Regularized Zero-Forcing Beam Design under Time-Varying Channels

Heejung Yu and Taejoon Kim

In this paper, an efficient beam tracking algorithm for a regularized zero-forcing (RZF) approach in slowly fading multiple-input and single-output (MISO) broadcast channels is considered. By modifying an RZF equation, an RZF beam tracking algorithm is proposed using matrix perturbation theory. The proposed algorithm utilizes both beams from the previous time step and channel difference (between the previous and current time steps) to calculate the RZF beams. The tracking performance of the proposed algorithm is analyzed in terms of the mean square error (MSE) between a tracking approach and an exact recomputing approach, and in terms of the additional MSE caused by the beam tracking error at the receiver. Numerical results show that the proposed algorithm has almost the same performance as the exact recomputing approach in terms of the sum rate.

Keywords: MISO broadcast channels, RZF beamforming, matrix perturbation, tracking algorithm.

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I. Introduction

As the throughput demand of wireless networks increases, beamforming technologies with multiple antennas at the base stations (BSs) have been adopted in various standards, such as LTE-A and WLAN. Due to a limitation of space, a mobile station (MS) can have only one antenna, in most cases, whereas a BS can have multiple antennas. In such an environment, multiple-input and single-output (MISO) broadcast channels become attractive, where a BS with multiple antennas simultaneously serves multiple MSs with a single antenna by using beamforming to mitigate interference. One of the simplest beamforming methods for MISO broadcast channels is that of the zero-forcing (ZF) approach, which pre-cancels any interference leakage to unintended receivers [1], [2]. This approach can achieve the best performance when the noise effect is neglected. The ZF approach is not optimal in terms of the sum rate; consequently, there have been several methods put forward in an attempt to enhance it. In the case of multiuser multiple-input and multiple-output (MIMO) broadcast channels, such methods include regularized channel inversion or regularized zero-forcing (RZF) [3]–[5] and signal-to-leakage-plus-noise ratio [6].

In the above beamforming methods, channels are assumed to be static in nature. If the opposite is assumed (that is, channels are time-varying in nature), then a corresponding beamforming method must recalculate beams through the use of updated channel information. This, however, entails a greater level of complexity because matrix inversion or decomposition operations are required at each time step.

In general, because the channel coefficients of a time-varying channel experience very little variation over time, a tracking algorithm that utilizes both beams from the previous time step and channel difference (between the previous and current time steps) to calculate the RZF beams can be developed. For a ZF approach, an efficient algorithm to update a beam under time-varying channels was proposed by Kim and others [7]. This algorithm can directly use matrix perturbation, because a beam is obtained by solving a least squares problem, for which the solution is given by either a singular vector or an eigenvector. However, an RZF approach considering noise effects has no direct relation with a singular vector of a channel matrix. If a modified channel matrix is defined by concatenating a channel matrix with the square root of a noise covariance matrix, then an RZF beam can be regarded as a ZF beam associated with the modified channel matrix. With such a modified formula, a beam tracking algorithm can be developed by using null-space perturbation in matrix perturbation theory.

An efficient algorithm that calculates RZF beams and has an exact calculation phase and a tracking phase dependent upon the tracking depth is proposed. The tracking error and additional mean square error in the received signal at an MS are analyzed. In detail, their behaviors depending on the minimum singular value of the modified channel matrix and mobile speed of the MS are examined.

The main contributions of this paper can be summarized as follows.

- A reformulation of an existing RZF equation is introduced by defining a concatenating matrix, which is obtained by concatenating a composite channel matrix and the square root of a noise covariance matrix.
- An RZF beam tracking algorithm is proposed by using matrix perturbation theory, and its performance is analyzed.
- The RZF beam tracking algorithm reduces the complexity of an RZF approach under the assumption of time-varying channels.

We will make use of standard notational conventions. Vectors and matrices are written in boldface lowercase characters and boldface uppercase characters, respectively. All vectors are column vectors. For matrix $\mathbf{A}$, $\mathbf{A}^{\mathrm{T}}$ and $\mathbf{A}^{\mathrm{H}}$ indicate the transpose and Hermitian transpose of $\mathbf{A}$, respectively. Additionally, $\mathbf{A}^{\dagger}$ means a pseudo inverse of matrix $\mathbf{A}$, and diag $(a_1, \ldots, a_K)$ denotes a diagonal matrix of which the ith diagonal component is a_i. Further, we denote the identity matrix and zero matrix by $\mathbf{I}$ and $\mathbf{0}$, respectively; the Frobenius norm of matrix $\mathbf{A}$ by $\|\mathbf{A}\|_F$; and the 2-norm of matrix $\mathbf{A}$ by $\|\mathbf{A}\|_2$. For matrix $\mathbf{A}$, Col($\mathbf{A}$) and Null($\mathbf{A}$) are the column space and null space of $\mathbf{A}$, respectively. The notation $\mathbf{a} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that vector $\mathbf{a}$ is a Gaussian-distributed random vector with mean $\boldsymbol{\mu}$ (vector) and covariance $\boldsymbol{\Sigma}$ (matrix).

II. System Model and Preliminaries

We consider a downlink MISO channel where a BS with multiple antennas transmits data to multiple MSs simultaneously. The BS is equipped with N_t antennas, and the K MSs have a single antenna. By using a properly designed beam vector at the BS, the interference leakage to the unintended MSs can be efficiently eliminated. At time n, the received signal by MS k can be

$$y_k[n] = \mathbf{H}_k[n]\mathbf{v}_k[n]s_k[n] + \sum_{l=1, l\neq k}^{K} \mathbf{H}_k[n]\mathbf{v}_l[n]s_l[n] + n_k[n], \quad (1)$$

where $\mathbf{H}_k[n]$ stands for the MISO channel from the BS to MS k, $\mathbf{v}_k[n]$ is the beamforming vector associated with the data symbol $s_k[n]$ (which is transmitted to MS k), and $n_k[n](\sim N(0, \sigma^2))$ is additive white Gaussian noise. By stacking the received signal for all MSs, the total signal can be expressed by

$$\mathbf{y}[n] = \left[y_1[n], \ldots, y_K[n]\right]^{\mathrm{T}} = \mathbf{H}[n]\mathbf{V}[n]\mathbf{s}[n] + \mathbf{n}[n], \quad (2)$$

where $\mathbf{H}[n] = \left[\mathbf{H}_1^{\mathrm{T}}[n], \ldots, \mathbf{H}_K^{\mathrm{T}}[n]\right]^{\mathrm{T}}$, $\mathbf{V}[n] = \left[\mathbf{v}_1[n], \ldots, \mathbf{v}_K[n]\right]$, $\mathbf{s}[n] = \left[s_1[n], \ldots, s_K[n]\right]^{\mathrm{T}}$, and $\mathbf{n}[n] = \left[n_1[n], \ldots, n_K[n]\right]^{\mathrm{T}}$. The ZF approach to design the beam for MS k is expressed as

$$\tilde{\mathbf{H}}_k[n]\mathbf{v}_k[n] = \mathbf{0}, \quad (3)$$

where $\tilde{\mathbf{H}}_k[n] = \left[\mathbf{H}_1^{\mathrm{T}}[n], \ldots, \mathbf{H}_{k-1}^{\mathrm{T}}[n]\ \mathbf{H}_{k+1}^{\mathrm{T}}[n], \ldots, \mathbf{H}_K^{\mathrm{T}}[n]\right]^{\mathrm{T}}$. This means that the beam vector $\mathbf{v}_k[n]$ lies in the null-space of the composite interference channel matrix $\tilde{\mathbf{H}}_k[n]$ for MS k. In the ZF approach, the noise effect is not considered. Thus, sum rate losses in low and medium signal-to-noise ratio (SNR) ranges cannot be avoided. As an alternative, and to enhance the performance in such SNR ranges, an RZF approach was introduced elsewhere [3]–[5] and is given by

$$\mathbf{V}[n] = \alpha \mathbf{H}^{\mathrm{H}}[n]\left(\mathbf{H}[n]\mathbf{H}^{\mathrm{H}}[n] + \rho \mathbf{I}\right)^{-1}, \quad (4)$$

where α is a normalizing constant and ρ is a regularization parameter. This approach is frequently referred to as minimum mean square error (MMSE) precoding, because it is similar to the MMSE receiver design. By adjusting ρ, an optimal tradeoff between the allowed interference and noise enhancement can be obtained. In practice, the regularization parameter is chosen by $\rho = N_t(\sigma^2/P_t)$, which is motivated by the results obtained by Peel and others [3]. It is also known that this parameter approximately maximizes the signal-to-interference-plus-noise ratio at each receiver.

To develop an efficient RZF beam tracking algorithm under time-varying channels, we define the pseudoinverse of a matrix obtained by concatenating the composite channel matrix and

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square root of the noise covariance, $\mathbf{G}[n] \triangleq \left[\mathbf{H}[n]\ \sqrt{\rho}\mathbf{I}\right]$, as follows:

$$\mathbf{W}[n] = \alpha' \mathbf{G}^{\mathrm{H}}[n]\left(\mathbf{G}[n]\mathbf{G}^{\mathrm{H}}[n]\right)^{-1}, \tag{5}$$

where α' is a normalizing constant. We can obtain $\mathbf{V}[n]$, which is the same as (4), by extracting the first N_t rows from $\mathbf{W}[n]$ and rescaling it. That is, the RZF beams can be calculated using the ZF approach with modified composite channel matrix $\mathbf{G}[n]$. The kth column of $\mathbf{W}[n]$, $\mathbf{w}_k[n]$, lies in the null-space of $\tilde{\mathbf{G}}[n]$, which is obtained by deleting the kth row from $\mathbf{G}$; that is, $\tilde{\mathbf{G}}_k[n] \triangleq \left[\tilde{\mathbf{H}}_k[n]\ \sqrt{\rho}\mathbf{I}_k\right]$, where $\mathbf{I}_k$ is a modified identity matrix (that is, $\mathbf{I}$ but with the kth row deleted). The kth column of $\mathbf{W}[n]$, $\mathbf{w}_k[n]$, can be expressed as a linear combination of basis vectors of the null-space of $\tilde{\mathbf{G}}_k[n]$ for all k; that is,

$$\tilde{\mathbf{G}}_k[n]\mathbf{w}_k[n] = \mathbf{0}. \tag{6}$$

To evaluate the tracking performance, we use the first-order Gauss-Markov channel model, which is widely used in the literature. For an (i, j) element of channel matrix $\mathbf{H}[n]$, its time-variation is given by

$$h_{ij}[n] = \beta_i h_{ij}[n-1] + \sqrt{1-\beta_i^2}\, u_{ij}[n] \qquad (n \geq 1), \tag{7}$$

using $h_{ij}[0] \sim N(0, 1)$ and $u_{ij}[n] \sim N(0, 1)$. A correlation coefficient of channel fading is denoted by β_i, and depends on both Doppler frequency f_{d} and symbol duration T_{s} [8]. In detail, the correlation coefficient is given by

$$\begin{aligned}\beta_i &= J_0(2\pi f_{\mathrm{d}} T_{\mathrm{s}}) = \sum_{r=0}^{\infty} \frac{(-1)^r}{2^{2r}(r)^2}(2\pi f_{\mathrm{d}} T_{\mathrm{s}})^{2r} \\ &\approx 1 - (\pi f_{\mathrm{d}} T_{\mathrm{s}})^2,\end{aligned} \tag{8}$$

where $J_0(\cdot)$ is a Bessel function of order zero. When channel coefficients slowly vary over time, that is, $f_{\mathrm{d}}\, T_{\mathrm{s}} \ll 1$, the last approximation holds.

III. Efficient RZF Beam Tracking Algorithm

In this paper, we consider an RZF approach under the assumption of time-varying channels. Under a straightforward approach (an exact recalculating approach), an RZF beam can be recalculated with current channel information; however, the recalculation is too complex to implement in real time. To reduce the computation complexity with a negligible loss of the achievable sum rate, an efficient beam tracking algorithm is required. Under a general ZF approach, matrix perturbation theory was adopted by Kim and others [7] to track the null-space of the composite interference channel matrix. A similar perturbation approach cannot be applied to the RZF approach given by (4). However, the RZF approach can be regarded as a ZF approach with a modified channel matrix when (4) is reformulated as (5). This reformulation makes it possible to apply null-space perturbation in developing an efficient beam tracking algorithm.

For more detail, we introduce null-space perturbation with a linear equation given by

$$\mathbf{A}_0\mathbf{P}_0 = \mathbf{0}, \tag{9}$$

where $\mathbf{A}_0$ is a full rank $M \times N$ ($> M$) matrix and $\mathbf{P}_0$ is a unitary $N \times (N - M)$ matrix; that is, the columns of $\mathbf{P}_0$ comprise the orthonormal basis vectors of the null-space of $\mathbf{A}_0$. When $\mathbf{A}_0$ is slightly perturbed by $\mathbf{A}(\varepsilon) = \mathbf{A}_0 + \varepsilon\mathbf{A}_1$, the solution to the perturbed linear equation can be approximated with a simple equation. In [9], null-space perturbation was introduced when $\mathbf{A}_0$ is a square matrix. The following theorem is for a fat matrix with $M < N$.

Theorem 1. Let $\mathbf{P}_0$ be a unitary matrix of a solution to (9); that is, a collection of orthonormal basis vectors. The new basis of the solution to $\mathbf{A}(\varepsilon)\mathbf{P}(\varepsilon) = 0$, where $\mathbf{A}(\varepsilon)$ is a rank-preserving perturbation of $\mathbf{A}_0$ (that is, the rank of $\mathbf{A}(\varepsilon)$ is the same as that of $\mathbf{A}_0$) is given by

$$\mathbf{P}(\varepsilon) = \mathbf{P}_0 - \varepsilon\mathbf{A}_0^{\dagger}\mathbf{A}_1\mathbf{P}_0 + O(\varepsilon^2). \tag{10}$$

Proof. The perturbed matrix $\mathbf{P}(\varepsilon)$ can be represented as a power series: $\mathbf{P}(\varepsilon) = \mathbf{P}_0 + \varepsilon\mathbf{P}_1 + \varepsilon^2\mathbf{P}_2 + \cdots$. Instead of $\mathbf{P}(\varepsilon)^{\mathrm{H}}\mathbf{P}(\varepsilon) = \mathbf{I}$, we suppose that the normalized condition for $\mathbf{P}(\varepsilon)$ satisfies quasi-orthonormality, as in [9]; that is, we have

$$\mathbf{P}_0^{\mathrm{H}}\mathbf{P}(\varepsilon) = \mathbf{I}. \tag{11}$$

Since $\mathbf{P}(\varepsilon)$ should satisfy $\mathbf{A}(\varepsilon)\mathbf{P}(\varepsilon) = \mathbf{0}$, we can obtain the following system of fundamental equations:

$$\sum_{i=0}^{1}\mathbf{A}_i\mathbf{P}_{k-i} = \mathbf{0} \qquad (k = 1, 2, \ldots). \tag{12}$$

From the normalization condition (11), we have

$$\mathbf{P}_0^{\mathrm{H}}\mathbf{P}_k = \begin{cases}\mathbf{I} & k = 0, \\ \mathbf{0} & \text{otherwise.}\end{cases} \tag{13}$$

The remaining derivation of the recursive formula for $\mathbf{P}_k$ is based on the following lemma [9]:

Lemma 1. The necessary and sufficient condition for the system of linear equations $\mathbf{By} = \mathbf{c}$ is feasible for any $\mathbf{x}$, $\mathbf{x}^{\mathrm{H}}\mathbf{c} = \mathbf{0}$ whenever $\mathbf{x}^{\mathrm{H}}\mathbf{B} = \mathbf{0}$. Moreover, the general solution for the feasible linear equation is $\mathbf{y} = \mathbf{B}^{\dagger}\mathbf{x} + \mathbf{d}$ if and only if $\mathbf{Bd} = \mathbf{0}$.

The general solution for (12) for $k = l$ ($l > 0$) is

$$\mathbf{P}_l = \mathbf{Q}_l - \mathbf{A}_0^{\dagger}\mathbf{A}_1\mathbf{P}_{l-1} \qquad (l = 1, 2, \ldots), \tag{14}$$

where $\mathbf{Q}_l \in \mathbb{C}^{N\times(N-M)}$ satisfying $\mathbf{A}_0\mathbf{Q}_l = \mathbf{0}$. Hence, $\mathbf{Q}_l$ can be represented as $\tilde{\mathbf{P}}\mathbf{M}_l$ with $\mathbf{M}_l \in \mathbb{C}^{(N-M)\times(N-M)}$. With

k = 0 in (12), $\mathbf{P}_0$ should satisfy $\mathbf{A}_0\mathbf{P}_0 = \mathbf{0}$ and normalized condition (13). Hence, we can choose $\mathbf{P}_0$ as $\tilde{\mathbf{P}}$. For l = 1, 2, … , (14) has to satisfy normalized condition (13); that is, we have

$$\tilde{\mathbf{P}}^{\mathrm{H}}\left(\tilde{\mathbf{P}}\mathbf{M}_l - \mathbf{A}_0^{\dagger}\mathbf{A}_1\mathbf{P}_{l-1}\right) = \mathbf{0}. \tag{15}$$

From the property of Col($\mathbf{A}_0^{\dagger}$)$^{\perp}$ = Null($\mathbf{A}_0$); that is, $\tilde{\mathbf{P}}^{\mathrm{H}}\mathbf{A}_0^{\dagger} = \mathbf{0}$, $\mathbf{M}_l$ is uniquely determined as 0 for l = 1, 2, … . Hence, the recursive solution for $\mathbf{P}_l$ is given by

$$\mathbf{P}_l = -\mathbf{A}_0^{\dagger}\mathbf{A}_1\mathbf{P}_{l-1} \qquad (l = 1, 2, \ldots), \tag{16}$$

with $\mathbf{P}_0 = \tilde{\mathbf{P}}$. With approximation, $\mathbf{P}(\varepsilon) = \mathbf{P}_0 + \varepsilon\mathbf{P}_1 + O(\varepsilon^2)$, where $\mathbf{P}_1 = -\mathbf{A}_0^{\dagger}\mathbf{A}_1\mathbf{P}_0$. ■

Based on the above theorem, we can derive an RZF beam tracking algorithm. The channel matrices for MS k at time m and n (> m) are given by $\mathbf{H}_k[n] = \mathbf{H}_k[m] + \Delta\mathbf{H}_k[m, n]$. With them, we can define the concatenated matrix for MS k to calculate an RZF beam as follows:

$$\tilde{\mathbf{G}}_k[n] = \tilde{\mathbf{G}}_k[m] + \Delta\tilde{\mathbf{G}}_k[m, n]. \tag{17}$$

We set $\mathbf{A}_0 = \tilde{\mathbf{G}}_k[m]$ and $\mathbf{A}(\varepsilon) = \tilde{\mathbf{G}}_k[n] = \tilde{\mathbf{G}}_k[m] + \varepsilon\frac{\Delta\tilde{\mathbf{G}}_k[m,n]}{\varepsilon}$, where $\varepsilon = \|\Delta\tilde{\mathbf{G}}_k[m, n]\|_2$ in Theorem 1. It is known that the concatenated matrix $\tilde{\mathbf{G}}_k[m]$ and perturbation matrix $\Delta\tilde{\mathbf{G}}_k[m, n]$ almost surely have the same rank because the entries of the channel matrices (that is, the left block of the concatenated matrix) are generated as an independent and identically distributed Gaussian distribution, and the right block has a column rank. Then, the solution to $\tilde{\mathbf{G}}_k[n]\mathbf{R}_k[n] = \mathbf{0}$ becomes $\mathbf{R}_k[n] = \mathbf{R}_k[m] + \Delta\mathbf{R}_k[m, n]$, where $\mathbf{R}_k[m]$ is the solution to $\tilde{\mathbf{G}}_k[m]\mathbf{R}_k[m] = \mathbf{0}$. By the perturbation theory, we have

$$\mathbf{R}_k[n] = \mathbf{R}_k[m] - \tilde{\mathbf{G}}_k^{\dagger}[m]\Delta\tilde{\mathbf{G}}_k[m, n]\mathbf{R}_k[m] + O(\|\Delta\tilde{\mathbf{G}}_k[m, n]\|_2^2). \tag{18}$$

The transmit beam for MS k at time n can be expressed by the first N_t elements in a vector of the linear combination of $\hat{\mathbf{R}}_k[n] = \mathbf{R}_k[m] - \tilde{\mathbf{G}}_k^{\dagger}[m]\Delta\tilde{\mathbf{G}}_k[m, n]\mathbf{R}_k[m]$. Here, the coefficients of the linear combination are determined with the projection of the kth row of $\mathbf{G}[m]$ onto the null-space of $\tilde{\mathbf{G}}_k[m]$; that is, $\mathbf{w}_k[m] = \mathbf{R}_k[m]\mathbf{f}_k$. For simplicity, the combining coefficient $\mathbf{f}_k$ is maintained during beam tracking. Then, $\mathbf{w}_k[n]$ is determined by $\mathbf{R}_k[n]\mathbf{f}_k$ with (18). By right-multiplying $\mathbf{f}_k$ in (18), we have

$$\begin{aligned}\mathbf{w}_k[n] &= \mathbf{w}_k[m] - \tilde{\mathbf{G}}_k^{\dagger}[m]\Delta\tilde{\mathbf{G}}_k[m, n]\mathbf{w}_k[m] \\ &\quad + O(\|\Delta\tilde{\mathbf{G}}_k[m, n]\|_2^2).\end{aligned} \tag{19}$$

As shown in (19), the tracking error of the beam is determined by $\|\Delta\tilde{\mathbf{G}}_k[m, n]\|_2$. Therefore, the tracking error becomes more significant as the time difference between m and n increases. For a more accurate tracking of RZF beams, we have to recalculate the beam with an operation of singular value decomposition when (n – m) is large. In practice, the tracking depth, L_t, should be defined and exact beams should be calculated with the concatenated matrix at every L_t time step. The overall process of RZF beam calculation can be summarized as follows:

Step 0. Initialization: Obtain CSI, $\{\mathbf{H}_k[n], k = 1, \ldots, K\}$, and SNR, σ^2.

At $n = iL_t$ for some integer L_t and i = 0, 1, … (Exact Calculation Phase)

Step 1. For MS k, construct $\tilde{\mathbf{G}}_k[iL_t]$ with $\{\mathbf{H}_l[iL_t], l = 1, \ldots, k-1, k+2, \ldots, K\}$ and σ^2.

Step 2. With $\tilde{\mathbf{G}}_k[iL_t]$, obtain basis vectors by subspace decomposition.

Step 3. With the basis vectors in the null-space of $\tilde{\mathbf{G}}_k[iL_t]$ ($\mathbf{R}_k[iL_t]$), $\mathbf{f}_k$, which is calculated with $\mathbf{H}_k[iL_t]$, and σ^2, obtain an RZF beam vector, $\mathbf{v}_k[n]$, from $\mathbf{w}_k[iL_t] = \mathbf{R}_k[iL_t]\mathbf{f}_k$ by taking the first N_t components.

For $n = iL_t + 1 : iL_t + L_t - 1$ (Tracking Phase)

Step 4. Obtain $\Delta\tilde{\mathbf{G}}_k[iL_t, n] = \tilde{\mathbf{G}}_k[n] - \tilde{\mathbf{G}}_k[iL_t]$.

Step 5. Calculate the perturbed solution as

$$\hat{\mathbf{w}}_k[n] = \mathbf{w}_k[iL_t] - \tilde{\mathbf{G}}_k^{\dagger}[iL_t]\Delta\tilde{\mathbf{G}}_k[iL_t, n]\mathbf{w}_k[iL_t].$$

Step 6. With $\hat{\mathbf{w}}_k[n]$, obtain an RZF beam vector, $\mathbf{v}_k[n]$, by taking the first N_t components from $\hat{\mathbf{w}}_k[n]$.

Because the accuracy of beam tracking depends on $\|\Delta\tilde{\mathbf{G}}_k[iL_t, n]\|_2$, it can be adjusted by changing the value of tracking depth L_t. The tracking depth can be regarded as a tradeoff between the complexity and accuracy of the beam calculation.

In this paper, we assume downlink MISO channels where MSs have a single antenna. However, the RZF beam tracking algorithm can be extended to downlink MIMO channels with MSs equipped with multiple antennas by defining $\tilde{\mathbf{G}}_k[n]$ with MIMO channel matrices, $\{\mathbf{H}_l[n], l = 1, \ldots, L\}$.

IV. Performance Analysis of Beam Tracking

To investigate performance loss caused by the proposed RZF tracking approach, the performance of RZF beamforming with inaccurate beam vectors is analyzed in this section. If the designed beam vector $\mathbf{w}_k[n]$ lies in the null-space of $\tilde{\mathbf{G}}_k[n]$ at time n, then we can achieve a good performance with RZF beamforming. However, the beam obtained by the proposed tracking algorithm $\hat{\mathbf{w}}_k[n]$ does not lie in the null-space of $\tilde{\mathbf{G}}_k[n]$ exactly. This misalignment can be considered a source of additional noise. With such a concept, we can derive the performance loss caused by the tracking algorithm. To this end, the detailed expression of the second term in the null-space perturbation (that is, $\varepsilon^2\mathbf{P}_2$) is used by neglecting the third and higher order terms.

Lemma 2. When the perturbed null-space of $\mathbf{A}(\varepsilon) = \mathbf{A}_0 + \varepsilon\mathbf{A}_1$ is approximated by the first two terms in (10) (that is, $\mathbf{P}_0 - \varepsilon\mathbf{A}_0^\dagger\mathbf{A}_1\mathbf{P}_0$), the approximation error is given by

$$\mathbf{E} = -\varepsilon^2\mathbf{A}_0^\dagger\mathbf{A}_1\mathbf{P}_1 + O(\varepsilon^3) = -\varepsilon^2(\mathbf{A}_0^\dagger\mathbf{A}_1)^2\mathbf{P}_0 + O(\varepsilon^3). \quad (20)$$

Proof. As shown in the proof of Theorem 1, a more exact expression for the perturbed null-space is given by

$$\mathbf{P}(\varepsilon) = \mathbf{P}_0 + \varepsilon\mathbf{P}_1 + \varepsilon^2\mathbf{P}_2 + \cdots. \quad (21)$$

Defining the perturbed null-space with $\mathbf{P}_0 + \varepsilon\mathbf{P}_1$, the error vector can be approximated with the second term, $\varepsilon^2\mathbf{P}_2$. From (16), $\mathbf{P}_2 = -\mathbf{A}_0^\dagger\mathbf{A}_1\mathbf{P}_1$. Then, the error matrix can be approximated with (20). ■

With the above lemma, the error in beam tracking based on the perturbed null-space at time n can be obtained by

$$\begin{aligned}\mathbf{e}_k[m,n] &= \mathbf{w}_k[n] - \hat{\mathbf{w}}_k[n] \\ &\approx \left(\tilde{\mathbf{G}}_k^\dagger[m]\Delta\tilde{\mathbf{G}}_k[m,n]\right)^2\mathbf{W}_k[m].\end{aligned} \quad (22)$$

For a more detailed investigation on the tracking error, we assume that

$$\begin{aligned}\tilde{\mathbf{G}}_k[m] &= \mathbf{\Phi}_k[m]\mathbf{\Lambda}_k[m]\mathbf{\Psi}_k^{\mathrm{H}}[m] \\ &= [\phi_1\cdots\phi_{(K-1)}][\mathrm{diag}(\lambda_1,\cdots,\lambda_{(K-1)})\mathbf{0}][\psi_1\cdots\psi_{N_t+K}]^{\mathrm{H}},\end{aligned} \quad (23)$$

where $\mathbf{\Phi}_k[m]$, $\mathbf{\Lambda}_k[m]$, and $\mathbf{\Psi}_k[m]$ are a $(K-1)\times(K-1)$ unitary matrix, a $(K-1)\times(N_t+K)$ diagonal matrix, and a $(N_t+K)\times(N_t+K)$ unitary matrix, respectively. Then, we can rewrite (22) as follows:

$$\begin{aligned}&\left(\tilde{\mathbf{G}}_k^\dagger[m]\Delta\tilde{\mathbf{G}}_k[m,n]\right)^2\mathbf{w}_k[m] \\ &= \sum_{j=1}^{K-1}\frac{1}{\lambda_j}\psi_j\phi_j^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\left(\sum_{i=1}^{K-1}\frac{1}{\lambda_i}\psi_i\phi_i^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\mathbf{w}_k[m]\right) \\ &= \sum_{j=1}^{K-1}\frac{1}{\lambda_j}\psi_j\phi_j^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\left(\sum_{i=1}^{K-1}\frac{1}{\lambda_i}g_i\psi_i\right) \\ &= \sum_{j=1}^{K-1}\frac{1}{\lambda_j}\left(\sum_{i=1}^{K-1}\frac{1}{\lambda_i}g_i\ \phi_j^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\psi_i\right)\psi_j.\end{aligned} \quad (24)$$

In the second equality, we define $g_i = \phi_i^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\mathbf{w}_k[m]$, which is a scalar. With this result, the upper bound of $\|\mathbf{e}_k[m,n]\|^2$ can be evaluated.

Lemma 3. The approximated error of beam tracking in (22), denoted by $\left(\tilde{\mathbf{G}}_k^\dagger[m]\Delta\tilde{\mathbf{G}}_k[m,n]\right)^2\mathbf{w}_k[m]$, is upper-bounded as

$$\left\|\left(\tilde{\mathbf{G}}_k^\dagger[m]\Delta\tilde{\mathbf{G}}_k[m,n]\right)^2\mathbf{w}_k[m]\right\|^2 \le \left(\sum_{i=1}^{K-1}\frac{1}{|\lambda_i|^2}\right)^2\|\Delta\tilde{\mathbf{G}}_k[m,n]\|_2^4. \quad (25)$$

Proof. By using sub-additivity and sub-multiplicity, we have

$$\begin{aligned}&\left\|\left(\tilde{G}_k^\dagger[m]\Delta\tilde{\mathbf{G}}_k[m,n]\right)^2\mathbf{w}_k[m]\right\|^2 \\ &\le \sum_{j=1}^{K-1}\frac{1}{|\lambda_j|^2}\left\|\sum_{i=1}^{K-1}\frac{1}{\lambda_i}g_i\phi_j^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\psi_i\right\|^2\|\psi_j\|^2 \\ &\le \sum_{j=1}^{K-1}\frac{1}{|\lambda_j|^2}\left(\sum_{i=1}^{K-1}\frac{1}{|\lambda_i|^2}|g_i|^2\|\Delta\tilde{\mathbf{G}}_k[m,n]\|_2^2\right) \\ &\le \sum_{j=1}^{K-1}\frac{1}{|\lambda_j|^2}\left(\sum_{i=1}^{K-1}\frac{1}{|\lambda_i|^2}\|\Delta\tilde{\mathbf{G}}_k[m,n]\|_2^4\right) \\ &= \left(\sum_{j=1}^{K-1}\frac{1}{|\lambda_j|^2}\right)^2\|\Delta\tilde{\mathbf{G}}_k[m,n]\|_2^4.\end{aligned} \quad (26)$$

Here, we use $\|\phi_i\| = \|\psi_i\| = 1$ and $|g_i| = \left|\phi_i^{\mathrm{H}}\Delta\tilde{\mathbf{G}}_k[m,n]\mathbf{w}_k[m]\right| \le \left\|\Delta\tilde{\mathbf{G}}_k[m,n]\right\|_2$. ■

From Lemma 3, the error in beam tracking depends on the distribution of singular values of $\tilde{\mathbf{G}}_k[m]$ as well as $\|\Delta\tilde{\mathbf{G}}_k[m,n]\|_2$. This means that the tracking error becomes significant when the minimum singular value is small.

Defining $\hat{\mathbf{w}}_k[n] = \mathbf{w}_k[n] - \tilde{\mathbf{w}}_k[n]$ and $\hat{\mathbf{v}}_k[n] = \mathbf{v}_k[n] - \tilde{\mathbf{v}}_k[n]$, where $\tilde{\mathbf{w}}_k[n]$ and $\tilde{\mathbf{v}}_k[n]$ denote the tracking errors in $\hat{\mathbf{w}}_k[n]$ and $\hat{\mathbf{v}}_k[n]$, respectively, we can write the received signal at MS k as follows:

$$\begin{aligned}y_k[n] &= \mathbf{H}_k[n]\tilde{\mathbf{v}}_k[n]s_k[n] \\ &\quad + \sum_{l=1,l\ne k}^{K}\mathbf{H}_l[n]\tilde{\mathbf{v}}_l[n]s_l[n] + n_k[n] \\ &= \mathbf{H}_k[n](\mathbf{v}_k[n] + \tilde{\mathbf{v}}_k[n])s_k[n] \\ &\quad + \sum_{l=1,l\ne k}^{K}\mathbf{H}_l[n](\mathbf{v}_l[n] + \tilde{\mathbf{v}}_l[n])s_l[n] + n_k[n] \\ &= \mathbf{H}_k[n]\mathbf{v}_k[n]s_k[n] + \sum_{l=1,l\ne k}^{K}\mathbf{H}_l[n]\mathbf{v}_l[n]s_l[n] \\ &\quad + \sum_{j=1}^{K}\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]s_j[n] + n_k[n].\end{aligned} \quad (27)$$

In the last equality, the third term, $\sum_{j=1}^{K}\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]s_j[n]$, is the interference increment due to beam tracking error. When there is no tracking error (that is, $\tilde{\mathbf{v}}_k[n] = \mathbf{0}$), the MMSE solution $\mathbf{v}_k[n]$ minimizes the mean square error (MSE) between the transmitted and received signal,

$\mathbb{E}\left[\left|y_k[n](\mathbf{v}_k[n])-s_k[n]\right|^2\right]$, where $y_k[n](\mathbf{v}_k[n])$ is the received signal with transmit beam of $\mathbf{v}_k[n]$. When a tracking error exists, the MSE is given by

$$\mathbb{E}\left[\left|y_k[n](\hat{\mathbf{v}}_k[n])-s_k[n]\right|^2\right] = \mathbb{E}\left[\left|y_k[n](\mathbf{v}_k[n])-s_k[n]\right|^2\right]+\mathbb{E}\left[\left|\sum_{j=1}^{K}\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]s_j[n]\right|^2\right], \tag{28}$$

where $y_k[n](\hat{\mathbf{v}}_k[n])$ denotes the received signal with the beam vector designed by the proposed method. Here, we assume that the tracking noise is a Gaussian vector that is independent of both the beam vector and the background noise. The second term in the right-hand side of (28) is the additional MSE made by a mismatch between the tracked beam vector and the exact vector. thus, it can cause a loss of system throughput and is regarded as one of the performance measures. The relation between this performance measure and other conditions, including mobility (v_m) and tracking depth ($n-m$), can be explained with the following theorem.

Theorem 2. Assume that the mobile speed of all K users with a single antenna is v_m, and two time indices of the exact inversion phase and the tracking phase are given by m and n ($> m$); that is, the tracking depth is ($n-m$). Under the Gauss-Markov channel model, the additional MSE due to tracking error is given by

$$\mathbb{E}\left[\left|\sum_{j=1}^{K}\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]s_j[n]\right|^2\right]\le c''K(n-m)^2v_m^4, \tag{29}$$

where c'' is a certain constant.

Proof. The additional MSE due to a tracking error is expressed as

$$\begin{aligned}\mathbb{E}\left[\left|\sum_{j=1}^{K}\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]s_j[n]\right|^2\right] &\le \sum_{j=1}^{K}\mathbb{E}\,\|\mathbf{H}_j[n]\|^2\;\mathbb{E}\,\|\tilde{\mathbf{v}}_j[n]\|^2\\ &\le \sum_{j=1}^{K}c\mathbb{E}\,\|\mathbf{H}_j[n]\|^2\;\mathbb{E}\,\|\Delta\tilde{\mathbf{G}}_j[m,n]\|_2^4\\ &\le \sum_{j=1}^{K}c\mathbb{E}\,\|\mathbf{H}_j[n]\|^2\;\mathbb{E}\,\|\Delta\tilde{\mathbf{G}}_j[m,n]\|_F^4 .\end{aligned} \tag{30}$$

In the first inequality, we use the triangular inequality, sub-multiplicativity of the norm, that is, $\mathbb{E}\left[|\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]|^2\right]\le \mathbb{E}\,\|\mathbf{H}_j[n]\|^2\;\mathbb{E}\,\|\tilde{\mathbf{v}}_j[n]\|^2$ and $\mathbb{E}\,|s_j[m]|^2=1$. Based on (19), we can see that $\|\tilde{\mathbf{v}}_j\|\le c\,\|\Delta\tilde{\mathbf{G}}_j[m,n]\|_2^2$. Then, the second inequality can be obtained. The third inequality holds because $\|\mathbf{A}\|_2 \le \|\mathbf{A}\|_F$ for a matrix. At first, we have to investigate the distribution of $\|\mathbf{H}_j[n]\|^2$. Each component of $\|\mathbf{H}_j[n]\|$ is a zero-mean complex Gaussian random a variable with a variance of 1. Then,

$$2\,\|\mathbf{H}_j[n]\|^2\sim\chi^2_{2N_t}, \tag{31}$$

where χ_k^2 denotes a chi-squared distribution with k degrees of freedom. Additionally, the distribution of $\|\Delta\tilde{\mathbf{G}}_j[m,n]\|_F^2$ can be obtained by the channel model given by (7).

$$\frac{1}{1-\alpha^{n-m}}\|\Delta\tilde{\mathbf{G}}_j[m,n]\|_F^2\sim\chi^2_{2N_t(K-1)}. \tag{32}$$

It is well known that the nth moment of a chi-squared random variable Z with k degrees of freedom is $\mathbb{E}(Z^n)=2^n\frac{\Gamma(n+k/2)}{\Gamma(k/2)}$, where $\Gamma(\cdot)$ is the gamma function. Then, we have

$$\mathbb{E}\left[\|\mathbf{H}_j[n]\|^2\|\tilde{\mathbf{v}}_j[n]\|^2\right]\le c'(1-\alpha^{n-m})^2, \tag{33}$$

where $c'=4c^2\frac{\Gamma(1+N_t)}{\Gamma(N_t)}\frac{\Gamma(2+N_t(N_t-1))}{\Gamma(N_t(N_t-1))}$. By using (8), we have

$$\begin{aligned}1-\alpha^{n-m} &\approx 1-\left(1-(\pi f_d T_s)^2\right)^{n-m}\\ &\approx (n-m)(\pi f_d T_s)^2\\ &=(n-m)v_m^2\left(\frac{\pi T_s}{\lambda}\right),\end{aligned} \tag{34}$$

where v_m is the velocity of mobile terminals, and λ is the wavelength of the carrier. Then, we can obtain the behavior of the upper bound of the additional MSE as follows:

$$\mathbb{E}\left[\left|\sum_{j=0}^{K}\mathbf{H}_j[n]\tilde{\mathbf{v}}_j[n]s_j[n]\right|\right]\le c''K(n-m)^2v_m^4, \tag{35}$$

where c'' is a constant. ■

Based on this observation, we can determine the tracking depth, L_t, depending on the velocity of the MSs.

The complexity of the proposed algorithm in terms of the number of complex multiplications can be estimated. The complexity for the exact beam calculation at every time is given by

$$L_t\left(2K^2(N_t+K)+\frac{2}{3}K^3\right) \tag{36}$$

because a matrix inversion is required at each time step. For the proposed algorithm, $2(N_t+K)(K-1)$ multiplications are required at each time step in the tracking phase. Additionally, the proposed algorithm requires more complexity to prepare the tracking phase; for example, the calculation of $\tilde{\mathbf{G}}_k^{\dagger}[iL_t]$ and $\tilde{\mathbf{G}}_k[iL_t]\mathbf{w}_k[iL_t]$. The total number of multiplications for the proposed algorithm is given by

$$\begin{aligned}&\left(2(N_t+K)\left\{K^2+(K-1)^2\right\}+\frac{2}{3}\left\{K^3+(K-1)^3\right\}\right)\\ &+(N_t+K)(K-1)+L_t\left(2(N_t+K)(K-1)\right).\end{aligned} \tag{37}$$

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V. Numerical Results

In this section, we show the numerical results under certain conditions to verify the analysis in the previous sections. Here, we assume that the carrier frequency is 2 GHz and the symbol duration is 66.7 µs, which is the OFDM symbol duration of 3GPP LTE. It is assumed that $N_t = 4$, $N_r = 1$, and $K = 4$. Time-varying channels are obtained by the Gauss-Markov model described in Section II. For a simple evaluation and verification of the results in the previous sections, the mobile speed of all K users is the same as v_m, and SNR = 20 dB. The channel variation over time is assumed to be known. In this paper, we focus solely on an RZF beam tracking algorithm. The channel variation over time can be obtained by optimal channel prediction based on the Kalman filter as in [10].

First, the beam-tracking error depending on the mobile speed and the singular values of $\tilde{\mathbf{G}}_k[m]$, which is denoted by $\lambda_{\min}$, is examined. Among the singular values, the minimum value has the most significant effect.

Figure 1 shows the MSE performance of the proposed RZF beam tracking algorithm with different minimum singular values of $\tilde{\mathbf{G}}_k[m]$. As examined in Lemma 3, the MSE of the beam tracking significantly depends on the minimum singular value of $\tilde{\mathbf{G}}_k[m]$; that is, $\lambda_{\min}$. As $\lambda_{\min}$ decreases (that is, $\tilde{\mathbf{G}}_k[m]$ is illconditioned), the accuracy of the beam tracking degrades and the MSE between the exact and tracked beams increases. With a given mobile speed, the gap between different channel realizations with different $\lambda_{\min}$ is a constant regardless of the time index $(n - m)$. This also coincides with Lemma 3.

Next, we investigate the behavior of the additional MSE caused by tracking error, $\mathbb{E}[|\sum_{j=1}^{K}[n]\tilde{\mathbf{v}}_j[n]s_j[n]|^2]$, after the exact calculation at time m as a function of mobile speed v_m. In Fig. 2, we show the additional MSE curves as a function of v_m with a given tracking time $(n - m)$ of 50 or 100. To verify the effect of v_m, the initial channels $\{h_{ij}[m]|i, j = 1, \ldots, 4\}$ are fixed, and $\{u_{ij}[n]\}$ are randomly chosen from the complex Gaussian distribution with zero mean and unit variance at each realization with (7) and (8). According to Theorem 2, the upper bound of the additional MSE behaves like v_m^4. We can see that the additional MSE is well matched with cv_m^4, where c is a certain constant in Fig. 2.

Additionally, we compare the sum rate performance of the proposed RZF tracking approach with that of the exact beam calculation at each time. We considered three different mobile speeds of 50 km/h, 100 km/h, and 200 km/h. To calculate the average sum rate, we take the average over the whole duration of tracking depth L_t with 1,000 channel realizations. In an exact

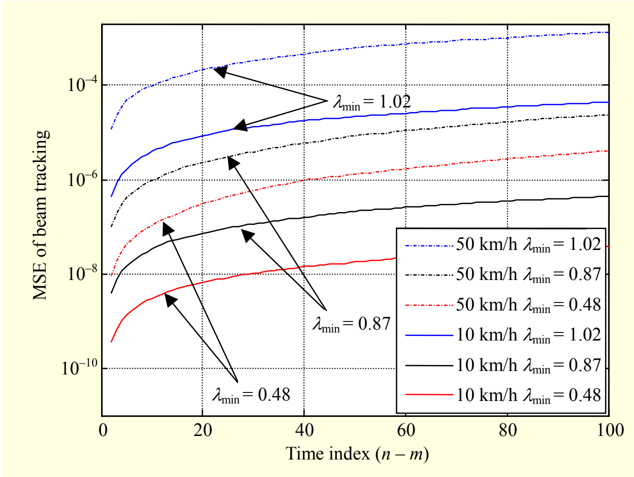

Fig. 1. MSE of beam tracking with $\lambda_{\min} \in \{1.02, 0.87, 0.48\}$, and $v_m \in \{10, 50\}$ km/h when $N_t = K = 4$ and $N_r = 1$.

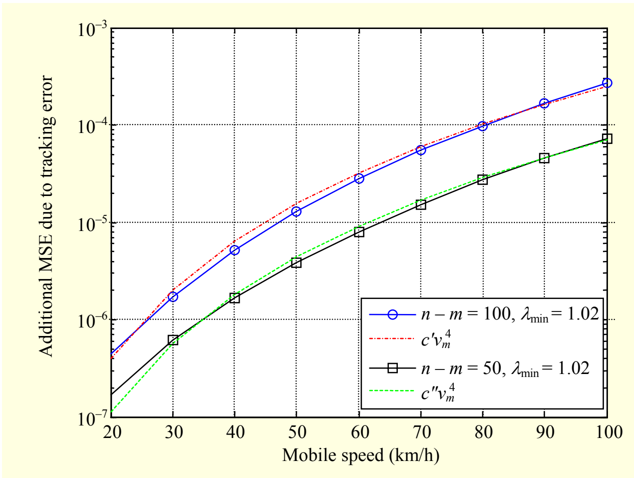

Fig. 2. Additional MSE due to tracking error at different mobile speeds with $\lambda_{\min} = 1.02$, $(n - m) \in \{50, 100\}$ when $N_t = K = 4$ and $N_r = 1$. Here, $c' = 2.5 \times 10^{-12}$ and $c'' = 7 \times 10^{-13}$.

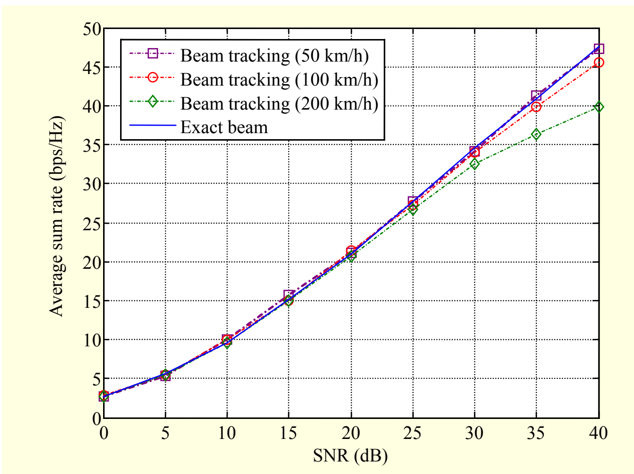

Fig. 3. Average sum rate at different mobile speeds when $N_t = K = 4$, $N_r = 1$, $L_t = 100$, and $v_m \in \{50, 100, 200\}$ km/h.

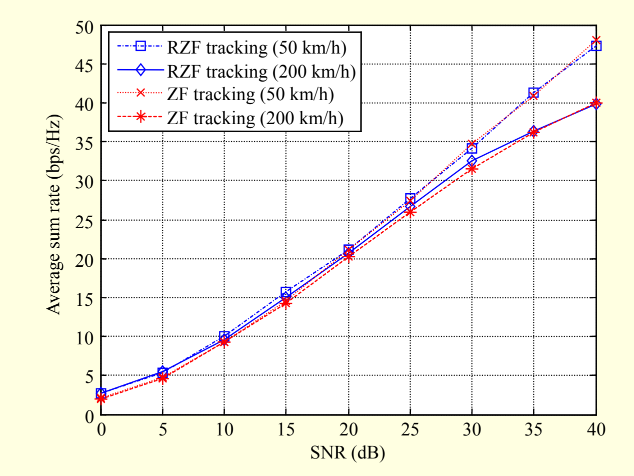

Fig. 4. Average sum rate of RZF and ZF tracking algorithms at different mobile speeds when $N_t = K = 4$, $N_r = 1$, $L_t = 100$, and $v_m \in \{50, 200\}$ km/h.

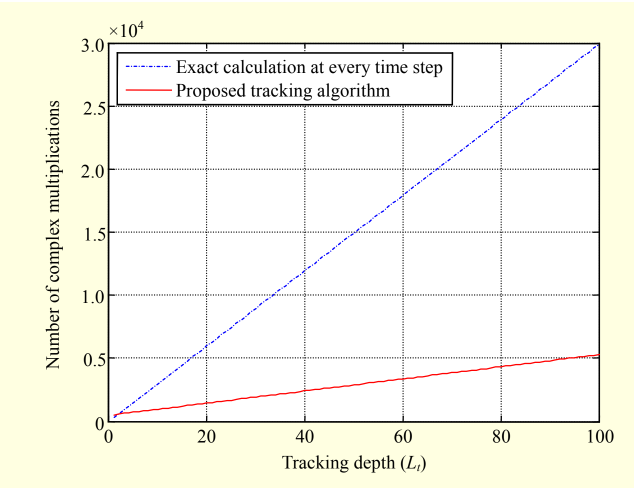

Fig. 5. Computational complexity of exact beam calculation and proposed tracking approaches when $N_t = K = 4$, $N_r = 1$.

beam calculation, the RZF approach given by (4) is applied. As we can see in Fig. 3, the proposed algorithm shows a negligible loss of the sum rate when the mobile speed is less than 100 km/h. Especially at a low or medium SNR, the sum rate loss becomes nearly zero because the noise at the receiver is dominant over the additional MSE caused by the tracking error. As shown in Fig. 2, the additional MSE is much less than 10^{-3} even with $(n - m) = 100$ and $v_m = 100$ km/h. This means the tracking error is much less than the noise when SNR ≤ 30 dB. This fact can be verified with the sum rates in Fig. 3.

Figure 4 shows the average sum rate of the proposed RZF tracking approach and the conventional ZF tracking approach in [7]. As expected, the RZF approach shows a better performance than the ZF approach in a low SNR region. As the SNR increases, the performance gap becomes negligible.

Finally, the computational complexity of the two different approaches is shown in Fig. 5 with different tracking depths. As expected, the proposed algorithm can reduce the beam design complexity over the exact calculation approach as the tracking depth increases.

VI. Conclusion

We proposed an RZF beam tracking algorithm for time-varying broadcast channels with multiple antennas. The algorithm has been developed with the null-space update formula based on matrix perturbation theory. We analyzed the performance of the proposed RZF beam tracking algorithm to obtain insights into the factors affecting its performance. Numerical results were provided to verify the analysis and show the performance of the proposed algorithm. In 5G networks, massive antennas can be equipped at the BSs, and the beam calculation becomes more complicated. Therefore, the complexity reduction in a beam calculation is an important issue. The proposed algorithm provides an efficient way to track beams with a comparable sum rate performance and less complexity.

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