

# Design of IIR Structure Active Mufflers using Stabilized Filter Algorithms

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## 안정화 필터 알고리즘을 적용한 IIR 구조 능동 머플러의 설계

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**Abstract** Active muffler is implemented by applying active noise control technique to reduce exhaust noise of automobile muffler. Conventional Filtered\_x LMS algorithm has a problem that the degree of control filter becomes very large and convergence deteriorates when acoustic feedback is present. The recursive LMS algorithm can compensate for this problem because it can be easily diverted in the adaptive filter adaptation process. In this paper, the structure of the primary path and the secondary path transfer function is designed as the IIR filter to improve the convergence performance and the computational burden, and the stabilization filter algorithm is applied to secure stability which is a disadvantage of the IIR filter structure. The stabilization filter algorithm plays a role of pulling the pole into the unit circle to prevent the pole of the transfer function corresponding to the acoustic feedback from diverging during the adaptation process. In this way, the computational burden of the active muffler system and the convergence performance can be improved. In order to show the usefulness of the proposed system, we compared the performance of the proposed Filtered\_x LMS algorithm with the performance of the proposed system for the exhaust sound of a diesel engine, which is a variable environment. Compared to conventional algorithm, proposed algorithm's computational burden is less than half, and convergence performances are more than 4 times.

요 약 능동 머플러는 자동차 머플러의 배기소음을 저감하기 위하여 능동 소음 제어 기법을 적용하여 구현된다. 기존의 보편적인 Filtered\_x LMS 알고리즘은 음향 궤환이 존재할 경우 제어필터의 차수가 매우 커지고 수렴성이 악화되는 문제가 있다. 이를 보완할 수 있는 Recursive LMS 알고리즘은 적응필터의 적응과정에서 쉽게 발산할 수 있어 적용이 제한되어 왔다. 본 논문에서는 수렴 성능과 계산량 부담이 개선되도록 1차 경로와 2차 경로 전달함수의 구조를 IIR 필터로 설계하였으며 IIR 필터 구조의 단점인 안정성 확보를 위해 안정화 필터 알고리즘을 적용하였다. 안정화 필터 알고리즘은 적응과정 중에 음향 궤환에 해당하는 전달함수의 극점이 발산하는 것을 방지하기 위하여 극점을 단위원 내부로 끌어 당기는 역할을 수행한다. 이러한 방법으로 능동 머플러 시스템의 계산량 절감과 수렴성능을 향상시킬 수 있다. 제안한 시스템의 유용성을 보이기 위하여 가변 환경인 디젤 엔진의 배기음향을 대상으로 기존의 Filtered\_x LMS 알고리즘과 제안한 시스템과의 성능을 비교하여 그 우수성을 보였으며, 계산량은 절반 이하, 수렴 특성은 4배 이상의 성능을 보였다.

**Keywords :** Adaptive Filter, Active Mufflers, Active Noise Control, Filtered\_U LMS Algorithm, Stabilized Filter

### 1. Active Muffler

Conventional passive muffler adopts a method in

which the exhaust gas is attenuated and absorbed while passing through the shielding path in the chamber in

order to reduce the exhaust noise of the internal

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combustion engine. This method interferes with the smooth emission of the exhaust gas, and thus has a negative effect on the reduction of the output of the engine and the decrease of the fuel consumption.

Active mufflers[1,2] are intended to solve the above-mentioned problems by straightening the exhaust pipe to solve the disadvantages of the passive muffler.

The Filtered\_x LMS (Least Mean Square) algorithm, which is mainly used for active noise control, mainly uses the FIR structure where the convergence is guaranteed and the amount of computation is proportional to the order of the control filter[3,4]. However, various studies have been made to improve the convergence characteristics due to problems such as degradation of system performance due to slow convergence characteristics.

In this paper, we propose an active muffler system designed with IIR filter structure for the control structure of the primary and secondary-paths so that the convergence performance and computational burden of the Filtered\_x LMS algorithm are improved. The IIR filter structure is problematic because of divergence in the adaptation process. In this paper, a stabilization filter algorithm is applied to secure the stability of the IIR structure, and an IIR filter structure corresponding to the first and second paths is designed. In order to show the usefulness of the proposed system, which is superior to the conventional Filtered\_x LMS algorithm.

## 2. Mathematical Modeling of Active Muffler

Fig.1 is an active noise control system for automobile mufflers. It is assumed that the primary sound source is located at a distance of  $l_0$  from the engine side exhaust end of the muffler, and the microphone for detecting the sound generated by the primary sound source is a primary. It is located at  $l_1$  distance from the sound source, and the secondary sound source is located at  $l_2$  distance from the

microphone[5,6].

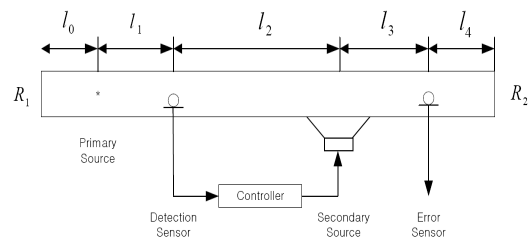


Fig. 1. Active Noise Canceling Systems for Mufflers

The microphone used to measure the error signal is located at a distance of  $l_3$  from the secondary sound source and  $l_4$  from the end of the muffler, and  $R_1$  and  $R_2$  are the sound wave reflection coefficients at both ends of the muffler.

Assuming that the major electro-acoustic transfer functions of Fig.1 are linear, it can be expressed as a superimposed model as Fig. 2.

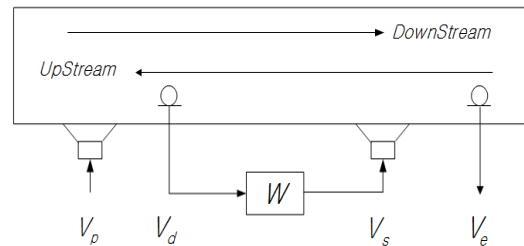


Fig. 2. Superposition Models for Active Mufflers

Fig. 1. is can be modeled into an acoustic system with two electrical inputs (ie, used to drive voltages  $V_p$  and secondary sources  $V_s$ ) and two electrical outputs (ie, output from the error sensor  $V_e$  from the detection sensor  $V_d$ ), as shown in Fig. 2. Assuming that the principle of superposition is established for each electric transfer function and the input voltage of the other side is set to 0 similarly to the 4 terminal network, the ratio of each input and output can be defined as Eq. (1).

$$P \cong \left[ \frac{V_e}{V_d} \right]_{V_s=0}, A \cong \left[ \frac{V_d}{V_p} \right]_{V_s=0} \quad (1)$$

$$S \cong \left[ \frac{V_e}{V_s} \right]_{V_p=0}, B \cong \left[ \frac{V_d}{V_s} \right]_{V_p=0}$$

In Eq. (1), transfer function B represents the acoustic feedback path. Since the transfer function of the controller is the principle of superposition, assuming all the components of the system (acoustic, electrical and electro-acoustic) to be linear, the two output voltages will have the form as Eq. (2).

$$V_e = PV_p + SV_s \quad (2)$$

$$V_d = AV_p + BV_s$$

Assuming that the transfer function of the controller W is linear in Fig. 2., block diagram of superposition models can be represented by Fig. 3.

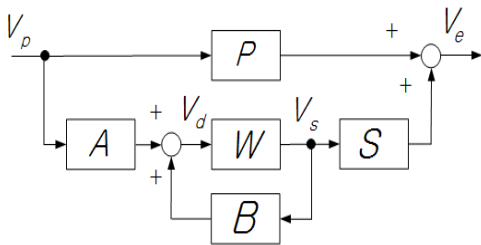


Fig. 3. Block Diagram of Superposition Models

In order to convert the transfer functions P, A, B and S to a discrete system, For example, Primary transfer function is treat the term for the net delay time as integer and replace the delay time with the backward difference operator  $z^{-1}$  as Eq. (2). In Eq. (2), the delay time coefficients are affected not only physical distance but also speed of sound  $c$  and sampling frequency  $w$ .

$$P(z^{-1}) = \frac{z^{-da_1} + R_1 z^{-da_3} + R_2 z^{-da_2} + R_1 R_2 z^{-da_4}}{(1 - R_1 R_2 z^{-ds_2})}$$

where  $(2)$

$$l = l_0 + l_1 + l_2 + l_3 + l_4$$

$$ds_2 = (l * w / c)$$

$$da_1 = (l_1 + l_2 + l_3) * w / c$$

$$da_2 = (l_1 + l_2 + l_3 + 2 * l_4) * w / c$$

$$da_3 = (2l_0 + l_1 + l_2 + l_3) * w / c$$

$$da_4 = (l + l_0 + l_4) * w / c$$

### 3. Design of Adaptive Filter

The adaptive filter algorithm shown in Fig. 4. which has a main purpose in achieving the control objective by estimating unknown system coefficients.

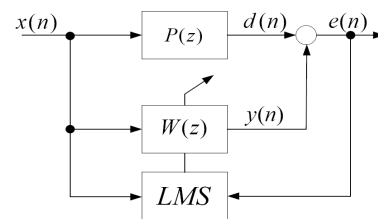


Fig. 4. Adaptive Filter Algorithms

$$W(n+1) = W(n) + \mu(n) e(n) X(n) \quad (3)$$

$$\mu(n) = \frac{v(n)}{(L + 1)} \quad (4)$$

where  $W(n)$  is LMS(Least Mean Square) control vector and  $X(n)$  is input signal vector,  $\alpha$  is arbitrary constant, L is Filter Tap

In Eq. (3),  $x(n)$  and  $d(n)$  are primary source and primary path output signal, and  $y(n)$  and  $e(n)$  are controller output, and control error signals of the system, respectively. In Eq. (4), the proper selection of the convergence factor  $\mu(n)$  to determine the control filter vector determines the performance of the system. There have been many studies to determine the optimal convergence factor, And the NLMS (Normalize LMS) algorithm that takes the norm of the input signal vector and reflects it on the convergence coefficient. Since the NLMS algorithm has a variable constellation coefficient, its proper choice depends on the performance of the system.

### 3.1 Filtered\_x LMS algorithms

In Fig. 4., the control output  $y(n)$  is physically generated through the secondary sound source from the control speaker, and is canceled with the primary sound source, which is referred to as a secondary path. Since the primary and secondary path transfer function are unknown or time-varying in many cases, it is necessary to obtain the optimal value by real-time estimation using the adaptive filter. Considering the existence of the secondary path, appropriate solution of control filter is called Filtered\_x LMS algorithm (in Fig. 3-2,  $y^*(n)$  is the secondary path output,  $S^{\wedge}(n)$  is pre-estimated secondary transfer function)[7].

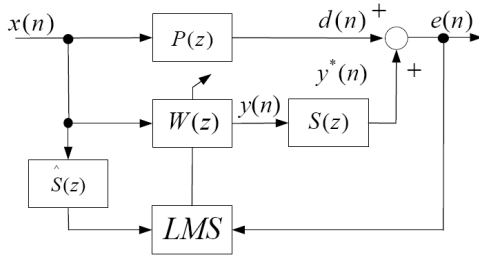


Fig. 5. Filtered\_x LMS Algorithms

### 3.2 Proposed algorithms

In the case of the acoustic feedback as shown in Fig. 5., since the order of the control filter should be very large because of the influence of the pole location of transfer function B in Eq. (2), Filtered\_x LMS algorithm has a disadvantage in that the order of control filter is very increased and the convergence speed is slowed down[4].

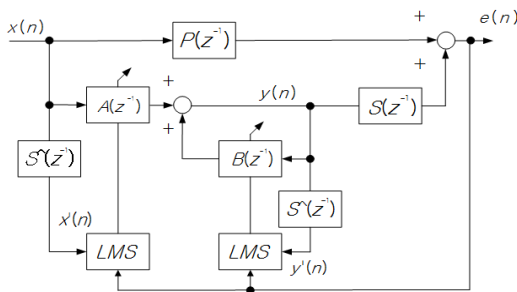


Fig. 6. Filtered\_U LMS Algorithms

Adaptive IIR filters structure which to solve above problems, its called recursive LMS (RLMS) algorithm or Filtered\_U LMS algorithm.(Fig. 6.)

In Eq. (3) shows the Filtered\_U LMS algorithms.

$$w(n+1) = w(n) + \mu u'(n)e(n) \tag{5}$$

where

$$u'(n) \equiv S^{\wedge}(n)*u(n)$$

Eq. (5) can be partitioned into two vector equations for adaptive filters  $A(z^{-1})$  and  $B(z^{-1})$  as follows.

$$\begin{aligned} a(n+1) &= a(n) + \mu x'(n)e(n) \\ b(n+1) &= b(n) + \mu y'(n-1)e(n) \end{aligned} \tag{6}$$

where

$$x'(n) = s^{\wedge}(n)*x(n)$$

$$y'(n-1) = s^{\wedge}(n)*y(n-1)$$

Even though IIR structure is more efficient than FIR structure, RLMS algorithms are may have stability problems especially when the adaptive algorithm for adaptive filters is not yet converged.

In this paper, we propose an active muffler system designed with IIR filter structure for the control structure of the primary and secondary-paths so that the convergence performance and computational burden of the Filtered\_x LMS algorithm are improved[8].

Eq. (6) shows stabilized procedure that before the RLMS algorithms converge, poles of the IIR filter are pulled to the center of the unit circle, and the poles are returned to their original positions after the filter converges. Acoustic feedback transfer function  $B(z^{-1})$  in Fig 6. is modified as

$$B(z^{-1}) = 1 + m(n)b_1z^{-1} + m^2(n)b_2z^{-2} + \dots + m^n(n)b_nz^{-n} \tag{7}$$

where

$$m(n) = 1 - e(-\alpha n/1000), 0 \leq \alpha \leq 5$$

Stabilized coefficient  $\alpha$  is determined by adaptation speed.

### 4. Computer Simulation

In order to show the usefulness of the proposed IIR type active muffler system, computer simulation was performed. Table 1, 2 shows the physical specifications of the muffler and common simulation parameters[9].

Table 1. Muffler Parameters

Variable	Symbol	Value
Reflection Coefficient	$R_1, R_2$	0.2, 0.1
Physical Length	$l_0$	0.3[m]
	$l_1$	0.5[m]
	$l_2$	1.9[m]
	$l_3$	0.5[m]
	$l_4$	0.3[m]

Table 2. Simulation Parameters

Variable	Value	
Sound Speed	340[m/s]	
Sampling Freq.	2[kHz]	
Execution Step	3,000[1.5Sec]	
Noise Source	Diesel Muffler Noise	
Secondary Path	Filtered_x	96
	IIR Stabilized	32+8
Control Filter	Filtered_x	192
	IIR Stabilized	96
Convergence Coefficient	Filtered_x	0.00005
	IIR Stabilized	0.003
Stabilized Coefficient	$1 - e^{-\frac{\alpha n}{2000}}$	$\alpha = 5$

The simulation target transfer functions of Eq. (8) are obtained by discrete transfer function modeling method of Eq. (2).

$$P(z^{-1}) = \frac{z^{-17} + 0.5z^{-21} + 0.06z^{-24}}{1 - 0.02z^{-21}} \tag{8}$$

$$S(z^{-1}) = \frac{z^{-3} + 0.2z^{-35} + 0.1z^{-6} + 0.02z^{-38}}{1 - 0.02z^{-21}}$$

$$A(z^{-1}) = \frac{z^{-3} + 0.2z^{-6} + 0.3z^{-35} + 0.02z^{-38}}{1 - 0.02z^{-21}}$$

$$B(z^{-1}) = \frac{z^{-11} + 0.3z^{-21} + 0.02z^{-30}}{1 - 0.02z^{-21}}$$

Fig. 8. shows the time evolution of exhaust noise when the diesel engine was varied from 1000 to 3000[RPM]. The z-axis shows the three-dimensional distribution of sound pressure level (SPL). It can be

seen that the frequency characteristics near 200, 300, and 400[Hz] vary with time.

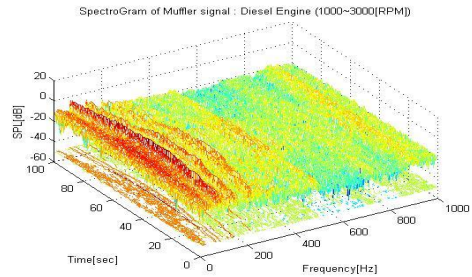
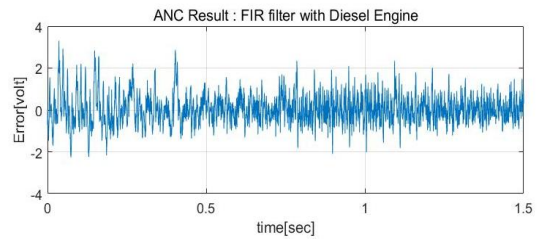
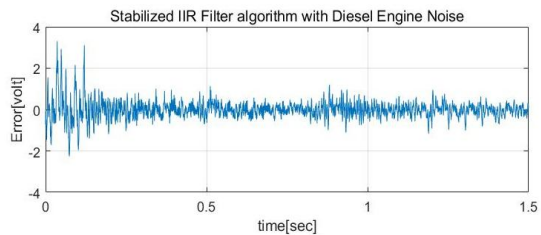


Fig. 8. Characteristics of Diesel Engine Noise Signal with Variable State

Fig. 9. shows the performance comparison with the proposed IIR type active muffler systems in the variable frequency situation of Fig. 8., and although the proposed algorithm reduces the control filter and the second-order path filter order of the control algorithm as shown in Table 2. The convergence and the noise attenuation performance are superior to those of the conventional Filtered\_x LMS algorithm. Compared to conventional algorithm, proposed algorithms's computational burden is less than half, and convergence performances are more than 4 times.



(a) Conventional Filtered\_x LMS Algorithms



(b) Proposed IIR type Filtered-U LMS Algorithms

Fig. 9. Comparison of Proposed Algorithms VS. Conventional Algorithms in Active Muffler System

## 5. Conclusion

The LMS algorithm applied in the adaptive filter algorithm of the active muffler is mainly composed of the FIR filter structure, so that there is a problem that the degree of the control filter becomes large and the convergence speed is slowed down when acoustic feedback exists. The corresponding IIR filter structure is difficult to use due to the stability problem of the adaptation process. In this paper, we propose an active muffler system with IIR structure using a stabilization algorithm and show its usefulness through computer simulation.

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<Research Interest>

Signal Processing, Adaptive Filter, Active Noise Control, DSP algorithm, Micro-Controller applications

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<Research Interest>

Digital Signal Processing, Active Noise Control, Instrumentation and Control