real (a) and imaginary (b) parts of (11) and (12) as a function of the normalized frequency f/f_0 for the cases of different resonant frequencies $f_0 = 10$ MHz, 10 GHz, and 10 THz, assuming that $m_1 = 0.1$, $m_2 = 0.4$, and $m_3 = 0.2$. Note that $\text{Re}[\mu_r] = -1$ when f/f_0 is roughly 1.06 irrespective of the resonant frequency (f_0). However, the imaginary parts (b) are shown to become large as the frequency increases as the loss-perturbing factor increases depending on $\sqrt{f_0}$.

In Fig. 4, we plot the loss tangent $|\mu''/\mu'|$ of bulk metamaterials made of RRs based on the same assumption as in Fig. 3,



(a)

(b)

Fig. 3. Relative effective permeability of the bulk ring resonators in Fig. 1 ($m_1 = 0.1$, $m_2 = 0.4$, and $m_3 = 0.2$). Lossless RR and lossy RRs are made of PEC ($\sigma \rightarrow \infty$) and copper ($\sigma = 5.8 \times 10^7$ S/m). (a) Real part and (b) imaginary part.

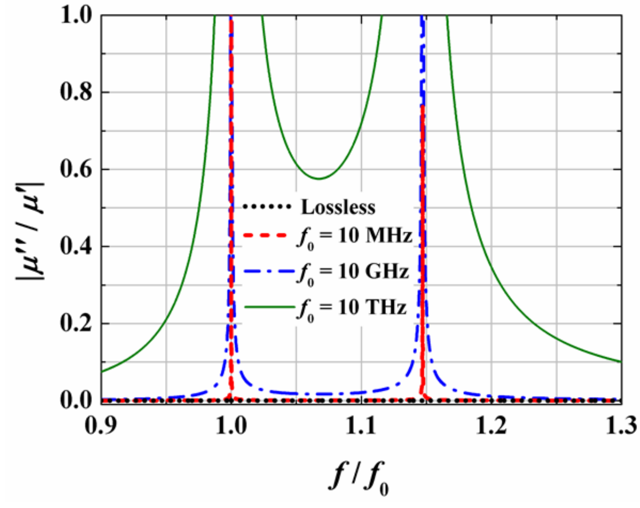


Fig. 4. Loss tangent $|\mu''/\mu'|$ of bulk ring resonators ($m_1 = 0.1$, $m_2 = 0.4$, and $m_3 = 0.2$) as a function of f/f_0. Lossless RR and lossy RRs are made of PEC ($\sigma \rightarrow \infty$) and copper ($\sigma = 5.8 \times 10^7$ S/m).

where μ' is the real part of the relative effective permeability and μ'' is its imaginary part. The loss tangent is observed to have two peaks. The first one is at the resonant frequency (f_0), and the second one is at the magnetic plasma frequency (f_{mp}), where Re[μ_r] = 0. The loss tangent in the frequency region between f_0 and f_{mp} is shown to be relatively larger than that in other regions. Especially in the case of f_0 = 10 THz, the loss tangent is approximately greater than 0.017.

III. Synthesis of an Effective Medium with Wide Range of Relative Permeability

If we want to find the resonant frequency (f_0) with which we have a specific relative permeability (μ_r) at f, by inverting (12), we obtain

$$f_0 = f\sqrt{\frac{(1-\mu_r)\ln\dfrac{2.78}{m_3} - \pi^2 m_2^{\,3}}{(1-\mu_r)\ln\dfrac{2.78}{m_3}}}\,. \qquad (13)$$

This formula can be used as a design equation for a bulk metamaterial to have a negative μ_r at any frequency f.

For a desired plasma frequency at f_{mp}, f_0 is obtained by

$$f_0 = f_{mp}\sqrt{\frac{\ln\dfrac{2.78}{m_3} - \pi^2 m_2^{\,3}}{\ln\dfrac{2.78}{m_3}}}\,. \qquad (14)$$

In the frequency range of f_0 to f_{mp}, μ_r is negative. We define the mu-negative (MNG) fractional bandwidth given by

$$BW_{MNG} = \frac{f_{mp} - f_0}{f_0} = \sqrt{\frac{\ln\dfrac{2.78}{m_3}}{\ln\dfrac{2.78}{m_3} - \pi^2 m_2^{\,3}}} - 1\,. \qquad (15)$$

Fig. 5 presents the theoretical and EM-simulated real parts of μ_r as a function of f/f_0 for some different values of m_2 when m_1 = 0.1 and m_3 = 0.2. The resonant frequency (f_0) of 10 GHz is used. As m_2 increases, BW_{MNG} also increases as expected in (15). The results in Fig. 5 are summarized in Table 1. When the m_2's are 0.35, 0.4, and 0.45, the theoretical BW_{MNG} are 9.15%, 14.7%, and 23.25%, respectively. The EM-simulated BW_{MNG} is 9.65% when m_2 = 0.35. The theoretical and EM-simulated BW_{MNG} are shown to be in good agreement.

Fig. 6(a) and (b) show the MNG fractional bandwidth (BW_{MNG}) and the magnetic loss tangent ($|\mu''/\mu'|$) of the medium made of ring-type RRs (Fig. 1) as a function of m_3 (= t/r) for

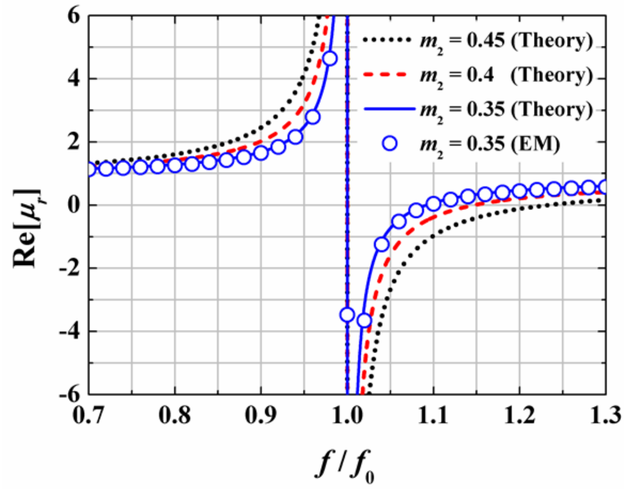


Fig. 5. Real part of relative effective permeability as a function of f/f_0 for different m_2's (f_0 = 10 GHz, m_1 = 0.1, and m_3 = 0.2).

Table 1. Theoretical and EM-simulated BW_{MNG} (f_0 = 10 GHz, m_1 = 0.1, and m_3 = 0.2)

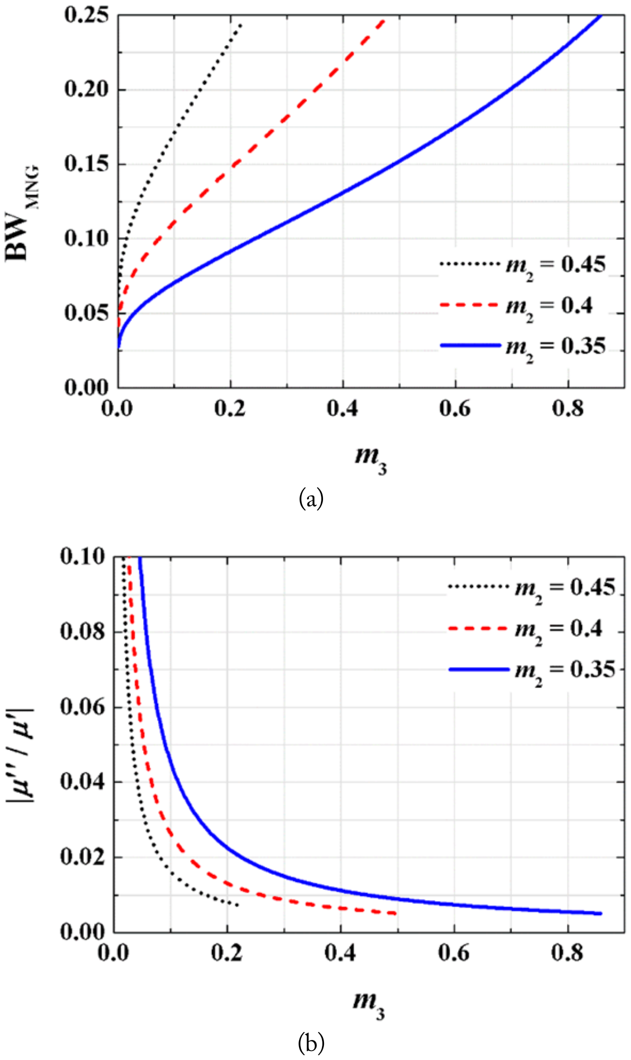
	Theory			EM
m_2	0.45	0.4	0.35	0.35
BW_{MNG}(%)	23.25	14.7	9.15	9.65

different m_2 (= r/a) values when f_0 = 60 MHz (used for MRI) and m_1 = 0.01. When the m_2's are 0.35, 0.4, and 0.45, m_3 must be less than 6/7, 1/2, and 2/9, respectively, to prevent contact among the resonators. The BW_{MNG} for each m_2 is shown to considerably increase from 5% to 25% as m_3 increases in the allowable range. m_3 (= t/r) is shown to play an important role in the enhancement of BW_{MNG}. The reason for this finding is that as m_3 increases, the inductance (7) and reactance slope parameter in (5) substantially decrease. The loss tangent ($|\mu''/\mu'|$) of the medium when Re[μ_r] = −1 also decreases because the aspect ratio in (6) decreases as m_3 increases. When m_2 is 0.35 or 0.4, the loss tangent is shown to be as small as 0.005.

The design Eq. (11) is used for a practical example. When an effective medium characterized by μ_r = −1 at f is desired, f_0 is given by

$$f_0 = f\sqrt{\frac{2\ln\dfrac{2.78}{m_3} - \pi^2 m_2{}^3}{2\ln\dfrac{2.78}{m_3}}}. \tag{16}$$

If a μ_r = −1 medium is required at f (63.87 MHz) for MRI applications with a reasonable choice of m_1 = 0.0064, m_2 = 0.4, and m_3 = 0.4, (16) and (7) give f_0 = 58.44 MHz and L = 3.17 nH, respectively. Then, the capacitance of the chip capacitor is determined as 2.34 nF.



(a)

(b)

Fig. 6. (a) Mu-negative fractional bandwidth (BW_{MNG}) and (b) magnetic loss tangent of the ring-type ring resonators ($|\mu''/\mu'|$ when Re[μ_r] = −1). Fixed f_0 = 60 MHz and m_1 = 0.01.

Table 2. Theoretical BW_{MNG} (f = 10 GHz, μ_r = −1, m_1 = 0.1, m_2 = 0.4, and m_3 = 0.4 at 10 GHz)

	Re[μ_r]	Im[μ_r]	f_0 (GHz)	BW_{MNG} (%)
Ring-type RR	−1	0.0083	9.43	18.4
Planar-type RR		0.0559	9.54	13.31
Planar-type BC-SRR		0.0415	9.51	15.09

Let us take more design examples at another resonant frequency. In Table 2, we summarize the characteristics of the media composed of ring-type RR, planar-type RR, and broadside-coupled SRR with their dimensions. The relative effective permeability is −1 at 10 GHz. The m_1, m_2, and m_3 are 0.1, 0.4, and 0.4 at 10 GHz, respectively. Using (16), we obtained the resonant frequencies for each case. The BW_{MNG} of the ring-type

RR is wider than that of the others. In addition, the loss of the ring-type RR is smaller than the others. The presented design procedures are scalable to other frequencies.

IV. CONCLUSION

Through the provided simple expressions of effective permeability for bulk metamaterials consisting of RRs, their characteristics have been systematically investigated by introducing some convenient geometrical factors. The MNG bandwidth and losses in the medium can be engineered for the best possible performance. The MNG bandwidth has been shown to be enhanced up to 25% by decreasing the reactance slope parameter with the possible widest width of the used RR. The loss tangent when $\mathrm{Re}[\mu_r] = -1$ has been shown to be as small as 0.005. Some convenient synthesis equations have also been derived to obtain a systematic design process with some examples.

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