International Journal of High-Rise Buildings
June 2020, Vol 9, No 2, 127-154
https://doi.org/10.21022/IJHRB.2020.9.2.127

International Journal of
High-Rise Buildings
www.ctbuh-korea.org/ijhrb/index.php

Review on Quantitative Measures of Robustness for Building Structures Against Disproportionate Collapse

Jian Jiang[1], Qijie Zhang[1], Liulian Li[2], Wei Chen[1], Jihong Ye[1], Guo-Qiang Li[3,†]

[1] *Jiangsu Key Laboratory of Environmental Impact and Structural Safety in Engineering, China University of Mining and Technology, Xuzhou 221116, China*
[2] *The First Construction Engineering Company Ltd. of China Construction Second Engineering Bureau, Beijing 100176, China*
[3] *State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China*

Abstract

Disproportionate collapse triggered by local structural failure may cause huge casualties and economic losses, being one of the most critical civil engineering incidents. It is generally recognized that ensuring robustness of a structure, defined as its insensitivity to local failure, is the most acceptable and effective method to arrest disproportionate collapse. To date, the concept of robustness in its definition and quantification is still an issue of controversy. This paper presents a detailed review on about 50 quantitative measures of robustness for building structures, being classified into structural attribute-based and structural performance-based measures (deterministic and probabilistic). The definition of robustness is first described and distinguished from that of collapse resistance, vulnerability and redundancy. The review shows that deterministic measures predominate in quantifying structural robustness by comparing the structural responses of an intact and damaged structure. The attribute-based measures based on structural topology and stiffness are only applicable to elastic state of simple structural forms while the probabilistic measures receive growing interest by accounting for uncertainties in abnormal events, local failure, structural system and failure-induced consequences, which can be used for decision-making tools. There is still a lack of generalized quantifications of robustness, which should be derived based on the definition and design objectives and on the response of a structure to local damage as well as the associated consequences of collapse. Critical issues and recommendations for future design and research on quantification of robustness are provided from the views of column removal scenarios, types of structures, regularity of structural layouts, collapse modes, numerical methods, multiple hazards, degrees of robustness, partial damage of components, acceptable design criteria.

Keywords: Robustness, Disproportionate collapse, Quantitative measure, Review, Deterministic, Probabilistic

1. Introduction

The study of disproportionate collapse under extreme or accidental loads can trace back to the 1940s, when Baker (1948) studied how buildings collapsed under bombing in London during the Second World War. Past design and research work for disproportionate and/or progressive collapse of structures has proceeded in waves initiated in the aftermath of best-known events, particularly the Ronan Point failure 1968 (Pearson and Delatte 2005), Murrah Federal Building bombing 1995 (Corley et al. 1998) and the World Trade Center collapse 2001 (NIST 2008). Disproportionate collapse is a relatively rare event as it requires both an abnormal load to initiate a local damage and a structure that lacks adequate continuity, ductility and redundancy to resist the spread of failure. However, for high-rise building structures (Chung et al. 2016; Zhou and Liu 2019), the consequence and hence the risk of collapse is immense (Li et al. 2018a; Chung and Yoo 2019). Therefore, it is of great importance to understand the collapse behavior of tall structures, and moreover to develop reliable and efficient methodologies to arrest collapse. It is generally recognized that increasing robustness is the most acceptable and effective method.

All buildings are vulnerable to disproportionate collapse in varying degrees (Ellingwood and Leyendecker 1978; Nair 2006; NIST 2007; Kwon et al. 2012), and it has been widely accepted that it is not safe to assume that a structure designed for normal conditions will withstand abnormal or accidental conditions (Ha et al. 2017). Therefore, in the last 20 years, great advances in design and research have been made on collapse resistance of building structures, supported by advanced computational tools. A detailed comparison of available design codes, standards and guides (GSA 2005; ASCE 7 2005; JSSC 2005; DoD 2013; EN 1991-1-7 2010) can be found in references (Dusenberry 2002; Ellingwood and Dusenberry 2005; Ellingwood 2006; Mohamed 2006; Nair 2006; Starossek 2006; Stevens et al. 2011; Marchand and Stevens 2015; Russell et al. 2019). The comparison highlights an apparent discrepancy between the observed structural disproportionate collapse of buildings and current guidelines to mitigate such outcomes. Research

---

[†] Corresponding author: Guo-Qiang Li
Tel: +86-21-6598-2975, Fax: +86-21-6598-3431
E-mail: gqli@tongji.edu.cn

advances have been summarized extensively in review papers (Starossek 2007; Agarwal et al. 2012; El-Tawil et al. 2013; Byfield et al. 2014; Qian and Li 2015; Adam et al. 2018; Jiang and Li 2018; Kunnath et al. 2018; Abdelwahed 2019; Bita et al. 2019; Russell et al. 2019; Subki et al. 2019; Stochino et al 2019; Azim et al. 2020; Kiakojouri et al. 2020), and academic books (Starossek 2009, 2018; Fu 2016; Isobe 2017).

To defend buildings against unpredictable accidental events, the general consensus in the structural engineering community is to ensure robustness so that spread of the initial damage is controlled and disproportionate collapse is ultimately prevented. A quantitative measure would be useful to examine the performance of a structure in terms of its robustness. The growing acceptance of performance-based design is accompanied by a need for evaluation, optimization and regulation of robustness under a set of extreme natural and man-made events. To achieve these tasks, robustness measures must meet the requirements of being expressive, objective, simple, calculable, and generally applicable (Starossek and Haberland 2011), which cannot all be satisfied to the same level at the same time. To date, the concept of robust structures is still an issue of controversy since there are no well established and generally accepted criteria for a consistent definition of robustness, which hinders development of quantitative measures of structural robustness. There are some review papers related to quantification of structural robustness (Ghosn et al. 2016; Zio 2016; Adam et al. 2018). However, they focus on eigher structural members, indicators (qualitative concept) of robustness, or a broad subject of civil engineering and infrastructure networks, but are not directly applicable for quantifying robustness of buildings which is the objective of this paper.

This paper presented a comprehensive review on quantitative measures (rather than indicator) of robustness for buildings against disproportionate collapse, especially focusing on steel framed buildings. It was started by recalling the definition of robustness (Section 2) and its difference from similar terms such as collapse resistance, vulnerability and redundancy. A classification of the existing quantitative measures of robustness was presented in Section 3. A detailed overview was given on the classified structural attribute-based measures in Secton 4, and structural performance-based deterministic and probabilistic measures in Section 5. Finally, the main issues and recommendations are provided in Section 6. This paper focused on steel framed structures, however, noting that similar findings are also applicable to reinforced concrete structures.

2. Definition of Disproportionate Collapse and Robustness

2.1. Disproportionate Collapse vs Progressive Collapse

It is necessary to first distinguish between "disproportionate collapse" and "progressive collapse" (Adam et al. 2018), as illustrated in Figure 1. The former concept was first described in the UK code (Building Regulation 2010) as "the building shall be constructed so that in the event of an accident the building will not suffer collapse to an extent disproportionate to the cause", while the latter was first systematically discussed in the US design guidelines (GSA 2005) as defined in ASCE 7 (2005) "the spread of an initial local failure from element to element, eventually resulting in the collapse of an entire structure or a disproportionately large part of it". A list of existing definitions of disproportionate/progressive collapse is shown in Appendix A. No matter what is the definition of disproportionate or progressive collapse, it indicates that large displacements, even failure of individual structural members are acceptable provided that structural collapse is prevented.

These two terms are often, mistakenly, used to be synonymous. This is partly because disproportionate collapse often occurs in a progressive manner, and progressive collapse can be disproportionate. However, a collapse may be progressive in nature but not necessarily disproportionate in its extents, for example if a collapse is arrested after it progresses through a limited extent of structural bays. Vice versa, a collapse may be dispro-

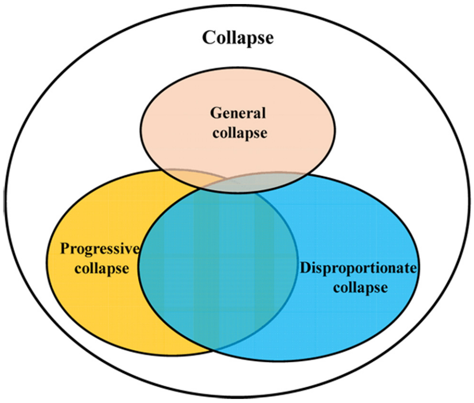


Figure 1. Relation between Disproportionate, Progressive and General Collapse.

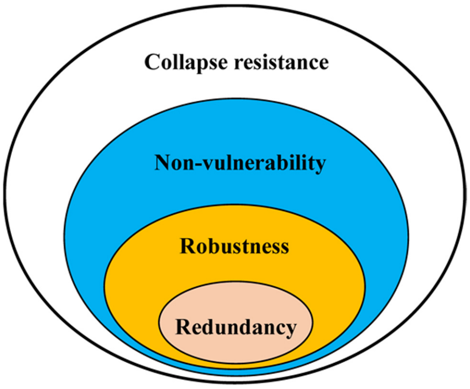


Figure 2. Relation between Robustness and Collapse Resistance, Vulnerability, Redundancy.

portionate but not necessarily progressive if the collapse is confined to a single but large structural bay. The Murrah Building and WTC 1 and 2 are progressive collapse but cannot be reasonably labeled as disproportionate collapse since the initial explosion or impact is also very large (Nair 2006). In other words, a disproportionate collapse is only a judgement on the consequences of the damage, while a progressive collapse describes the characteristics and mechanism of collapse behavior. Therefore, dispro-portionate collapse is more appropriate in the context of design to accommodate specified design objectives, while progressive collapse is more suitable in the context of research when referring to the physical phenomenon and mechanism of collapse. Moreover, disproportionate collapse must not be confused with general or global collapse due to strong earthquake, wind, hurricane, etc. The former is usually with respect to abnormal loads such as blast, impact, and fire where the initial damage is localized, while the latter involves a global series of simultaneous failure. There are some exceptions that earthquake may cause the damage of several corner columns of a building, and thus there is some overlap between general collaspe and disproportionate/progressive collapse as shown in Figure 1. The term "disproportionate collapse" is used herein to facilitate a review on design-oriented application of robustness.

2.2. Definition of Robustness

The term "robustness" has been well-defined in various fields beyond structural engineering (e.g., software engineering, quality control, design optimization, medicine, social sciences or finances). For example, robustness in quality control is defines as the degree to which a system is insensitive to the effects that are not considered in design. A robust statistical technique is insensitive to small deviations in the assumptions. In optimization theory, a robust system is that whose performance is insensitive to uncertainties or random perturbations in design parameters. The awareness of the significance of robustness in structural engineering has intensified over the years, but robustness is still an issue of controversy for its definition and quantification despite substantial research and useful recommendations. This poses difficulties with regard to interpretation as well as regulation (Faber 2006).

A list of available definitions of robustness in design codes and research studies is provided in Appendix B. A definition is given in EN 1991-1-7 (2010) as "the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause". A similar definition is provided in the U.S. General Services Administration (GSA) that describes robustness as the ability of the structure to resist failure "due to its vigorous strength and toughness". The workshop on robustness of structures held by Joint Committee on Structural Safety (JCSS) in 2005 (Faber 2006) concluded that "robustness is broadly recognized to be a property which can not only be associated with the structure itself but must be considered as a product of several indicators: risk, redundancy, ductility, consequences of structural component and system failures, variability of loads and resistances, dependency of failure modes, performance of structural joints, occurrence probabilities of extraordinary loads and environmental exposures, strategies for structural monitoring and maintenance, emergency preparedness and evacuation plans and general structural coherence".

By summarizing these definitions, robustness is generally defined as insensitivity of a structure to its initial damage or local failure, i.e. a structure is robust if an initial damage does not lead to disproportionate collapse. In the literature, a large number of similar terms with various meanings are used associated with robustness, such as collapse resistance, vulnerability, redundancy, reliability, integrity, damage tolerance, resilience, stability, ductility, toughness, susceptibility, fragility, etc. (Starossek et al. 2011). A distinction between robustness and the first three terms (the most confusing ones) is presented in this section to help understand the meaning of robustness, and to provide a basis for various quantifications as presented in the next sections.

2.2.1. Robustness vs Collapse Resistance

Robustness is associated with the probability use [ ], i.e. P[CollapseD] and collapse resistance (the short form for resistance against disproportionate collapse) with the probability use [ ] (Starossek and Haberland 2010), as shown in Figure 3 (*H* is the abnormal event; *D* is local damage; P[] is probability). This indicates that collapse resistance can be influenced in various ways, and one possibility is through the structural robustness. In other words, a robust structure is collapse resistant but not vice-versa. Robustness is a property of the structure alone and independent of the possible causes and probabilities of the initial local failure, while collapse resistance is a property dependent on both structural features and the possible causes of initial local failure. A robust structure is collapse resistant because an initial damage does not spread dispropor-tionately, being achieved by alternative load paths or segmentation (isolation of a failure in a segment). A non-robust structure can be made collapse resistant by reducing the vulnerability of structural components to weaken or prevent initial damage. A disproportionate collapse can be prevented by providing robustness (system behavior) or by reducing the exposure to abnormal actions (event control). This is in contrast to a broader definition of robustness as given in EN 1991-1-7 (2010), where robustness is referred to insensitivity to abnormal events rather than initial damage. In a word, the collapse resistance is a broader concept, involving robustness of the global structure, resistance of local components and hazard events.

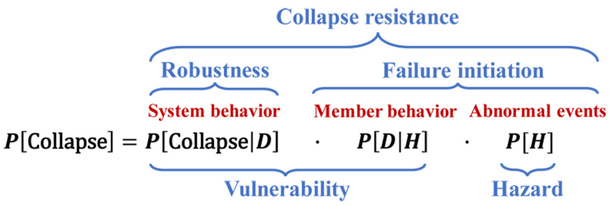


Figure 3. Definitions in the Context of Disproportionate Collapse.

2.2.2. Robustness vs Vulnerability

Basically, vulnerability is not the antonym of robustness. Vulnerability decribes the sensitivity of a member or a structure to damage events, rather than induced local damage as used in robustness. On one hand, robustness is related to system behavior, while vulnerability refers to either member or system behavior (Starossek and Haberland 2010), as shown in Figure 3. On the other hand, robustness is a quality of the structural system alone, and is independent of the cause of the damage, while vulnerability is used for evaluating the consequences of a given hazard (typically floor damage extent, economic losses or number of fatalities), considering the type of action and the structural response to that action (Felipe et al. 2018). The vulnera-bility of a structure will vary between different hazards, e.g. a structure may be vulnerable to vehicle impact but not to seismic loading. However, the robustness of structures should work for both hazards.

2.2.3. Robustness vs Redundancy

Structural robustness the robustness and redundancy are often used inter-changeably as synonymous terms. However, they indicate different properties of a structural system. Structural redundancy is defined as the capability of the structural system to redistribute the loads among its members through alternate loading path. In general, redundancy tends to enable a more robust structure and to ensure that alternate load paths are available by means of structural ties, strength and ductility. This means structural robustness strongly depends on redundancy. Alternatively, if alternate paths are not available, the robustness of structues can also be ensured by introducing a discontinuity in the structure (segmentation) or by designing some critical elements to resist the extreme event (key element design). In this context, continuity (tie strength), redundancy (alternate load path), ductility, segmentation, energy absorption capacity are identified as the means of accomplishing robustness. In addition, structure redundancy is related to a specific and prospective damage scenaro, while robustness is a comprehensive measure based on a series of possible damage scenarios. The lower the structural redundancy, the worse the robustness of a structure.

3. Classification of Quantitative Measures of Robustness

Robustness refers to the ability of a structure to resist to either abnormal events or an initial damage without disproportionate collapse. Before quantification, it is necessary to first distinguish three common terms: indicator, measure, index. An indicator describes the satisfication or not to certain conditions and circumstances, which is a qualitative or quantitative description of a property. The other two are used for quantitative description of a property. An index is a ratio for the change of values of a quantity, used for quantitative description. In a word, an indicator is the most general form used to describe a property, and each measure can also be regarded as an indicator and an index is a special form of a measure.

Measures of robustness should be linked to a series of requirements such as expressiveness, objectivity, simplicity, calculability, and generality (Lind 1995; Starossek 2018). These requirements are partly in conflict with each other, and it may not be possible to meet them all to the same level at the same time. For example, it is possible to achieve strong expressiveness but at the cost of calculability. Therefore, the above requirements may have to be limited. There are other requirements for a quantitative measure of robustness, such as that it should be a decision-making tool for design or redesign, valid to actually express the tolerance of damage, reliable to distinbuish different damages among different systems, and reproducible. Any candidate for a measure of robustness should be assessed against these requirements.

To quantify robustness it is necessary to consider possible scenarios of abnormal events, structural collapse, and collapse-induced consequences. However, there is no general rule to quantify the robustness of structures. Starossek and Haberland (2008) divided the quantifications into structural attribute-based and structural behavior-based measures. Sørensen (2011) and Chen et al. (2016b) divided the robustness measures of structures divid into three categories with decreasing complexity: risk-based, reliability or probability-based and deterministic performance-based measures. Adam et al. (2018) classified the quantificaiton of robustness into threat-dependent and threat-independent methods. The former included all reliability/risk-based measures and some deterministic measures. The structural attribute-based measures belonged to threat-independent group. Lin et al. (2019) classified robustness assessment methods into probabilistic and deterministic theory. The probabilistic methods were further divided into probability redundancy-based and vulnerability-based methods.

In this paper, the existing quantitative methods were classified into two groups: structural attribute-based measures and structural performance-based measures, as shown in Figure 4. The former is evaluated based on the structural topology (geometry, configuration, connectivity of structures) and structural stiffness. Based on whether considering the load, it can be further classified into two categories: change to attribute alone and change to attribute and load. The structural performance-based methods

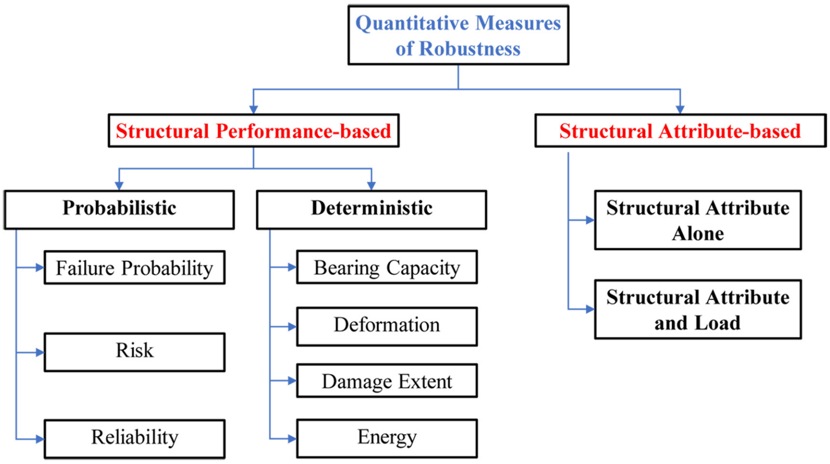


Figure 4. Classification of the Existing Measures of Structural Robustness.

involve structural behaviour such as bearing capacity, ductility, redundancy, energy dissipation, probability of failure, risk, reliablity, which can be further divided into deterministic and probabilistic measures. The deterministic measures can be further divided into bearing capacity, deformation, damage extent, and energy-based measures. They focus on the responses of the system against collapse, without considering the randomness in abnormal events, system properties and consequences. In contrast, the probabilistic measures can be further classified into failure probability-based measures (involving randomness of abnormal events and structural system), risk-based measures (involving randomness of abnormal events (causes) and consequences) and reliability-based measures (involving randomness of structural system alone).

4. Review on Structural Attribute-based Quantitative Robustness Measures

The structural attributes include structural topology and structural stiffness. The former represents the configuration or hierarchical model of structural components and their connectivity, while the latter involves important information of the structure such as geometry dimension, member sizes, material properties, etc. The stiffness matrix can be easily computed for simple structures and its properties can be used to indicate the failure behavior of structural systems. Based on whether the loading condition was considered in the quantification, the structural attribute-based measures of robustness were further divided into attribute alone-based and attributed and load-based measures, which are presented in the following subsections.

4.1. Attribute Alone-based Robustness Index

A structural vulnerability theory was proposed by researchers from University of Bristol (Agarwal et al. 2001, 2003; Blockley 2002; Pinto et al. 2002), aiming at indentifying the possible failure modes and assessing the system vulnerability focusing only on the structural topology and stiffness. The topology of a structure was represented by a hierarchical model of clusers as a container of mimimum resisting substructures (“structural ring” or “structural round”). The sequence of grouping members into clusters was determined by ensuring an increasing “well-formedness” of the target cluster. The well-formedness Q of a structural round in Eq. (1-1) is a measure of the quality of the structural form, which is the mean value of the well-formedness q_i of each node in the round as expressed in Eq. (1-2) or Eq. (1-3).

$$Q = \frac{1}{N}\sum q_i \tag{1-1}$$

$$q_i = \det(k_{ii}) \quad \text{(for a static system)} \tag{1-2}$$

$$q_i = \det(k_{ii} + \omega_n^2 M_{ii}) \quad \text{(for a dynamic system)} \tag{1-3}$$

where Q and q_i is the well-formedness of a structural round and a node in the round, respectively; N is the total number of the connections in the round; K_{ii} is and M_{ii} is the stiffness and mass sub-matrix associated with any node i in the round, respectively; ω_n is the this superscript natural frequency of the structure; det() means determinant calculation.

This concept of well-formedness was used to identify failure scenarios (sequence of damage events) by three quantitative measures: separateness γ in Eq. (2-1) (ratio of well-formedness loss of the damaged structure to well-formedness of the intact structure), relative damage demand D_r in Eq. (2-2) (ratio of damage demand D of a failure scenario to the maximum possible damage demand D_{max} of the system), vulnerability index φ in Eq. (2-3) (ratio of separateness to relative damage demand). The damage demand is a measure of the effort required to cause damage, which is proportional to the loss of structural stiffness due to a damage. The separateness is a measure of failure consequence, with a zero value representing no collapse and a unity value representing total collapse.

While the vulnerability index is a measure of the disproportionateness of the consequences (γ) to the damage (D).

$$\gamma=\frac{Q(S)-Q(S')}{Q(S)} \tag{2-1}$$

$$D_r=\frac{D}{D_{max}} \tag{2-2}$$

$$\varphi=\frac{\gamma}{D_r} \tag{2-3}$$

where $Q(S)$ and $Q(S')$ is the well-formedness of the intact structure S and the damaged structure S', respectively.

Among a large number of possible failure scenarios, five important types of failure scenarios were figured out such as total, maximum, minimum, minimum demand and specific (or interested) failure scenarios. The total failure scenario requires the least effort to cause the total collapse of the structure, and the maximum failure scenario has the maximum damage consequence from the relative least effort. The minimum demand scenario has the damage event easiest to cause any damage. The specific failure scenario is the one of designer's interest. The total failure scenario has a separateness equal to one and the highest vulnerability index, while the maximum failure scenario is the one having highest vulnerability index.

Basically, this structural vulnerability theory is applicable to linear elastic analysis of framed structures, and can be used to find different levels of failure scenarios (total, maximum, minimun failure scenarios) and the key elements of structures. Further work is needed to extend this theory for other types of structures and plastic deformation. Ye and Jiang (2018) improved this theory by considering the transition from rigid connection to pinned connection through defining a rigid connection as a basic element in the traditional vulnerability theory (only components are treated as basic element). It was found that the improved method can accurately predict the damage location and collapse mode, which cannot be achieved by the traditional method.

A similar method was proposed by Liu and Liu (2005) and Gao and Liu (2008), based on the the theory of minimum of potential energy and structural stiffness. A series of unit equivalent axial force, shear force and moment were imposed at the ends of individual structural member, and the sum of the induced axial force, shear force and moment in the member was used to measure the importance of the member. A component-level coefficient of importance B_{ii} is expressed in Eq. (3). This method was proposed for truss structures and was applicable for elastic conditions, which is its limitation. The lower the importance coefficient, the higher the level of redundancy, and thus the better the robustness. This method depends on the loading path and structural stiffness distribution. However, a simple sum of these internal forces lacks theoretical basis.

$$B=a^T K^{-1} a K^n \tag{3-1}$$

$$\sum_{i=1}^{n}(1-B_{ii})=r \tag{3-2}$$

where a is the transition matrix between member internal forces and external forces; K is structural stiffness matrix; K'' is the member stiffness matrix; B is a transition matrix between member deformation terms and nondeformation terms; B_{ii} is the term on the diagonalline of matrix B, i.e. the importance coefficient of the member i; r is the redundancy factor.

Hu (2007) assessed the structural stiffness through fundamental frequency of a structure, and used the ratio of stiffness degradation after member removal as the measure index. However, use of fundamental frequence cannot account for other modal frequencies and the sequence change of structural frequencies after damage. In addition, the member contributes less to the fundamental frequency may have great contribution to structural safety.

Nafday (2008, 2011) proposed two redundancy factors denoted as system integrity distance matric δ_s (or system safety performance metric) in Eq. (4-1) and system integrity volume metric Δs (or degree of linear dependency in stiffness matrix) in Eq. (4-2). The former was defined as the reciprocal of the condition number κ(K) of the (n×n) stiffness matrix K, representing the shortest distance from the stiffness matric (initial state) to set of singular matrices (limit state). This is based on the concept that a singular stiffness matrix represents an unstable structure and it is desirable to have the stiffness matrix "far way" from the set of noninvertible singular matrices. The δ_s ranges between 0 and 1, with a higher value indicating more stable system. The Δs was defined as the determinant of normalized stiffness matrix K_N, ranging between 0 and 1 (higher value means higher robustness). The singularity of a stiffness matrix increased as the degree of linear dependency of its row or column vectors increased.

An importance measure I was also proposed by Nafday (2008) to represent the contribution of a structural member to system safety, as expressed in Eq. (4-3). It was defined as the ratio of the determinant of normalized stiffness matrix of the intact structure K_N to the damaged structure K_N^*, ranging from 1 to infinity. The higher the importance measure, the more critical the member for survival of the structure. A similar member consequence factor C_f^i was proposed by Nafday (2011) to search for key members, as shown in Eq. (4-4). The factor ranges from 0 to 1 with a lower value means a more critical member.

$$\delta_S = 1/\kappa(K), \delta_S = n/\|K\|\|K^{-1}\| \quad \text{(system integrity distance matric)} \quad (4\text{-}1)$$

$$\Delta S = |K_N| \quad \text{(system integrity volume metric)} \quad (4\text{-}2)$$

$$\mathrm{I} = \frac{K_N}{K_N^*} \quad \text{(Importance measure)} \quad (4\text{-}3)$$

$$C_f^i = |K_N^i| / |K_N| \quad \text{(member consequence factor)} \quad (4\text{-}4)$$

Where κ is the condition number of a matrix; $\|\cdot\|$ represents Euclidean matrix norm; K_N^i is the normalized stiffness matrix of the damaged structure after removal of the i^{th} member.

Starossek and Haberland (2011) suggested a similar stiffness-based measure R_s of robustness in Eq. (5) by comparing the stiffness matrix K_0 of the intact structure and that K_j of the damaged structure after removing a structural element or a connection j. This measure can be normalized to be in a range between 0 and 1, where a value of one represents the maximum possible robustness, while zero corresponds to a total lack of robustness. This index has a low level of expressiveness. Furthermore, the reduction in load-bearing capacity due to member removal is not well correlated with R_s, and thus R_s should be regarded as a connectivity measure instead of a complete robustness measure. This is because system connectivity (redundancy) provides only a partial contribution to robustness.

$$R_S = \min_j \frac{\det(K_j)}{\det(K_0)} \quad (5)$$

Generally, the above topology and stiffness-based vulnerability methods have the advantages of simplicity and ease of calculation. However, they are applicable to identify weaknesses in a structure at an early stage in the design process (elastic analysis), rather than being a substitute for a full dynamic analysis under extreme actions. The matrix determinant-based method lacks clear physical meaning. Further work is needed to extend them to account for elastoplastic behaviour and dynamic effects due to sudden failure.

4.2. Attribute and Load-based Robustness Index

The effective load path depends on not only the distribution of geometry and stiffness, but loading action. The above methods in Section 4.1 fail to consider the effect of member failure on the load redistribution path. In fact, the structural stiffness matrix used in the robustness measurement excluding load is not the effective stiffness of a structure to resist the load, where a member contributing less to the loading path may have a higher importance than that contributing more to loading path. Therefore, attribute and load-based methods have been proposed.

England et al. (2008) exteneded Eq. (1) for specific loading conditions. The potential for damage propagation was examined through a new measure of hazard potential H in Eq. (6). Hazard potential is a measure of the potential for the progression of damage through a particular failure scenario under a set of loads. For a set of failure scenarios in a structure, the lowest hazard potential denotes the most vulnerable failure scenario.

$$H_i = \frac{U_i/U_0}{F_i/F_0} \quad (6)$$

Where U_i and F_i represent strain energy and well-formedness of a structure after the i^{th} event, respectively; U_0 and F_0 correspond to the undamaged state.

Ye et al. (2010) used generalized structural stiffness to measure the importance of members. The generalized structural stiffness K_stru can be calculated from the system internal energy U given a generalized loading force F_max, as given in Eq. (7-1). The measure index I was defined in Eq. (7-2) as the ratio of damage-induced loss of generalized structural stiffness to that of the intact structure. This index has a clear physical meaning for elastic behavior of structures that the importance of a member is measured by its contribution to generalized structural stiffness. Note that the generalized structural stiffness depends on both the structural topology and load.

$$K_{stru} = \frac{1}{2} F_{max}^2 \frac{1}{U} \quad (7\text{-}1)$$

$$I = \frac{K_{stru,0} - K_{stru,f}}{K_{stru,0}} \quad (7\text{-}2)$$

Where $K_\text{stru,0}$ and $K_\text{stru,f}$ is the generalized structural stiffness of the intact and damaged structure, respectively. The index I ranges between 0 and 1. The larger the index, the more important the member.

The above structural attribute-based methods (with and without load) consider robustness as a fixed property of the system in terms of structural configuration (topology) and stiffness. These stiffness-based indices are applicable to the performance objective of stability (rather than bearing capacity or deformation limit), and and are more applicable for for an indicator rather than a measure for structural robustness since the algebraic properties of structural stiffness matrix lack clear physical meaning, which is difficult to be used in practical design. Most attribute-based methods deal with individual members and their impact on the performance of the system. They are straightforward for simple structures like truss structures. However, in dealing with complex structural system, different aspects should be considered including location of each individual component, safety level of each member, stiffness sharing of each member, and material behavior of each member, etc. In this case, the quantification of the importance of all components and ranking them in a proper order should be addressed on a system performance perspective but not on individual component checking.

5. Review on Structural Performance-based Quantitative Robustness Measures

5.1. Deterministic Robustness Index

Deterministic structural performance-based methods define robustness as the ratio of some specific structural properties (capacity, deformation, damage or energy) of undamaged (or intact) and damaged structures. They are physically meaningful and convenient to calculate. However, the inherent randomness involved in the structural properties and external loads cannot be reflected, which may greatly affect the robustness of the structures. The following subsections present the robustness measures based on bearing capacity, deformation, damage extent and energy flow.

5.1.1. Bearing Capacity-based Robustness Index

Based on redundancy and robustness of bridge structures (Frangopol and Nakib 1991; Ghosn and Moses 1998; Wisniewski et al. 2006; Ghosn et al. 2010), three deterministic capacity-based measures of robustness were proposed in Eq. (8). The robustness or redundancy was defined as the capability of a structure to continue to carry loads after the failure of the most critical member. The load factors LF_i provide deterministic estimates of critical limit states that describe the safety of a structural system, as shown in Figure 5. These load multipliers were usually obtained by performing an incremental nonlinear finite element analysis. The overall safety of the system was assessed by a redundancy factor, defined as the minimum of the three redundancy ratios. The proposed methodology is a combination of partial safety factor method and nonlinear static analysis.

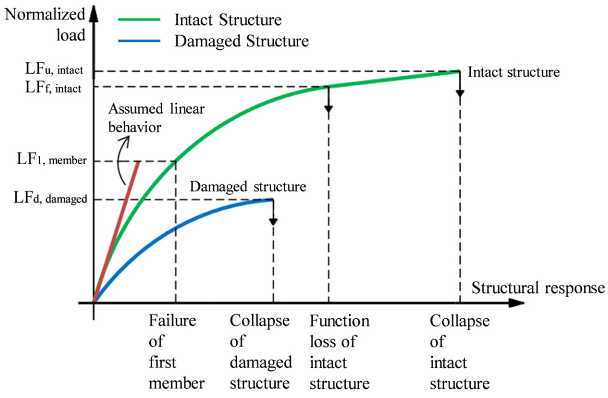


Figure 5. Definitions of Capacities of an Intact and Damaged Structure Under Different Limit States.

$$R_u=\frac{LF_u}{LF_1},R_f=\frac{LF_f}{LF_1},R_d=\frac{LF_f}{LF_1} \quad (8)$$

Where R_u is the system reserve ratio for the ultimate limit state of the intact structure; R_f is the system reserve ratio for the functionality limit state of the intact structure; R_d is the system reserve ratio for the ultimate limit state of the damaged structure; LF_u, LF_f, LF_d, are load factors of these three cases, defined as the ratio of system capacity to the applied load; LF_1 is the load ratio of the most critical member.

Similar redundant factors were proposed in Eq. (9) by Feng and Moses (1986), Frangopol and Curley (1987) as:

(1) Reserve redundant factor:

$$R_1=\frac{L_{intact}}{L_{design}} \quad (9\text{-}1)$$

(2) Residual redundant factor:

$$R_2=\frac{L_{damage}}{L_{intact}} \quad (9\text{-}2)$$

(3) Strength redundant factor:

$$R_3=\frac{L_{intact}}{L_{intact}-L_{damage}} \quad (9\text{-}3)$$

Where L_{intact} is the collapse load of the intact structure; L_{design} is the design load; L_{damage} is the collapse load of the damaged structure.

It is interesting to note that the product of R_2 and R_3 indicates whether the damaged structure will survive the design load (i.e. design load survivability, and survival implies $R_2R_3>1$).

$$R_2R_3=\frac{L_{damage}}{L_{design}} \quad (9\text{-}4)$$

Although the reserve redundant factor in Eq. (9-1) and design load survivability in Eq. (9-4) are descriptive of the collapse capacity relative to the design load, they do not quantify robustness by comparing the capacity of the undamaged and damaged structure as required by the definition of robustness. The residual redundant factor in Eq. (9-2) and strength redundant factor in Eq. (9-3) makes this comparison, but is not helpful for design purpose because it does not incorporate the design load. Fallon et al. (2016) proposed a new relative robustness index (*RRI*) to relate the capacity of the damaged structure to that of the intact structure and design load as shown in Eq. (10). A negative *RRI* indicates that the damaged structure does not meet the design load requirement. A positive *RRI* between 0 and 1 indicates that the capacity of the damaged structure exceeds the design load but is less than the capacity of the undamaged structure. A value of 1 indicates no loss of capacity because of the local damage. Eq. (10) offers a distinct improvement over the expressions in Eq. (9) and can be easily incorporated into a uniform pushdown evaluation for progressive collapse resistance. In addition, the consequence of collapse was quantified in terms of the ratio of damaged floor area to the total area of the structure, making the method available for practical design.

$$RRI=\frac{L_{damaged}-L_{design}}{L_{intact}-L_{design}}=\frac{\frac{L_{damaged}}{L_{dasign}}-1}{\frac{L_{intact}}{L_{design}}-1}=\frac{\lambda_{damaged}-1}{\lambda_{undamaged}-1} \tag{10}$$

The above methods consider the system robustness by comparing the resistance of damaged and intact structures and design load. To examine the relationship between member capacity and system capacity, Hendawi and Frangopol (1994) proposed a system safety factor *SSF* as:

$$SSF=\frac{\sum_i \bar{R}_i}{\bar{P}} \tag{11}$$

Where $\bar{R}_i$ is the mean resistance of the member *i*, and $\bar{P}$ is the mean of the total applied load.

To consider the probability of failure, a probabilistic damage factor $\overline{D.F.}$ was proposed in Eq. (12) by Frangopol and Curley (1987), where $\bar{S}_{intact}$ and $\bar{S}_{damaged}$ are the mean of the strength of the intact and damaged structure, respectively. This damage factor represents the percent reduction in the mean strength of a given member.

$$\overline{D.F.}=\frac{\bar{S}_{intact}-\bar{S}_{damaged}}{\bar{S}_{intact}} \tag{12}$$

The above factors are meaures of overall system strength for intact (R_1) and damaged structures (R_2 and R_3). A generalized redundancy was proposed by Pandey and Barai (1997) as expressed in Eq. (13) assuming that the structural redundancy was inversely proportional to the response sensitivity. This redundancy measure is quite general and is applicable to a discrete or a continuum structure. The response sensitivity of each element can be computed using the finite element method. However, the effectiveness of this method depends on the sensitivity defined in various structural responses of displacement, strain, stress, frequency, etc., leading to different degrees of redundancies. This method can be extended to probabilistic domain if the probabilistic sensitivity is available.

$$GR_j=\frac{\sum_{i=1}^{ne}\left[\frac{V_i}{S_{ij}}\right]}{V},\ j=1,2,\ldots,damage\ parameters \tag{13-1}$$

$$GNR_j=\frac{GR_j}{\max(GR_1,GR_2,\ldots,GR_{ne})} \tag{13-2}$$

Where GR_j is the generalized redundancy of the structure for j^{th} damage; GNR_j is the normalized vertion of GR_j; S_{ij} is the response sensitivity of the i^{th} element for j^{th} damage; *V* is the total volume of the structure; V_i is the volume of the i^{th} element; *ne* is the number of elements in the structure.

Similar to the above residual redundant factor in Eq. (9-2), Khandelwal and El-Tawil (2011) proposed an overload factor, defined as the ratio of failure load to the nominal gravity load of a damaged structure. The overload factor determined from pushdown analyses with different load imposing schemes together with different collapse modes were used to measure robustness of structures.

Coefficients of importance of components (or member sensitivity index) were proposed (Gao 2009; Huang and Li 2012; Choi and Chang 2009) to evaluate the robustness of structures. The coefficient of importance is defined as:

$$\eta_i=\frac{\lambda_0-\lambda_i}{\lambda_0}\ \text{or}\ \gamma_i=\frac{R_0-R_i}{R_0} \tag{14}$$

Where η_i or γ_i is the coefficient of importance; λ_0 is the capacity ratio of the intact structure, defined as the ratio of the ultimate loading capacity to the applied load; λ_i is the capacity ratio of the damaged structure due to the failure of the component *i*; R_0 is the ultimate loading capacity of the intact structure; R_i is the ultimate loading capacity of the damaged structure.

The lower the loading capacity of the damaged structure, the larger the coefficient of importance and the higher the sensitive of the structure to the local failure. The member having such large importance coefficient or member sensitivity is called a key member. The importance coefficient γ_i represents the ratio of the lost of loading capacity due to local failure to the initial capacity of an intact structure. Choi and Chang (2009) proposed the idea of a safety ratio φ to serve as a acceptance criterion that collapse is prevented when $\lambda_i / \lambda_0 \geq 1/\varphi$. Based on γ_i, Huang and Li (2012) proposed a robustness index I_{rob} as the product of reserve strength and residual strength, as expressed in Eq (15).

$$I_{rob}=\min_i\{k(1-\gamma_i)\} \tag{15}$$

Where *k* is the ratio of ultimate capacity R_0 to the applied load *P*. The robustness index is equivalent to the ratio of ultimate capacity of damaged structure to the applied load. If $I_{rob} \geq 1$, the robustness requirement is satisfied.

Based on the performance objective of maintaining sufficient system structural resistance under an extreme load, Maes et al. (2006) proposed a measure of robustness based on reserve strength ratio (*RSR*) defined as the ratio of the collapse load to the design load of a structure. This measure has the advantage that it can be increased only by a structural optimization aimed at maximizing damage tolerance, and not simply by enlarging the cross section of the elements.

$$R_1 = min_i \frac{RSR_i}{RSR_0} = \frac{RSR\ based\ on\ member\ i\ impaired}{RSR\ based\ on\ no\ impaired\ members} \tag{16}$$

Where RSR_i is for the damaged structure due to the failure of the member *i*; RSR_0 is for the intact (undamaged) structure. The measure R_1 is taken as the minimum of the ratios.

The above robustness measure R_1 was used by Masoero et al. (2013) to quantify the collapse resistance of structures under bending or pancake collapse mechanism. A "mechanism parameter" was defined to indicate the collapse initiation mechanism, based on the mechanical and geometric properties of a structure. It was found that R_1 was minimum for structures under bending collapse both before and after damage, while R_1 was maximum in the case of global pancake collapse.

Husain and Tsopelas (2004) proposed two redundancy indices, the deterministic redundancy-strength index r_s in Eq. (17-1) and probabilistic redundancy-variation index r_v in Eq. (17-2). The two indices were to measure the effects of element strength on the structural system strength (i.e. overall effect of redundancy). The former was defined as the ratio of mean ultimate to mean yield strength of a structure, and the latter was a function of the number of plastic hinges *n* and their average correlation coefficient $\overline{\rho}$ between their strengths. The two redundancy indices can be calculated for a specific structure and a particular loading condition by performing a static nonlinear analysis on the structure which is the limitation of this study that dynamic effects were not accounted for. The index r_v is probabilistic because it is defined as the ratio of coefficient of variation of the system strength to the member strength.

$$r_s = \frac{\overline{S_u}}{\overline{S_{nr}}} = \frac{\overline{S_u}}{\overline{S_y}} \tag{17-1}$$

$$r_v = \sqrt{\frac{1+(n-1)\overline{\rho}}{n}} \tag{17-2}$$

Where $\overline{S_u}$ is the ultimate strength or maximum resistance of the structure; $\overline{S_{nr}}$ is the strength of the same structure system as if it were nonredundant by assuming that it consists of ideal elastic-brittle elements. In such a nonredundant structure, the first yielding will lead to collapse if the strength reserves of the undamaged elements have been exhausted. Accordingly, assuming elastic-brittle behavior of structural elements, the point of first significant yielding in a structural system can be considered as a reasonable approximation of the strength of the nonredundant structure. Therefore, $\overline{S_{nr}}$ in Eq. (17-1) can be substituted by $\overline{S_y}$, which is the strength of the structural system at the point of the first "yielding". In Eq. (17-2), to evaluate the yield strength and the ultimate strength of a structural system, the average strengths of individual elements were considered during the pushover analysis. Then both $\overline{S_u}$ and $\overline{S_y}$ can be easily identified on the load-deflection curve resulting from the nonlinear pushover analysis.

Izzuddin et al. (2008) proposed a single measure of robustness in terms of system pseudo-static capacity (P_f) defined as the maximum bearing capacity by comparing the maximum dynamic displacement u_d to ductility limit. This measure provides a practical means to assess structural robustness by accounting for dynamic effect, ductility, redundancy and energy absorption capacity. This approach was also used by Vlassis et al. (2009) for progressive collapse assessment of multi-storey buildings subject to impact from an above failed floor.

$$p_f = max_{u_d}\left[\frac{1}{u_d}\int_0^{u_d} P_{nS} du\right] \tag{18}$$

Where P_{ns} is the applied load in nonlinear static analysis.

Tsai (2012) proposed an increase factor for collapse resistance defined as the ratio of brace-contributed resistance to that of unbraced frame. This factor can be related to ductility demand, and can be used to determine the design strength and stiffness of added braces to enhance the robustness of retrofiteed buildings.

Chen et al. (2016b) developed three vulnerability indices for robustness of structures under a single discrete event (e.g. only impact), under multiple discrete events (e.g. earthquake and impact), under continuous events (e.g. event during lifetime). In the first case, the importance coefficient of components γ_{ki}, as shown in Eq.(14) representing the relation between component behavior and system behavior, was used as a weighting coefficient for vulnerability coefficient of components (representing the component behavior alone). These indices depended on the damage of components and its effect on the structure, and number of possible local damage scenarios.

$$RI_i = 1 - \frac{1}{C_n^1}\sum_{k=1}^{n} \gamma_{ki} \cdot v_{ki} \quad \text{(single discrete event)} \tag{19-1}$$

$$RI = \sum_i \omega_i \left(1 - \frac{1}{n}\sum_{k=1}^{n} \gamma_{ki} \cdot v_{ki}\right) \quad \text{(multiple discrete event)} \tag{19-2}$$

$$RI = \int_{-\infty}^{+\infty} \omega(x) \cdot \left(1 - \frac{1}{n}\sum_{k=1}^{n} \gamma_{kx} \cdot v_{kx}\right) \quad \text{(continuous event)} \quad (19\text{-}3)$$

Where RI_{i} is the robustness index of structures under the event i; C_n^1 is the number of all possibilities when one out of n components is removed, and its reciprocal, i.e., $1/C_n^1$, is a factor to control RI_{i} to take the value between 0 and 1. Variables γ_{ki} and v_{ki} denote the importance coefficient and vulnerability coefficient of component k under event i, respectively; ω_{i} is the probability of occurrence of the event i; ω(x) is the occurrence probability density function; n is the number of components;

Li et al. (2018b) proposed a bearing capacity-based robustness measure in Eq. (20) for steel frames, by comparing the load imposed on the intact structure and the residual loading capacity of the damaged structure. This robustness index accounted for the dynamic effects and plastic internal force redistribution. $I_{\mathrm{rob}} \leq 0$ represents occurrence of collapse since the critical load of the damaged structure is smaller than the load imposed on the intact structure, while $I_{\mathrm{rob}} > 0$ means no collaspe. A hypothetical upper limit for I_{rob} is equal to 1, and between 0 and 1, the larger the I_{rob} value, the better the robustness of the frame.

$$I_{rob} = \frac{(q_{2m} - \gamma q_{1m})}{q_{2m}} = \frac{(q_{2dm} - q_{1m})}{q_{2dm}} \quad (20)$$

Where $q_{1\mathrm{m}}$ is the load on the intact structure, which can be represented by the load imposed on the two middle bays adjacent to the removed column, while load imposed on the side bays is denoted as $q_{1\mathrm{s}}$ as shown in Figure 6a. The maximum values of $q_{1\mathrm{m}}$ and $q_{1\mathrm{s}}$ can be considered as the elastic limit load of intact structure; $q_{2\mathrm{m}}$ is the static critical load of the middle bays for the damaged structure until failure (Figure 6b) and $q_{2\mathrm{s}}$ is the corresponding static critical load of the side bays for the damaged structure; γ is a dynamic amplification factor, and $q_{2\mathrm{dm}}$ is the dynamic critical load of the middle bays for the damaged structure until failure, which may be obtained by $q_{2\mathrm{dm}} = q_{2\mathrm{m}}/\gamma$.

Different from most above methods for frame structures, Yan et al. (2019) proposed an index to identify the critical member in single-layer latticed domes. This was based on the governing collapse mechanism of nodal snap-through buckling at either end of the initially removed member. The index estimated the relative vulnerability to node buckling by comparing all indices of all members, without explicitly calculating the exact node-buckling load. The index was derived by considering influence of the load on the node, the stiffness of the connecting members, the boundary condition of the connecting members, and the angle of the gap created by the member removal.

The bearing capacity or collapse mode of an intact or damaged structure is always determined by finite element analysis. Pantidis and Gerasimidis (2017) proposed analytical closed-form solutions of collapse load for yielding-type collapse mode (due to failure of beams above the removal column) and stability collapse mode (due to buckling of adjacent columns), respectively. A collapse limit state factor was defined in Eq. (21) as the ratio of the two collapse loads to determine which collapse mode was triggered first. This factor serves as a indicator of the collapse mode: $R(\mathrm{a}) > 1$ applies for yielding-type collapse modes and $R(\mathrm{a}) < 1$ applies for stability collapse modes. A unity value means simultaneous occurrence of the two collapse modes.

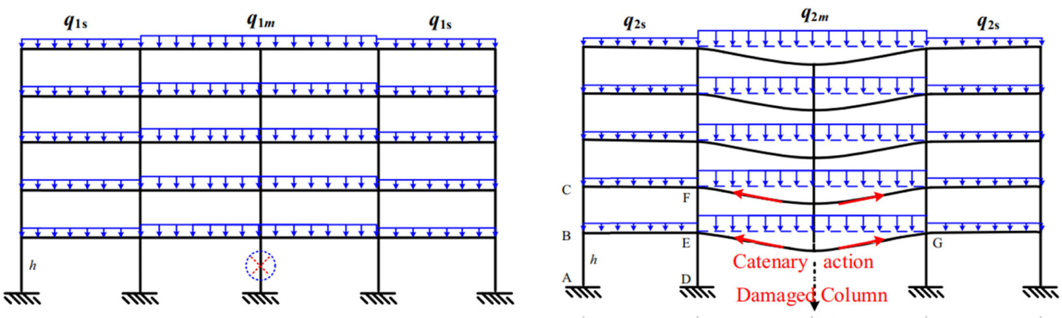
$$R(a) = \frac{C_c(a)}{C_b(a)} \quad (21)$$

Where $C_{\mathrm{c}}(\mathrm{a})$ and $C_{\mathrm{b}}(\mathrm{a})$ is the collapse load for the stability and yielding-type collapse mode, respectively; a is the location of the column removal.

5.1.2. Deformation-based robustness index

Biondini et al. (2008) presented a robustness index associated with the displacements of the system as shown in Eq. (22). This index is related to the load ratio and damage degree of a structure.

$$\rho = \frac{\|s_0\|}{\|s_d\|} \quad (22)$$



(a) (b)

Figure 6. Schematic of structural load on a structure: (a) intact structure; (b) damaged structure.

Where s is the displacement vector, $\|\cdot\|$ denotes the euclidean scalar norm, and the subscripts "0" and "d" refer to the intact and damaged state of the structure, respectively.

Huo et al. (2012) proposed a ductility coefficient of rotation capacity of beam-to-column connections to measure the robustness of steel structures, defined as the ratio of ultimate rotation capacity $\theta_{\max}$ at failure to the rotation θ_c at the formation of catenary action in beam, as given in Eq. (23). When $\varphi_j > 1$, it suggests that the connecton is able to develop catenary in the beam. The higher the ductility coefficient, the higher the reserve capacity of the connection and the structure.

$$\varphi_j = \frac{\theta_{max}}{\theta_c} \tag{23}$$

5.1.3. Damage extent-based robustness index

Based on the quantification of the damage progression resulting from initial damage, a dimensionless damage-based measure R_d of robustness was proposed by Starossek and Haberland (2011) as:

$$R_d = 1 - \frac{p}{p_{lim}} \tag{24}$$

where p is the maximum total damage resulting from the initial damage; $p_{\lim}$ is the acceptable total damage. Note that p and $p_{\lim}$ refer to damage in addition to the initial damage. The quantification of damage required herein can be performed with regard to the affected masses, volumes, floor areas or even the resulting costs.

Another formulation $R_{d,\mathrm{int}}$ of a damage-based measure of robustness was proposed by Starossek and Haberland (2011) as the complement of the integral of the dimensionless damage progression caused by various extents of initial damage i. A value of one stands for maximum possible robustness and a value of zero stands for a total lack of robustness. In Eq. (25), initial damage of any size is considered. However, an initial damage larger than a limit value specified by design objectives can no longer be considered as local, which is one limitation of this method.

$$R = 1 - 2\int_0^1 [d(i) - i]\,di \tag{25}$$

Where $d(i)$ is the maximum total damage resulting from the initial damage of extent i. Both $d(i)$ and i are dimensionless variables obtained by dividing the respective reference value (mass, volume, floor area or cost) by the corresponding value of the intact structure.

In addition, the measure R_d in Eq. (24) is expressive since it directly refer to the design objectives of "acceptable total damage", while the integral measure $R_{d,\mathrm{int}}$ in Eq. (25) is thus not expressive. The requirements of expressiveness and simplicity, and in principle also the requirement of generality, can be met by these damage-based approaches. The question of calculability, on the other hand, is more difficult to answer. This is because determination of the total damage resulting from initial damage requires an examination of the failure progression which is always achieved by dynamic analysis, taking into account geometric and material non-linearity, as well as separation and falling down of structural components. Depending on the type of structures and the governing type of collapse, the damage-based approaches will become too complex and intractable for practical design purposes.

5.1.4. Energy-based robustness index

The above discussion shows that stiffness-based (Section 4) and damage-based measures of robustness are either easy to calculate or expressive, but not both. Approaches based on energy considerations might be capable of meeting both requirements to the same degree. Energy-based methods are based on the assumption that total external work done by the applied load energy flowing into a system must equal the total amount of energy in the system, as a sum of internal energy (strain related energy) and kinetic energy (velocity related energy). If a collapsing structure is capable of attaining a stable energy state through absorption of gravitational energy, then collapse will be arrested. Otherwise, if a deficit in energy dissipation develops, the unabsorbed portion of released gravitational energy is converted into kinetic energy and collapse propagates from unstable state to unstable state until total failure occurs.

Zhang and Liu (2007) proposed a network of energy flow for framed structures. An energy-base member importance coefficient γ^j was defined in Eq. (26) as the ratio of energy stored in the damaged structure $U^{(i)}$ after member removal to the total energy U of the intact structure. The more the deformation energy in the damaged structure, the more importance the removed member. The network can be used to find the key members and also the critical load redistribution path. However, it is more applicable to be a qualitative indicator rather than quantitative measure of structural robustness since the coefficient varies in a range of (1, +∞) and its relation to the member importance is nonlinear.

$$\gamma^i = \frac{U^{(i)}}{U} \tag{26}$$

Huang and Wang (2012) proposed an importance index CI_{i} of structural members in Eq. (27-1) based on their contributions to the structural energy distribution and the damage-affected areas. The larger the coefficient, the more important the member and the more critical the initial damage scenario. They insisted that redundancy depended on the response sensitivity and the reserve strength of the members (safety margin). A redundancy index R_{j} was also proposed in Eq. (27-3) based on the strain energy-based sensitivity of individual members and the loading capacity redundancy. The measure of robustness was taken as the minimum of all the redundancy indices for different member removal scenarios. However, this method is applicable to removal of one member and does not consider the failure of connections.

$$CI_i = \frac{\alpha_i}{max\{\alpha_i\}} \cdot \frac{S_i}{max\{S_i\}} \tag{27-1}$$

$$S_i = (N - n + 1) \cdot S_{i0} \tag{27-2}$$

$$R_j = \frac{\sum_{i=1(i \neq j)}^{ne} \beta_{i,j}}{\sum_{i=1(i \neq j)}^{ne} (\beta_{i,j}/c_{i,j})} \tag{27-3}$$

Where CI_i is the importance index of the i^{th} member; α_i is the variation of the total strain energy of the structure before and after the damage of the i^{th} member; N and n is the total number of storeys and the storey of the i^{th} member, respectively; S_{i0} is the affected area due the damage of the i^{th} member; R_j is the redundancy index for the damage of the j^{th} member; β_{ij} is strain energy sensibility defined as the change of strain energy in the i^{th} member due to the damage of the j^{th} member; $c_{i,j}$ is the capacity redundancy factor taken as the minimum of the reserve strength-based value and stability-based value.

Fang and Li (2007) proposed an energy-based robustness index I_{rob1} by comparing the total internal energy E_u of an intact structure at the ultimate limit state and that E_d at the design limit state (e.g. yielding), expressed as:

$$I_{rob1} = \frac{E_u}{E_d} \tag{28}$$

This index can only represent the performance of the intact structure, rather than the robustness of the structure in the case of local damage. To this end, this index was improved by Lv et al. (2011) to consider the internal energy E_r of the damaged structure at its ultimate limit state:

$$I_{rob2} = \frac{E_r}{E_d}, \quad I_{rob3} = \frac{E_r}{E_u}, \quad I_{rob4} = \frac{E_u}{E_u - E_r} \tag{29}$$

By comparing the energy released during an initial failure and the energy required for progression of failure, Starossek and Haberland (2011) proposed an energy-based measure R_e of robustness. This simple formulation is to assess the possibility of a total collapse with a value of one indicating perfect robustness. Values between one and zero are acceptable to a greater or lesser degree, while negative values indicate failure progression to complete collapse.

$$R_e = 1 - \max_j \frac{E_{r,j}}{E_{f,k}} \tag{30}$$

Where $E_{r,j}$ is the energy released during the initial failure of a structural element j and contributing to damaging a subsequently affected structural element k; $E_{f,k}$ is the energy required for the failure of the subsequently affected structural element k.

The Eq. (30) has a simple form and also appears to be expressive with regards to the possibility of a total collapse. A difficulty, however, arises in determining the value of $E_{r,j}$, which can be overderestimated or underestimated (Haberland 2007). The energy released by the initial failure of the structural element consists of several parts. In structures vulnerable to pancake-type or domino-type collapse, the gravitational potential energy of separating, overturning or impact elements is transformed into kinetic energy in a discrete and concentrated manner, which makes a major and even dominant contribution to the total released energy. In such cases, a numerical assessment can be easy and effective. However, in structures susceptible to other types of collapse (e.g. yielding-type collapse), the value of $E_{r,j}$ can only be determined through a complete structural analysis. Furthermore, $E_{r,j}$ should include only the energy portion that contributes to damaging the subsequently affected element k. This estimation again is relatively easy in structures having pancake-type or domino-type collapse modes. Therefore, it is expected that such structures are the most suitable ones for the application of energy-based measures of robustness.

Bao et al. (2017) conducted nonlinear static pushdown analysis to determine the ultimate loading capacity of a damaged structure given a column loss scenario, and applied an energy-based analysis to account for the dynamic effect due to sudden column loss. The dynamic effect was considered by assuming that the external work was totally transformed into internal work at the peak dynamic displacement where the kinetic energy is zero, and the static load-displacement curve from pushdown analysis was thus modified to obtain a dynamic load-dynamic peak displacement curve. The dynamic ultimate capacity of the damaged structure, representing the peak vertical load that can be sustained under sudden column loss, was determined corresponding to the peak static load. A robustness index R was proposed in Eq. (31) as the minimum value of the normalized ultimate capacity $\lambda_{d,u}^i$ over all damage scenarios, which is similar to the concept R_2 in Eq. (9-2). A unity value of R indicates no collapse for all sudden column loss scenarios. This method avoids the requirement of complex dynamic analysis, and enables the use of various dynamic increase factors in the same structure (different dynamic increase factors used for different column removal locations). The proposed index is more readily calculable than the damage-based approaches in Section 5.1.3 that require calculating the extent of damage. This is because the analysis can be terminated at the ultimate load for determining R and it is not necessary to analyze the spread of damage following the formation of a collapse mechanism. However, this is also the limitation of this method. The energy-based procedure was based on the assumption of an unchanging deformation mode for both static loading and sudden column loss, which may give nonconservative results in the post-ultimate response when failures caused the change of collapse modes.

$$R = min_i(\lambda^i_{d,u} \,|\, i \in column\ removal\ scenarios) \tag{31}$$

Beeby (1999) proposed using of energy dissipation per unit volume of the intact structure as a measure of robustness. Smith (2006) defined a measure of robustness as the minimum energy required to destroy enough members to cause the collapse of a structure. Different structural arrangements can be compared in this way and critical sequence of collapse can be identified. The buckling energy and failure energy of columns were distinguished by Szyniszewski (2009) and Szyniszewski and Krauthammer (2012), considering that the column buckling did not always lead to column failure and collapse propagation. An energy-based demand capacity ratio was proposed based on the post-buckling deformation energy of damaged columns. A comparison to force-based demand capacity ratio showed that the force-based ratio was not very sensitive to the fundamental changes in structural behavior, and the column deformation energy was a better stability indicator under dynamic loading than the maximum dynamic force. Recently, Wilkes and Krauthammer (2019) investigated the correlation of energy flow and rate of energy flow to member failure and structural collapse of mid-rise steel framed structures. This was achieved from energy-time curves obtained by nonlinear dynamic analysis.

5.2. Probabilistic Robustness Index

The deterministic robustness or redundancy indices are a fixed value used to identify critical members and to assess performance of a structural system for given damage scenarios. It is difficult for them to include all the possible damage scenarios and potential failure paths. To account for the random nature of the required information such as uncertainties in abnormal loads, member and system responses, consequences of failure, probabilistic measures should be used to quantify robustness of structures, from a more practical view. A common probabilistic measure of safety is the reliability index β which is related to the probability of failure P_f through Eq. (32). The probability of failure P_f can be used to directly measure the robustness of structures. When it is difficult or impossible to calculate the failure probability, the reliability index β can be used to indirectly measure the failure probability and structural robustness. The following subsections present a review on three main categories of probabilistic robustness measures, i.e. failure probability-based, risk-based and reliability-based measures, respectively. The first category considers the uncertainties in abnormal events and structural system, while the second involves the probability of the causes (abnormal events) and consequences of structural collapse. The third one, as a trade-off, accounts for radomness in structural system alone.

$$P_f = \Phi(-\beta) \tag{32}$$

Where Φ is the cumulative Gaussian probability distribution function.

5.2.1. Failure Probability-based Robustness Index

Failure probability-based robustness indices take into account probability of abnormal events and local failure as well as probability of collapse given the local failure. They are always derived by comparing the probabilities of the system failure for an undamaged and a damaged structure.

Ellingwood (2005, 2006) proposed that the probability of building collapse must be limited to a socially accepted value (e.g. 10^{-6}/year). Let H be the abnormal event and D be local damage. The probability of structural collapse due to H is expressed as (Ellingwood 2005; Starossek and Haberland 2008):

$$P[\text{Collapse}] = P[\text{Collapse}|D]P[D|H]P[H] \tag{33}$$

Where $P[H]$ is probability of H; $P[D|H]$ is conditional probability of damage state D, given H; $P[\text{Collapse}|D]$ is probability of collapse, given damage state D.

For multiple hazards and damage states, Eq. (33) can be rewriten as:

$$P[\text{Collapse}] = \sum_H \sum_D P[\text{Collapse}[D]P[D|H]P[H] \tag{34}$$

The probabilities P[D|H] and P[Collapse|D], rather than controlling abnormal events $P[H]$, are within the control of a structural engineer. The common design standards usually describe safety of a structure as a function of safety of all elements against local failure, and therefore only take into account the probability P[D|H]. The reaction of a structure to the possible occurrence of local failures in terms of P[Collapse|D] is seldom investigated. Only important buildings are investigated regularly for this case. A summary of the probability of terrorist threat, hazard, damage, fatality, and economic and social loss for progressive collapse is described by Stewart (2017) and Adam et al. (2018).

To overcome the limitation of deterministic measures that only the ultimate limit state is considered, Lind (1995) proposed complementary concepts of damage tolerance in Eq. (35-1) and vulnerability in Eq. (35-2) to capture the reduction in reliability of a system that was damaged without collapse. The damage tolerance T_d is a function of a set of system states (different levels of damage) and loading conditions. For a specific damage state and loading condition, a damage factor was defined as the ratio of failure probability of a damaged state to an undamaged state. The vulnerability V was defined as the reciprocal of the damage tolerance. The proposed measure is a step toward differentiation of a set of possible system states rather than simply considering "failure" and "no failure" in normal probabilistic analysis.

$$T_d = P(R_0, S)/P(R_d, S) \tag{35-1}$$

$$V = P(R_d, S)/P(R_0, S) \tag{35-2}$$

Where T_d is the damage tolerance; V is the vulnerability; $P()$ is the probability; R_0 and R_d is a set of undamaged and damaged states, respectively; S is a set of prospective loadings.

Frangopol and Curley (1987) and Fu and Frangopol (1990) proposed a probabilistic measure related to structural redundancy (RI), which also indicated the level of robustness:

$$RI = \frac{P_{f(damaged)} - P_{f(intact)}}{P_{f(intact)}} \tag{36}$$

Where $P_{f\,(\mathrm{damaged})}$ and $P_{f\,(\mathrm{intact})}$ is the failure probability of a damaged and intact structural system, respectively; This redundancy index provides a measure of redundancy of a structural system, ranging between zero and infinity with a lower value indicating a higher robustness. However, it could be difficult to assess this index in a practical application due to the wide range of the values that the index can take.

By replacing $P_{f\,(\mathrm{intact})}$ in Eq. (36) with acceptable failure probability $P_{f\,(\mathrm{acceptable})}$, a new robustness index was proposed by Chen et al. (2016a), as expressed in Eq. (37) where $P_{f\,(\mathrm{collapse})} = P[\mathrm{Collapse}]$. Based on the robustness index, the performance of a structure can be classified into four levels: collapse ($RI \leq 0$), low robustness ($0 < RI \leq 1/3$), general robustness ($1/3 < RI \leq 2/3$), high robustness ($2/3 < RI \leq 1$). This classification is important for design purpose.

$$RI = \frac{P_{f(damaged)} - P_{f(intact)}}{P_{f(intact)}} \tag{37}$$

Based on the performance objective of maintaining sufficient system reliability under an extreme load, Maes et al. (2006) proposed a measure of robustness R_2 expressed as the minimum of system failure probability ratio for each member. The ratio was defined by comparing the system failure probability P_{s0} of the intact structure to that P_{si} of the damaged structure due to one impaired member i. This measure is related to the criticality of individual members and their role in a redundant system, which can be related to a variety of importance-based measures commonly used in system reliability.

$$R_2 = \min_i \frac{P_{S0}}{P_{Si}} \tag{38}$$

Felipe et al. (2018) proposed a systematic reliability-based approach to identify key elements in a redundant structure (the element most likely to cause collapse) and to simplify the design process. The approach involved a threat-specific analysis, in which probabilities of initial damage were evaluated for a given loading event. Damage propagation and ultimate collapse were distinguished to rank elements in a system in terms of their vulnerability regarding structural collapse. The key element was identified as the one presenting the largest intersection between vulnerability and importance with respect to collapse. A coefficient of vulnerability (CV) of an element was defined as the ratio of failure probability of the element $P[f_i]$ to the sum of failure probabilities for all elements in the system. A coefficient of importance (or disproportionate) CID for damage propagation of an element was defined as the ratio of the probability of failure progression $P[f_j|f_i]$ to the maximum possible probability of damage propagation. A coefficient of importance CIC for progressive collapse of an element was defined as the ratio of the probability of occurrence of the failure path c_i, starting with the failure of the i^{th} element to the maximum probability of failure path. A coefficient of vulnerability CVD to damage progression of an element was defined by the product of CV and CID. A coefficient of vulnerability CVC to structural collapse of an element was defined as the product of CV and CIC. The values of the above coefficients are always between 0 and 1 from their definitions. The key element of a system was defined as the element having the largest vulnerability with respect to structural collapse. Design improvements toward structural integrity should focus on the key element.

However, the proposed methodology is a threat-dependent method (depend on failure probabilities of the structural elements), which is rather complex in the sense that it requires full probabilistic analysis of all possible failure paths and identifies element vulnerability and the key element. This may render the applicability to large structural systems limited.

$$CV_i = \frac{P[f_i]}{\sum_{j=1}^{n} P[f_j]} \tag{39-1}$$

$$CID_i = \frac{P_i[f^p]}{\max_m[P_m[f^p]]} \tag{39-2}$$

$$CIC_i = \frac{P[C_i]}{\max_m[P[c_m]]} \tag{39-3}$$

$$\mathrm{KE} = \max_i[CVC_\mathrm{i}] = \max_i[CV_\mathrm{i} \cdot CIC_i] \tag{39-4}$$

Where

$$P_m[f^p] = \sum_{j=1,i\neq-1}^{n} P[f_j | f_i] \cdot P[f_i] \ and \ m = 1,\ldots,n \tag{39-5}$$

$$P[C_i] = \sum_{j=1,j\neq i}^{n} P[f_i] \cdot P[f_j | f_i] \cdot P[(\mathrm{U}_{k,j\neq i,k\neq j})(f_k | f_{i,j})] \tag{39-6}$$

Lin et al. (2019) proposed a novel methodology to quantitatively evaluate the structural robustness of offshore platforms against progressive collapse. A generalized bearing capacity ratio ($\lambda_i^{(j)}$) was defined as the change of internal stress in the i^{th} component before and after failure of the j^{th} component. The component with the maximum ratio was regarded as the candidate failure component at the j^{th} failure step of the m^{th} failure path, and thus the total failure paths and all failure steps in each path were obtained. An incremental loading method was used to calculate the reliability of each failure path, based on which the probability of each failure path was determined. Three robustness indices were proposed: path and state-dependent robustness index R_I, overall robustness index R_P, and comprehensive robustness index R_W. The R_I describes the robustness of the structure at each step of each failure path, and can illustrate the effect of each local damage on the failure probability of a structure. The overall robustness index R_P is expressed as a weighted sum of R_I using a path weight η_k, demonstrating the overall effect of various potential failure paths on structural robustness. Performing twice weighting on R_I by path weight η_k and step weight $\omega_\mathrm{k,l}$ leads to the comprehensive robustness index R_W, which includes both path information and state information at the same time. It was found that the robustness varied significantly under different failure paths, and an unexpected accident occurring in a small-probability path would have a more serious impact on structural robustness. One problem of this study is that the deduction of probability of failure paths is not clear, and it seems that no dynamic effect was included in the analysis.

$$\begin{Bmatrix} \lambda_i^{(j)} = \dfrac{a_i^{(j)}}{R_i}, \ j=1 \\ \lambda_i^{(j)} = \left(\dfrac{a_i^{(j)}}{R_i} \Big/ \dfrac{a_i^{(j-1)}}{R_i}\right) = \dfrac{a_i^{(j)}}{a_i^{(j-1)}}, (j>1) \end{Bmatrix}, \ (\mathrm{i}=1,2,\ldots,\mathrm{n}) \tag{40-1}$$

$$R_I(k,l) = \frac{P_{k,l} - P_k}{P_k}, (1 \leq l \leq m) \tag{40-2}$$

$$R_P(l) = \sum_{k=1}^{s} \eta_k R_I(k,l), \ \eta_k = \frac{p_k}{\sum_{k=1}^{s} p_i}, (k=1,2,\ldots,s) \tag{40-3}$$

$$R_w = \sum_{k=1}^{s} \eta_k \sum_{l=1}^{m} \omega_{k,l} R_I(k,l), \ \omega_{k,l} = \frac{p_{k,l}}{\sum_{j=1}^{m} p_{k,j}}, \quad (k=1,2,\ldots,s; l=1,2,\ldots m) \tag{40-4}$$

Where R_i is the strength of the component r_i; $a_i^{(j)}$ is the internal stress of component r_i when imposing an unit load on the damaged structure consisting of r_j, $r_\mathrm{j}+1$, …, r_n; P_k is the occurrence probability of the k^{th} failure path; $P_\mathrm{k,l}$ is the structural failure probability at the l^{th} failure step of the k^{th} failure path; n is the number of components; m is the number of failure steps for a failure path; s is the number of failure paths.

5.2.2. Risk-based Robustness Ratio

Risk-based robustness assessment offers a powerful and full probabilistic framework, by considering the probability of the causes (i.e. how likely is the hazard event) and consequences (i.e. what happens when such events do occur such as loss of safety and economy). However, complexity and subjectivity reduce the calculability and application potential of the risk-based approaches. Without some appreciation for these risks, it is difficult to judge the effectiveness of various strategies to mitigate structural collapse.

Based on the definition of risk, Pinto et al. (2002) defined the risk SR of a failure as the product of the probability P_f of its occurrence and its consequence C, as given in Eq. (41). The consequence C can be defined in the context of ultimate limit state or servicebility limit state.

$$\mathrm{SR} = P_f \times C \tag{41}$$

Risk-based quantifications of system robustness were first proposed by Baker et al. (2008). A robustness index in Eq. (42) based on the definition that “a robust system is considered to be one where indirect risks do not contribute significantly to the total system risk” (Formisano et al. 2015). The approach divided consequences into direct consequences associated with the damage of a local element and indirect consequences associated with the additional and subsequent system failure. The robustness measure resulted from the comparison of direct risks with the total risks (sum of the direct and indirect risks). The system risk was computed by multiplying the consequence of each scenario by its probability of occurrence, and then integrating over all of the random variables. The fewer indirect risks involved in the total risk, the more robust a structure is. The index takes values between zero and one, where $I_{\mathrm{Rob}} = 1$ represents a completely robust structure since there is no risk due to indirect consequences, while $I_{\mathrm{Rob}} = 0$ denotes a completely vunerable structure that all risk is due to indirect consequences. This measure of robustness allows the robustness of different systems to be compared.

However, one problem of this method is that it measures the relative direct risk due to indirect risk, resulting in a false impression that a system might be deemed robust if its direct risk is extremely large (relative to its indirect risk), but that system should be rejected based on reliability criteria (i.e. predefined acceptable direct risk) rather than robustness criteria. Sørensen (2011) argued that the robustness index in Eq. (42) was not always fully consistent with a full risk analysis, although it was a helpful indicator based on risk analysis principle. The direct risks can be estimated with higher accuracy than the indirect risks, since the direct risk typically are related to code-based limit states.

$$I_{\mathrm{Rob}}=\frac{R_{\mathrm{Dir}}}{R_{\mathrm{Dir}}+R_{\mathrm{Ing}}} \tag{42}$$

Where I_{Rob} is the robustness index; R_{Dir} is the direct risk; R_{Ind} is the indirect risk.

Faber et al. (2017) revised Eq. (42) and proposed a more general and consistent scenario-based approach to quantify robustness. The revised formulation took the ratio between direct consequences and total consequences as scenario wise, and the robustness index with respect to a given scenario I is expressed as in Eq. (43). The selection of the formulations depended on the focus of the system assessment. Note that the robustness index $I_{\mathrm{R}}(i)$ itself is a random variable which may be analysed further by categorization and ordering of the different scenarios in accordance with the hazard, damage, failure and consequences.

$$I_R(i)=\frac{c_D(i)}{c_T(i)} \tag{43-1}$$

$$I_R(i)=\frac{c_{D,I}(i)}{c_{D,I}(i)+c_{D,P}(i)} \tag{43-2}$$

$$I_R(i)=\frac{c_{D,I}(i)+c_{D,P}(i)}{c_{D,I}(i)+c_{D,P}(i)+c_{ID}(i)} \tag{43-3}$$

Where $C_{\mathrm{D}}(i)$ and $C_{\mathrm{T}}(i)$ is the direct and total consequences, respectively; $C_{\mathrm{D,I}}(\mathrm{i})$ and $C_{\mathrm{D,P}}(\mathrm{i})$ is the direct consequences due to the initial damage and propagated damage, respectively; $C_{\mathrm{ID}}(\mathrm{i})$ is the indirect consequences.

To avoid the difficulty in quantifying the probability of the exposures (e.g. human/gross errors), a conditional robustness index was proposed by Sørensen (2011) using risks $R_{\mathrm{Dir|exposure}}$ and $R_{\mathrm{Ind|exposure}}$ conditioned for a given exposure as:

$$I_{\mathrm{rod|exposure}}=\frac{R_{\mathrm{Dir|exposure}}}{R_{\mathrm{Dir|exposure}}+R_{\mathrm{Ind|exposure}}} \tag{44}$$

A vulneribility index Iv was also proposed by Baker et al. (2008) as the ratio of total direct risks to the total direct consequences. This vulnerability index provides an indicator of the risks associated with structural damage, normalized by the direct risk exposure.

$$I_V=\frac{\sum_i R_{Dir_i}}{\sum_j C_{Dir_{elj}}} \tag{45}$$

Where R_{Dir_i} is the direct risk due to the ith damage; $C_{Dir_{elj}}$ is the direct consequence due to the j^{th} damage.

Maes et al. (2006) proposed a risk-based robustness measure R_3 that served as an indicator for the performance objective of containing the costs associated with the consequences of failure. It was quantified as the inverse of the tail heaviness H of the log-exceedance curve based on the relation of failure consequences versus probability of exceedance. The tail heaviness is a frequently used quantitative measure of risk, and can be easily computed numerically for a specific consequence-logexceedance probability curve (Maes 1995). $H < 1$ represents robustness (i.e. a fully contained consequences), while $H > 1$ means non-robustness (i.e. out-of-control consequences). Note that the measure R_3 is appropriate to evaluate the robustness of a system subjected to an exceptional hazard, since it considers both the consequences of failure including follow up consequences and their likelihood. In practical situations, it is usually both appropriate and efficient to consider "easy" measures of robustness such as R_1 in Eq. (16) and R_2 in Eq. (38). A more general method based on a full consequence analysis is valuable if one wishes to give a quantitative meaning to "robustness" in the case of more complex or continuous human, environmental and engineered systems.

$$R_3=1/H \tag{46}$$

5.2.3. Reliability-based robustness index

As a trade-off, reliability-based robustness assessment approaches only account for the randomness of structural systems by ignoring the probabilities of the accidental events and the corresponding initial damages. Reliability-based measures focus more on progressive collapse resistance of a structure itself, and define the robustness as a function of the failure probabilities. Once the reliabilities of the intact and damaged structure subjected to initial damages are obtained, the structural robustness can be computed easily.

If the resistance R and the applied load P follow common probability distributions (known probability density function) such as normal or lognormal distributions, the reliability index can be mathmatically determined based on the mean value and standard deviation of R and P. For random variables R and P with unknown probability density function, a reliability analysis is needed. The reliability index can be evaluated on a member-by-member basis or on a structural system basis.

Based on the system reliability, a probabilistic redundancy index β_R was proposed (Frangopol and Curley 1987; Fu and Frangopol 1990; Sørensen 2011) as:

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}} \tag{47}$$

Where β_{intact} and β_{damaged} is the reliability index for the collapse limit state of the intact and damaged system, respectively. This index takes values between zero and infinity, with larger values indicating larger robustness.

Ghosn and Moses (1998) measured the robustness of bridge structures in terms of the reliability index margin between system reliability and the reliability of the key member.

$$\Delta\beta_u = \beta_{intact} - \beta_{member} \tag{48-1}$$

$$\Delta\beta_f = \beta_{functionality} - \beta_{member} \tag{48-2}$$

$$\Delta\beta_d = \beta_{damaged} - \beta_{member} \tag{48-3}$$

Where $\beta_{\text{functionality}}$ is the reliability index for the functionality (servicebility) limit state of the intact structure; β_{member} is the reliability index of the most critical member.

An equivalent reliability-based robustness index ranging between zero and unity was proposed by Sørensen (2011) as:

$$I_{rob} = \frac{\beta_{\text{damaged}}}{\beta_{\text{intact}}} \tag{49}$$

Feng et al. (2020) applied probability density evolution method (PDEM) to determine the system reliability of reinforced concrete structures subjected to progressive collapse. The structural uncertainties including geometric properties, material properties and applied loads were considered. The PDEM was employed to calculate the probability density function of the collapse resistances for these uncertainties obtained from pushdown analysis. The critical damage scenario together with PDEM was used to determine the failure reliability. The robustness index in Eq. (47) was used to measure the robustness of structures. One limitation is that independent random variables were used which cannot reflect interaction between coupled uncertainties.

Gharaibeh et al. (2002) presented a system reliability-based methodology to identify and rank important members in structural systems under different material behaviors (brittle or ductile) and for different stiffness sharing factors. The importance of a member was defined as its impact on the system reliability. Two member importance factors were proposed: member reliability importance factor in Eq. (50-1) and member post-failure importance factor in Eq. (50-2). The former represents the sensitivity of system reliability β_{system} to changes in reliability of the member $\beta_{\text{m,i}}$ (with a normalized form of $I^0_{m,i}$), while the latter measures the sensitivity of system reliability to changes in member post-failure behavior (residual strength). The residual strength of the damaged member was measured by a strength factor η_{i} depending on the ductility of the member ($\eta_{\text{i}} = 0$ for perfect brittle behavior; $\eta_{\text{i}} = 1$ for perfect ductile behavior). A simple expression of reduction in reliability index of the structure given a member failure was also proposed. Although these factors can be used for complex structures, they needs reliability analysis based on statistic data and probability distrubiton of load and capacity, and are difficult to be used in practical engineering.

$$I_{m,i} = \frac{\partial\beta_{system}}{\partial\beta_{m,i}}, I^0_{m,i} = I_{m,i} / \sum_{j=1}^{N} I_{m,j}$$

(reliability importance factor) (50-1)

$$I_{\eta,i} = \beta_{system} | \eta_i = 1.0 - \beta_{system} | \eta_i = 0.0$$

(post-failure importance factor) (50-2)

The deterministic redundancy-strength index r_{s} in Eq. (17-1) and probabilistic redundancy-variation index r_{v} in Eq. (17-2) proposed by Husain and Tsopelas (2004) correspond to the mean value and coefficient of variation, respectively, in the statistic theory. These two indices were further used to evaluate the redundancy response modification factor and reliability index by Tsopelas and Husain (2004). The modification factor was used to modify the static analysis results by considering the effect of redundancy. The reliability index β is expressed as Eq. (51). This index is a function of the system redundancy (r_{s}, r_{v}), coefficient of variation (COV) of the strength v of the elements in the structure, COV of the load l on the structure. This index represents a good step to measure the relation of reliability and redundancy. Further work is needed to extend it using nonlinear dynamic analysis.

$$\beta = \frac{(r_s - l)}{v_e \cdot \sqrt{r_v^2 \cdot r_s^2 + l^2 \cdot v^2}} \tag{51}$$

Liao et al. (2007) proposed a uniform-risk redundancy factor to modify the design lateral load for steel moment frames under earthquake. The redundancy factor R_{R} was defined as the ratio of elastic spectral acceleration at the fundamental period causing collapse at the two probability levels (P_{ic} is the actual probability of collapse; $P_{\mathrm{ic}}^{\mathrm{all}}$ is the allowable probability of collapse). Although this method is seismic-purpose, it can be potentially applied to progressive collapse by replacing the spectral acceleration with dyamic increment factor.

$$R_R = \begin{cases} 1 & \text{当}\ P_{\mathrm{ic}} \le P_{\mathrm{ic}}^{\mathrm{all}} \\ \dfrac{S_a^{\mathrm{ic}}}{S_a^{\mathrm{all}}} & \text{当}\ P_{\mathrm{ic}} \ge P_{\mathrm{ic}}^{\mathrm{all}} \end{cases} \tag{52}$$

The largest challenge in reliability-based robustness quantifications is the calculation of system reliability. The most widely adopted method is the Monte Carlo simulation, which is efficient but suffers from a large computational cost. An alternative method is to use the probability density evolution method (PDEM) (Feng et al. 2020), which is derived based on the probability conservation principle. The PDEM establishes a new method to reach the balance between computational efficiency and accuracy in system reliability assessment.

6. Summary and Recommendations

6.1. Summary of Existing Quantification Approaches

Insensitivity to local failure is referred to as robustness. Different structural systems subjected to different local failures may suffer from different collapse modes, and thus exhibit different degrees of robustness. Such differences are not included in modern probability-based design procedures using partial safety factors. Additional considerations are therefore necessary to ensure structural robustness after an initial local failure. Such considerations have been made in the past mostly in qualitative form rather than quantitative manner. This paper presented a comprehensive review on quantitative measures of structural robustness, and the following findings can be drawn:

1. A clear definition of robustness is the precondition of a better quantitative measure of robustness. Robustness is a property of the structural system alone, i.e. a system behavior independent of the possible abnormal events and probabilities of the induced initial local failure. Robustness can be achieved by continuity (tie strength), redundancy (alternate load path), ductility, segmentation, energy absorption capacity, etc. It is different from collapse resistance (a broader concept also dependent on initial damage), vulnerability (system or component behavior for a given initial failure) and redundancy (one aspect of robustness to provide multiple alternate load paths).
2. A total of about 50 quantitative measures of structural robustness are found in terms of various robustness or redundancy indices, which can be classified into two main groups: structural attribute-based and structural performance-based measures. The former is based on the structural topology (configuration) and structural stiffness (with or without load), while the latter depends on the structural performance reponses such as bearing capacity, deformation, damage extent, energy flow etc. Based on whether considering the probability concept, the structural performance-based measures can be further classified into deterministic and probabilistic measures.
3. The deterministic measures predominate in quantifying structural robustness, which are always derived by comparing the structural responses of an intact and damaged structure. They are mostly expressed by dimensionless factors in terms of bearing capacity, displacement or rotation, damage extent or energy. They are largely characterized by a systematic analysis for a range of column removal scenarios (alternate load path method) using nonlinear static and dynamic analysis or energy-based method.
4. The structural attribute-based measures of robustness are derived from algebraic properties (determinant, condition number, matrix norm) of system stiffness matrices. Some of them lack clear physical meanings, which can only be used as indicators of robustness in a qualitative manner. Most attribute-based measures are applicable to the elastic state of simple structural form (e.g. truss structures) and their extention to other types of structures (e.g. frame structures) and plastic state needs further work.
5. The probabilistic measures are receiving growing interest because it is difficult to consider all foreseeable damage events in deterministic measures. They involve the probability of uncertainties in abnormal events, structural system and failure-induced consequences, and can be further classified into failure probability-based measures (include uncertainties in abnormal events and structural system), risk-based measures (include probability of the causes and consequences) and reliability-based measures (a trade-off method involving radomness in structural system alone).
6. Nonlinear static analyses dominate the determination of peformance of intact and damaged structures, serving as a trade-off between linear static and nonlinear dynamic analysis. The dynamic increase factors are always needed to consider the dynamic effects due to sudden removal of columns. It is also needed to select bay pushdown analysis (increasing gravity loads only on the damaged bays) or uniform pushdown analysis (increasing gravity loads on all bays) to determine the capacity of structures.
7. Robustness is still an issue of controversy for its quantification. The purposes of existing quantitative measures of robustness differ, focusing on ranking structural members, identifying critical components, failure paths and collaspe modes in a structural system, accounting for aspects of the mebers including location

in the system, reserve strength, residual strength, stiffness sharing, and material behavior (ductile or brittle). Other purposes are to check the usefulness of robustness recommendations, indicators and prescriptions. There is no generalized form of quantitative measures to comprehensively assess the robustness of structures with a single robustness index.

6.2. Issues and Recommendations

1. A generalized quantification of robustness needs to start from the generally accepted definition and design objectives, and to focus on the response of a structure to local damage as well as the associated consequences of collapse. The inclusion of collapse consequences is the requirement to be used as a decision-making tool. A generalized quantification can be achieved by comparing a set of deterministic and probabilistic robustness indices rather than using a single index.
2. Many quantitative measures of robustness are applicable to one column removal scenarios, simple structural types idealized into a set of parallel-series components (e.g. truss, frame structures), regular structural layouts, predefined single collapse modes (yielding-type model or stability mode). Therefore, further research is needed for multiple column removal scenarios (realistic damaged components in the blast-affected zones), complex structural types (discrete or continuum), irregular structural layouts (vertical or horizontal), and multiple or coupled collapse modes.
3. The existing design methods and quantitative measures of structural robustness are proposed and applicable for steel framed structures and reinforced concrete structures. These are not directly applicable to large-span structures, prefabricated steel structures, precast/prestressed concrete structures, modular struc-tures, timber structures, offshore platforms, which are inherently susceptible to progressive collapse due to lack of continuity because of discrete and potentially brittle connections, large spans, and heavy elements. In addition, these types of structures have different collapse modes and resisting mechanisms, and thus needs different measures to quantify their robustness.
4. Many quantitative measures of robustness are applicable to single hazard such as earthquake, blast, impact or fire. Progressive collapse design requirements can add major cost to building construction if it is achieved without consideration of other design requirements. By taking a multi-hazard approach progressive collapse mitigation can often be achieved with minimal additional construction costs. It is therefore recommended to extend the existing robustness measures for multiple hazards, in an attempt to reduce risk of progressive collapse, improve structural life safety and save economic costs. On one hand, this means to develop different forms of robustness indices for these individual hazards, respectively, and choose a optimized structural system by comparing these measures. On the other hand, it means to develop a robustness measure to quantify robustness of a structure under a realistic combination of hazards (e.g. fire after earthquake, fire after blast).
5. The robustness indices always fall in a range between 0 and 1, representing two extreme situation of completely robust and completely vurnerable. Most measures are not related to performance objectives (i.e acceptable structural response) such as acceptable capacity limit, deformation limit, extent of collapse. Thus, a further classification of the degree of robustness is lacking (e.g. collapse, low robustness, moderate robustness, high robustness).
6. For the design purpose, deterministic measures are predicated on the assumption that robustness is a variable property of the structure that can be calibrated to meet a fixed design load via increased strength and improved load path redundancy. One limitation of deterministic measures is that only the ultimate limit state is considered since robustness changes throughout the entire collapse process (each failure step in each failure path). Another limitation is that they cannot cover all the local damage scenarios, and do not consider the consequences of collapse at overload, which cannot be used for decision-making. Therefore, it is recommended that deterministic measures are used to evaluate the robustness of the structural system alone in the early design stage through alternate path method. Probabilistic measures, as more reasonable and practical tools, can be used for decision-making on the selection of enhancing strategies for robustness from the view of safety and economic costs.
7. The energy-based measures of robustness are more applicable to structures susceptible to pancake-type or domino-type collapse where the energy flow can be easily determined since the energy transform from gravitational potential energy to kinetic energy in a discrete and concentrated manner. For other types of collapse such as yielding-type collapse, the energy transformation can only be determined through a complete structural analysis.
8. The quantitative measures of robustness are always derived by conducting nonlinear static or dynamic analysis, which is complex and time consuming. It is recommended to use machine learning method (i.e. artificial neural network) for fast prediction of the collapse mode and failure patterns of structures as well as subsequent progressive collapse potential assessment. A systematic methodology based on both Monte Carlo simulation and probability density evolution method should be developed to generate a robust and suffificient large dataset for training and testing.
9. Large-scale experiments by loading structures to

failure are needed to estimate the load-carrying and deformation capacities, in order to ensure adequate load distribution while undergoing large deformations without failure. These work will provide practical and economic collapse-thresholds and define collapse acceptance criteria (e.g. maximum allowable deflection or rotation) to ensure structural robustness after extreme load events.

10. Existing apporaches for quantifying robustness focus on whether a structure collapses, how likely it collapses and in which mode it collapses, which capture the key characteristic of blast/impact-induced collapse. However for fire, the issue of when it collapse is of great interest. There is no such measure to assess the fire-related collapse time.
11. Robustness measures largely depend on the accuracy of finite element analysis, but the effect of uncertainty in the analysis is not considered in the robustness quantification. Therefore, the accuracy of the developed robustness measures should be double checked before practical application.
12. Most robustness measures depend on the assumption of completely removal of columns, which is not true in reality that some residual strength left in the damaged column. It has been found that complete column removal in the alternate path method can be less conservative, and predict collapse mechanisms and collapse loads which are not the most critical. Partial damage in one or multiple components should be considered as a local failure scenario.
13. More powerfull computation tool as a combination (coupled) of computational fluid dynamic (CFD), finite element method (FEM), discrete element method (DEM) should be developed to validate the quantitative measures of robustness. The CFD can better simulate the blast load and its affected region, the DEM can better simulate the seperation and debris impact. Such a coupled numerical approach requires very extensive numerical analyses capabilities, involving switch between Eulerian domain and Lagrangian domain, implicit and explicit integration scheme, static and dynamic analysis.
14. The robustness measures should incorporate design criteria (acceptable threshold). For example, the initial damage beyond a predefined limit should not be used, the bearing capacity should be determined when some deformation limit is achieved, or the disproportionate collaspe occurs when some design damage extent is reached.

Acknowledgments

The work presented in this paper was supported by the National Natural Science Foundation of China with grant 51538002.

References

Abdelwahed B. (2019). “A review on building progressive collapse, survey and discussion”. Case Studies in Construction Materials, 11, e00264.

Adam JM, Parisi F, Sagaseta J, Lu X. (2018). “Research and practice on progressive collapse and robustness of building.”structures in the 21st Century. *Eng Struct*,173, 122-49.

Agarwal, J., D. Blockley, and N. Woodman. (2001). “Vulnerability of 3-dimensional trusses.” *Structural Safety*, 23(3), 203-220.

Agarwal, J., D. Blockley, and N. Woodman. (2003). “Vulnerability of structural systems.” *Structural Safety*, 25(3), 263-286.

Agarwal J., England J., (2008). “Recent Developments in Robustness and Relation with Risk.” *Proc. Inst. Civ. Eng., Struct. Build*., 161, 4, 183-188.

Agarwal J, Haberland M, Holickỳ M, Sykora M, Thelandersson S. (2012), “Robustness of structures: Lessons from failures.” *Struct Eng Int*, 22, 105-11.

ASCE 7. (2010). “American Society of Civil Engineers Standard 7 Minimum Design Loads for Buildings and Other Structures.” American Society of Civil Engineers.

Azim I., Yang J., Bhatta S. et al. (2020). “Factors influencing the progressive collapse resistance of RC frame structures.” *Journal of Building Engineering*, 27, 100986.

Baker, J.W., Schubert, M. and Faber, M.H. (2008), “On the assessment of robustness.” *Structural Safety*, 30, 253-267.

Baker, J. F., Williams, E. L., and Lax, D. (1948). “The civil engineer in war—Volume 3: Properties of materials, structures, hydraulics, tunneling and surveying.” Symp. of Papers, Institution of Civil Engineers, London.

Bao Y.H., Main J.A. Noh S.Y. (2017). “Evaluation of Structural Robustness against Column Loss: Methodology and Application to RC Frame Buildings.” *Journal of Structural Engineering*, 143(8), 04017066.

Beeby, A. (1999). “and a new approach to robustness. Structural Engineer.”, *Safety of structures*, 77(4).

Biondini F., Frangopol D. M., Restelli S. (2008). “On Structural Robustness, Redundancy and Static Indeterminacy.” Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, Reston, Va.

Bita H.M., Huber J.A.J., Voulpiotis K., and Tannert T. (2019). “Survey of contemporary practices for disproportionate collapse prevention.” *Engineering Structures,* 199, 109578.

Blockley, D., et al. (2002). Structural vulnerability, reliability and risk. P*rogress in Structural Engineering and Materials*, 4(2), 203-212.

Bontempi F., Giuliani L., Gkoumas K. (2007).” Handling the Exceptions: Robustness Assessment of a Complex Structural System.” Proc. of the 3rd Int. Conf on Struct. Engng., Mech. A. Comp. (SEMC 2007), Millpress, Rotterdam, *The Netherlands*, 1747-1752.

Brett C, Lu Y. (2013). “Assessment of robustness of structures: current state of research.” *Front Struct Civ Eng*, 7(4): 356-68.

Byfield M, Mudalige W, Morison C, Stoddart E. (2014). “A review of progressive collapse research and regulations.”

Proc ICE-Struct Build, 167,447-56.

Canisius TDG, Sorensen JD, Baker JW. (2007). “Robustness of structural systems - a new focus for the Joint Committee on Structural Safety (JCSS).” Proceedings of the 10th international conference on applications of statistics and probability in civil engineering (ICASP10)., London, Taylor and Francis.

Chen C.H., Zhu Y.F., Yao Y. et al. (2016a). “An evaluation method to predict progressive collapse resistance of steel frame structures.” *Journal of Constructional Steel Research*, 122, 238-250.

Chen Y.L., Huang L. and Lu Y.Q. et al. (2016b). “Assessment of Structural Robustness under Different Events according to Vulnerability.” Journal of Performance of Constructed Facilities, 30(5), 04016004.

Choi J.H. and Chang D.K. (2009). “Prevention of progressive collapse for building structures to member disappearance by accidental actions.” J*ournal of Loss Prevention in the Process Industries*, 22, 1016-1019.

Chung K., Park C., and Kim D. (2016). “Design Considerations for Concrete High-Rise Buildings.” *International Journal of High-Rise Buildings*, 5(3), 187-193.

Chung K. and Yoo S. (2019). “Structural Design and Construction of Mega Braced Frame System for Tall Buildings.” *International Journal of High-Rise Buildings*, 8(3), 169-175.

Corley, W. G., Mlakar, P., Sozen, M., and Thornton, C. (1998). “The Oklahoma City bombing: Summary and recommendations for multihazard mitigation.” *J. Perform. Constr. Facil.*, 123, 100-112.

DoD. (2013) Design of Buildings to Resist Progressive Collapse. Unified Facilities Criteria (UFC) 4-023-03, US Department of Defense (DoD).

EN 1991-1-7. (2010).Actions on structures - Part 1-7: General actions -Accidental actions.

Ellingwood, B. & Leyendecker, E.V., (1978), “Approaches for design against progressive collapse”, *J. Struct. Div.*, 104(3), 413-423.

Ellingwood B.R., (2005). “Strategies for mitigating risk of progressive collapse.” Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, 2005.

Ellingwood, B.R., and Dusenberry D.O., (2005). “Building design for abnormal load and progressive collapse”, *Computer-Aided Civil and Infrastructure Engineering*, 20, 194-205.

Ellingwood BR., (2006). “Mitigating risk from abnormal loads and progressive collapse.” *J Perform Constr Facil*. 20(4):315-23.

Ellingwood B.R. (2009). “Abnormal loads and disproportionate collapse: risk mitigation strategies.” Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, 2009.

El-Tawil S, Li H, Kunnath S. (2013). “Computational simulation of gravity-induced progressive collapse of steel-frame buildings: Current trends and future research needs.” *J Struct Eng*, 140, A2513001.

England, J., J. Agarwal, and D. Blockley., (2008). “The vulnerability of structures to unforeseen events.” *Computers & Structures*. 86(10), 1042-1051.

Faber M.H. (2006). “Robustness of Structures: An Introduction.” *Structural Engineering International*, 16(2), 101.

Fallon CT, Quiel SE, Naito CJ., (2016), “Uniform pushdown approach for quantifying building-frame robustness and the consequence of disproportionate collapse.” *J Perform Constr Facil*. 30(6), 04016060

Fang Z.X. and Li H.Q. (2007). “Safety and robustness of structures from the viewpoint of energy.” *Journal of Building Structures*, 28, 269-273. (in Chinese)

Felipe T.R.C., Haach V.G., Beck A. (2018). “Systematic reliability-based approach to progressive collapse.” *Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 4(4), 04018039

Feng D.C., Xie S.C., Xu J., and Qian K. (2020). “Robustness quantifification of reinforced concrete structures subjected to progressive collapse via the probability density evolution method.” *Engineering Structures*, 202, 109877.

Feng, Y.Y. and F. Moses, (1986). “Optimum design, redundancy and reliability of structural systems.” *Computers & structures*, 24(2),239-251.

Formisano A., Landolfo R. and Mazzolani F.M. (2015). “Robustness assessment approaches for steel framed structures under catastrophic events.” *Computers and Structures*, 147, 216-228.

Frangopol, D. M., and Nakib, R. (1991). “Redundancy in highway bridges.” *Engineering. Journal.*, 28(1), 45-50.

Frangopol, D. and J. Curley, 1987, “Effects of Damage and Redundancy on Structural Reliability.” *Journal of Structural Engineering,* 113(7): 1533-1549.

Fu F. (2016). “Review of Structural Analysis and Design to Prevent Disproportionate Collapse.” CRC Press, Taylor & Francis Group, Boca Raton.

Fu, G. and D.M. Frangopol, (1990). “Balancing weight, system reliability and redundancy in a multiobjective optimization framework.” *Structural Safety*, 1990. 7(2-4): 165-175.

Gao Y. (2009). “Importance coefficients of components in quantitative evaluation of structural robustness.” PhD Thesis, Shanghai Jiao Tong University, Shanghai (in Chinese)

Gao Y., Liu X.L.(2008). “Importance coeffificients of components in evaluation of structure robustness.” *J. Rock Mech. Eng*. 27(12), 2575-2584 (in Chinese)

Gharaibeh E., Frangopol D.M. and Onoufriou T. (2002). “Reliability-based importance assessment of structural members with applications to complex structures.” *Computers and Structures*, 80, 1113-1131.

Ghosn, M., and Moses, F. (1998). “Redundancy in highway bridge superstructures.” National Cooperative Highway Research Program (NCHRP), Washington, DC.

Ghosn M., Moses F., Frangopol D.M. (2010). “Redundancy and robustness of highway bridge superstructures and substructures.” *Structure and Infrastructure Engineering*, 6(1), 257-278.

Ghosn M., Duenas-Osorio L., Frangopol D.M. et al. (2016a). “Performance indicators for structural systems and infrastructure networks.” *Journal of Structural Engineering*, 142(9), F4016003.

Ghosn M., Frangopol D.M., McAllister T.P. et al. (2016b). “Reliability-based performance indicators for structural members.” *Journal of Structural Engineering*, 142(9), F4016002.

Gross J.L. and McGuire W. (1983). “Progressive collapse resistant design.” *Journal of Structural Engineering*, 109(1), 1-15.

GSA. (2005). “Progressive Collapse Analysis and Design Guidelines for New Federal Office Buildings and Major Modernization Projects.” US General Services Administration (GSA).

Ha T., Kim S., and Lee S. (2017). “Prediction of Time-dependent Lateral Movement Induced by Differential Shortening in Tall Buildings Using Construction Stage Analysis.” *International Journal of High-Rise Buildings*, 6(1), 11-19.

Haberland, M. (2007). “Progressiver Kollaps und Robustheit (Progressive collapse and robustness).” Thesis (diploma). Hamburg University of Technology, Structural Analysis and Steel Structures Institute, Hamburg, Germany.

Hansen E., Wong F. Lawver D. et al. (2005). “Development of an analytical database to support a fast running progressive collapse assessment tool.” Structures.

Hendawi, S. and Frangopol, D.M. (1994). “System reliability and redundancy in structural design and evaluation.” *Structural Safety*, 16 (1-2), 47-71.

Hu X.B. (2007). “Studies on the progressive collapse resistance behavior of the new type polyhedral space frame.” Thesis, Tsinghua University, Beijing. (in Chinese)

Huang L. and Li L.(2012). “A quantification method of structural robustness.” *Engineering Mechanics*, 29(8): 213-219. (in Chinese)

Huang Y.Z. and Wang Z. (2012). “An approach for evaluation of the structural robustness of steel frames.” *China Civil Engineering Journal*, 45(9), 46-54. (in Chinese)

Huo J.S., Hu C.L., Zhang J.Q. et al. (2012). “Analysis of dynamic behaviors and ductility of steel moment frame connections.” *Journal of Civil, Architectural & Environental Engineering*, 34, 149-154. (in Chinese)

Husain M. and Tsopelas P. (2004). “Measures of Structural Redundancy in Reinforced Concrete Buildings.” I: Redundancy Indices. *Journal of Structural Engineering*, 130(11), 1651-1658.

Isobe D. (2017). “Progressive collapse analysis of structures: numerical codes and applications.” Butterworth-Heinemann.

IStructE. (2002). “Safety in tall buildings and other buildings with large occupancy.” London: Institution of Structural Engineers.

Izzuddin, B.A., Vlassis, A.G., Elghazouli, A.Y. and Nethercot, D.A., (2008). “Progressive Collapse of Multi-Storey Buildings due to Sudden Column Loss – Part I: Simplified Assessment Framework.” *Engineering Structures*, Vol. 30, No. 5,308-1318.

JCSS (2008) Risk Assessment in Engineering. Principles, System Representation and Risk Criteria. Joint Committee of Structural Safety (JCSS), Zürich, Switzerland.

Jiang J. and Li G.Q. (2018). “Progressive Collapse of Steel High-Rise Buildings Exposed to Fire: Current State of Research.” *International Journal of High-Rise Buildings*, 7(4), 375-387.

JSSC. (2005). “Guildlines for Collapse Control Design: Construction of Steel Buildings with High Redundancy Design.” Japanese Society of Steel and Construction (JSSC) and Council on Tall Buildings and Urban Habitat(CTBUH), Japan Iron and Steel Federation.

Khandelwal K. and El-Tawil S. (2005). “Progressive collapse of moment resisting steel frame buildings.” Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, 2005.

Khandelwal K., El-Tawil S. (2011). “Pushdown resistance as a measure of robustness in progressive collapse analysis.” *Engineering Structures*, 33, 2653-2661.

Kiakojouri F., Biagi V.D., Chiaia B., et al. (2020). “Progressive collapse of framed building structures: Current knowledge and future prospects.” *Engineering Structures*, 206, 110061.

Kokot S, Solomos G. (2012). “Progressive collapse risk analysis: literature survey, relevant construction standards and guidelines.” Ispra: Joint Research Centre, European Commission; 2012.

Kong D.Y., Yang Y., Yang B., Zhou X.H. (2020). “Experimental Study on Progressive Collapse of 3D Steel Frames under Concentrated and Uniformly Distributed Loading Conditions.” *Journal of Structural Engineering*, 146(4), 04020017.

Krauthammer T. Modern protective structures. Boca Raton: CRC Press; 2008.

Kunnath S.K., Bao Y.H. and El-Tawil S. (2018). “Advances in Computational Simulation of Gravity-Induced Disproportionate Collapse of RC Frame Buildings.” *Journal of Structural Engineering*, 144(2), 03117003.

Kwon K., Park S., and Kim J. (2012). “Evaluation of Progressive Collapse Resisting Capacity of Tall Buildings.” *International Journal of High-Rise Buildings*, 1(3), 229-235.

Li G.Q., Zhang C. and Jiang J. (2018a). “A Review on Fire Safety Engineering: Key Issues for High-Rise Buildings.” *International Journal of High-Rise Buildings*, 7(4), 265-285.

Li L.L., Li G.Q., Jiang B.H., Lu Y. (2018b). “Analysis of robustness of steel frames against progressive collapse.” *Journal of Constructional Steel Research*, 143, 264-278.

Li S., Petrone F. and Kunnath S. (2019). “Robustness of RC buildings to progressive collapse: Influence of building height.” *Engineering Structure*s, 183, 690-701.

Liao, K.-W., Y.-K. Wen, and D. Foutch. (2007). “Evaluation of 3D steel moment frames under earthquake excitations. II: Reliability and redundancy.” *Journal of structural engineering*, 133(3), 471-480

Lin H., Yang L., Chen G.M. et al. (2019). “A novel methodology for structural robustness assessment of offshore platforms in progressive collapse.” *Journal of Loss Prevention in the Process Industries*, 62, 103966.

Lind, N.C. (1995), “A measure of vulnerability and damage tolerance.” *Reliability Engineering and System Safety*, Vol. 48, No. 1, 1-6.

Liu C.M., and Liu X.L. (2005). “Stiffness-based evaluation of component importance and its relationship with redundancy.” *Journal of Shanghai Jiao Tong University*, 39(5), 746-750. (in Chinese)

Lv D.G., Song P.Y., Cui S.S., and Wang M.X. (2011). “Structural robustness and its assessment indicators.”

Journal of Building Structures, 32(11), 44-54. (in Chinese)

Maes, M. A. (1995). "Tail heaviness in structural reliability Proceedings of the 7th International Conference on Applications of Statistics and Probability in Civil Engineering Reliability and Risk Analysis." eds. Lemaire, M. et al, Paris, France, 1995.

Maes, M.A., Fritzsons, K.E. and Glowienka, S. (2006), "Structural robustness in the light of risk and consequence analysis", IABSE, *Structural Engineering International*, Vol. 16, No. 2, 101-107.

Marchand K. and Stevens D.J. (2015). "Progressive Collapse Criteria and Design Approaches Improvement." Journal of Performance of Constructured Facilities, 29(5), B4015004.

Masoero E., Daro P. and Chiaia B.M. (2013). "Progressive collapse of 2D framed structures: an analytical model." Engineering Structures, 54, 94-102.

Menchel K., Massart T.J., Rammer Y. and Bouillard P. (2009). "Comparison and Study of Different Progressive Collapse Simulation Techniques for RC Structures." *Journal of Structural Engineering*, 135(6), 685-697.

Mohamed O.A. (2006). "Progressive Collapse of Structures: Annotated Bibliography and Comparison of Codes and Standards." *Journal of Performance of Constructured Facilities*, 20(4), 418-425.

Nafday A.M. (2008). "System Safety Performance Metrics for Skeletal Structures." *Journal of Structural Engineering*, 134(3), 499-504.

Nafday AM. (2011). "Consequence-based structural design approach for black swan events." *Struct Saf*, 33(1):108-14.

Nair R.S. (2006). "Preventing disproportionte collapse." *Journal of Performance of Constructured Facilities*, 20(4), 309-314.

Nazri F.M., Zolkifle M. Mansor H. et al. (2017). "Vulnerability Assessment of Building Frames Subjected to Progressive Collapse Caused by Earthquake." *MATEC Web of Conferences*, 103, 02019.

NIST. (2007). "Best practices for reducing the potential for progressive collapse in buildings", NISTIR 7396, National Institute of Standards and Technology, Gaithersburg, MD (194 pp).

Pandey P.C. and Barai S.V. (1997). "Structural sensitivity as a measure of redundancy." *Journal of Structural Engineering*, 123(3), 360-364.

Pantidis P. and Gerasimidis S. (2017). "New Euler-Type Progressive Collapse Curves for Steel Moment-Resisting Frames: Analytical Method." *Journal of Structural Engineering*, 143(9), 04017113

Parisi F, Augenti N. (2012), "Inflfluence of seismic design criteria on blast resistance of RC framed buildings: A case study." *Eng Struct*, 2012;44:78-93.

Pearson C. and Delatte N. (2005). "Ronan Point apartment tower collapse and its effect on building codes." *Journal of Performance of Constructured Facilities*, 19(2), 172-177.

Pinto J.T., Blockley D.I., Woodman N.J. (2002). "The risk of vulnerable failure." *Structural Safety*, 24, 107-122.

Qian K. and Li B. (2015). "Research Advances in Design of Structures to Resist Progressive Collapse." *Journal of Performance of Constructured Facilities*, 29(5), B4014007

Russell J.M., Sagaseta J., Cormie D., Jones A.E.K. (2019). "Historical review of prescriptive design rules for robustness after the collapse of Ronan Point." *Structures*, 20, 365-373.

Smilowitz R. and Tennant D. (2001). "Multi-Hazard Design to Resist Progressive Collapse." Structures.

Smith, J.W.(2006), "Structural robustness analysis and the fast fracture analogy", IABSE, *Structural Engineering International*, Vol. 16, No. 2, 118-123, 2006.

Starossek U. (2006). "The 2006 Annual Conference of the Structural Engineering Committee of the Korean Society of Civil Engineers." Progressive collapse of structures. Seoul, Korea, May 25, 2006

Starossek, U., (2007). "Typology of progressive collapse." *Engineering Structures*, 29 (9), 2302-2307.

Starossek U. and Haberland M. (2008). "Measures of structural robutness-requirements and applications." Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, 2008.

Starossek, U. (2009). "Progressive collapse of structures." Thomas Telford Ltd., London.

Starossek U, Haberland M. (2010) "Disproportionate collapse: terminology and procedures." *J Perform Construct Facil*; 24(6): 519-28.

Starossek U, Haberland M. (2011). "Approaches to measures of structural robustness." *Structure and Infrastructure Engineering*, 7, 625-631.

Starossek U, Smilowitz R., Waggoner M. (2011). "Report of the Terminology and Procedures Sub-Committee (SC1): Recommendations for Design against Disproportionate Collapse of Structures." Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, 2011.

Starossek U. (2018). Progressive collapse of structures. 2nd ed. London: ICE Publishing,.

Stevens D., Crowder B., Sunshine D. et al. (2011). "DoD Research and Criteria for the Design of Buildings to Resist Progressive Collapse." *Journal of Structural Engineering*, 137(9), 870-880.

Stewart M.G. (2017). "Risk of Progressive Collapse of Buildings from Terrorist Attacks: Are the Benefits of Protection Worth the Cost?" *Journal of Performance of Constructured Facilitie*s, 31(2), 04016093.

Stochino F, Bedon C, Sagaseta J, Honfifi D. (2019). "Robustness and resilience of structures under extreme loads." *Adv Civil Eng*, 2019;2019:1-14.

Subki N.E.A., Mansor H., Hamid Y.S. Parke G. (2019). "Progressive Collapse Assessment: A review of the current energy-based Alternate Load Path (ALP) method." MATEC Web of Conferences, 258, 02012.

Szyniszewski S. (2009). "Dynamic energy based method for progressive collapse analysis." Proc. of ASCE/SEI Struct. Congress, ASCE/SEI, 2009.

Szyniszewski S. and Krauthammer T. (2012). "Energy flow in progressive collapse of steel framed buildings." *Engineering Structures*, 42, 142-153.

Sørensen JD. (2011). "Framework for robustness assessment of timber structures." *Eng Struct* 2011; 33(11):3087-92.

Tsai M.H. (2012). "A performance-based design approach for retrofifitting regular building frames with steel braces against sudden column loss." *Journal of Constructional Steel Research*, 77, 1-11.

Tsopelas P. and Husain M. (2004). "Measures of Structural Redundancy in Reinforced Concrete Buildings. II: Redundancy response modification factor RR." *Journal of Structural Engineering*, 130(11), 1659-1666.

Vlassis, A.G., Izzuddin, B.A., Elghazouli, A.Y. and Nethercot, D.A., (2009) "Progressive Collapse of Multi-Storey Buildings due to failed floor impact", *Engineering Structures*, Vol. 31, 1522-1534.

Vrouwenvelder T. (2008). "Treatment of Risk and Reliability in the Eurocodes." *Proc. Inst. Civ. Eng*., *Struct. Build*., 161, 4, 209-214.

Wilkes J. and Krauthammer T. (2019). "An energy flow apporach for progressive collapse assessment." *Engineering Structures*, 190, 333-344.

Wisniewski, D., Casas, J.R. and Ghosn, M. (2006), "Load capacity evaluation of existing railway bridges based on robustness quantification", IABSE, *Structural Engineering International*, Vol. 16, No. 2, 161-166.

Xiao H. and Hedegaard B. (2018). "Flexural, Compressive Arch, and Catenary Mechanisms in Pseudostatic Progressive Collapse Analysis." *Journal of Performance of Constructured Facilities*, 32(1), 04017115.

Xu G.Q. and Ellingwood B.R. (2011). Disproportionate collapse performance of partially restrained steel frames with bolted T-stub connections. Engineering Structures, 33, 32-43.

Yan S., Zhao X.Z., Rasmussen K.J.R. and Zhang H. (2019). "Identifification of critical members for progressive collapse analysis of single-layer latticed domes." *Engineering Structures*, 188, 111-120.

Ye J.H. and Jiang L.Q. (2018). "Collapse mechanism analysis of a steel moment frame based on structural vulnerability theory." *Archives of Civil and Mechanical Engineering*, 18, 833-843.

Ye L.P., Lin X.C., Qu Z. et al. (2010). "Evaluating method of element importance of structural system based on generalized structural stiffness." *Journal of Architecture and Civil Engineering*, 27(1), 1-6. (in Chinese)

Zhang L.M. and Liu X.L. (2007). "Network of energy transfer in frame structures and its preliminary application." *China Civil Engineering Journal*, 40(3), 45-49. (in Chinese)

Zhou X.H. and Liu J.P. (2019). "Application of Steel-tubed Concrete Structuresin High-rise Buildings." International Journal of High-Rise Buildings, 8(3), 161-167.

Zio E. (2016). "Challenges in the vulnerability and risk analysis of critical Infrastructures." *Reliability Engineering and System Safety*, 152, 137-150.

Appendix A. List of definitions of disproportionate and progressive collapse

Source	Definition of disproportionate or progressive collapse
ASCE 7 2010	The spread of an initial local failure from element to element, eventually resulting in the collapse of an entire structure or a disproportionately large part of it.
GSA 2005	Progressive collapse is a situation where local failure of a primary structural component leads to the collapse of adjoining members which, in turn, leads to additional collapse. Hence, the total damage is disproportionate to the original cause.
UFC 4-010-01 DoD 2013	Progressive collapse. A chain reaction failure of building members to an extent disproportionate to the original localized damage.
NISTIR 7396 NIST 2007	Progressive collapse-The spread of local damage, from an initiating event, from element to element, resulting, eventually, in the collapse of an entire structure or a disproportionately large part of it; also known as disproportionate collapse.
Building Regulations 2010	The building shall be constructed so that in the event of an accident the building will not suffer collapse to an extent disproportionate to the cause.
JSSC 2005	In cases when loads are larger than assumed in the design wok, the base metal will not show plastic deformation and will cause collapse. Because peripheral members including connections are then subjected to increased loads, the collapse further advances. This chain-reaction fracture and collapse phenomenon are called progressive collapse.
Allen and Schriever 1972	Progressive collapse can be defined as the phenomenon in which local failure is followed by collapse of adjoining members which in turn is followed by further collapse and so on, so that widespread collapse occurs as a result of local failure.
Gross and McGuire 1983	A progressive collapse is characterized by the loss of load-carrying capacity of a relatively small portion of a structure due to an abnormal load which, in turn, triggers a cascade of failure affecting a major portion of the structure. Progressive collapse is a situation in which a localized failure in a structure, caused by abnormal load, triggers a cascade of failure affecting a major portion of the structure.
Smilowitz and Tennant 2001	Progressive collapse occurs when an initiating localized failure causes adjoining members to be overloaded and fail, resulting in an extent of damage that is disproportionate to the originating region of localized failure.
Hansen et al. 2005	When an initiator event causes a local failure in a building, the resulting failure front will propagate throughout the structure until the failure front is arrested, or until the remaining structure becomes geometrically unstable.
Khandelwal and El-Tawil 2005	Progressive collapse occurs when local failure of a primary structural component leads to the failure and collapse of adjoining members, possibly promoting additional collapse.
Ellingwood 2005	A progressive collapse is a catastrophic partial or total collapse that initiates from local structural damage and propagates, by a chain reaction mechanism, into a failure that is disproportionate to the local damage caused by the initiating event.
Ellingwood and Dusenberry 2005	A catastrophic partial or total structural failure that ensues from an event that causes local structural damage that cannot be absorbed by the inherent continuity and ductility of the structural system
Ellingwood 2006	A progressive collapse initiates as a result of local structural damage and develops, in a chain reaction mechanism, into a failure that is disproportionate to the initiating local damage.
Mohamed 2006	Progressive collapse of building structures is initiated by the loss of one or more load-carrying members. As a result, the structure will seek alternate load paths to transfer the load to structural elements, which may or may not have been designed to resist the additional loads. Failure of overloaded structural elements will cause further redistribution of loads, a process that may continue until stable equilibrium is reached. Equilibrium may be reached when a substantial part of the structure has already collapsed. The resulting overall damage may be disproportionate to the damage in the local region near the lost member.
Nair 2006	"Disproportionate collapse" is structural collapse disproportionate to the cause. In structures susceptible to this type of collapse, small events can have catastrophic consequences. Disproportionate collapse is often, though not always, progressive, where "progressive collapse" is the collapse of all or a large part of a structure precipitated by damage or failure of a relatively small part of it.
Starossek 2006	Progressive collapse is characterized by a distinct disproportion between the triggering spatially-limited failure and the resulting widespread collapse.

Appendix A. Continued.

Canisius et al. 2007	Progressive collapse, where the initial failure of one or more components results in a series of subsequent failures of components not directly affected by the original action is a mode of failure that can give rise to disproportionate failure.
Agarwal and England 2008	Disproportionate collapse results from small damage or a minor action leading to the collapse of a relatively large part of the structure. Progressive collapse is the spread of damage through a chain reaction, for example through neighboring members or storey by storey. Often progressive collapse is disproportionate but the converse may not be true.
Krauthammer 2008	Progressive collapse is a failure sequence that relates local damage to large scale collapse in a structure.
Ellingwood 2009	A disproportionate (or progressive) collapse of a structure is one that initiates from local damage and, rather than being arrested by the capability of the structural system to redistribute forces and bridge around the damaged area, propagates to a final damage state that involves a major portion of the structure.
Menchel et al. 2009	one or several structural members suddenly fail, whatever the cause accident or attack. The building then collapses progressively, every load redistribution causing the failure of other structural elements, until the complete failure of the building or of a major part of it.
Starossek and Haberland 2010	Disproportionate collapse: A collapse that is characterized by a pronounced disproportion between a relatively minor event and the ensuing collapse of a major part or the whole of a structure. Progressive collapse: A collapse that commences with the failure of one or a few structural components and then progresses over successively affected other components.
Xu and Ellingwood 2011	A disproportionate (or progressive) collapse of a structure is initiated by local damage, that cannot be contained and that propagates throughout the entire structure or a large portion of it, to the point where the extent of final damage is disproportionate to the initiating local damage.
Kokot and Solomos 2012	Progressive collapse of a building can be regarded as the situation where local failure of a primary structural component leads to the collapse of adjoining members and to an overall damage which is disproportionate to the initial cause.
Parisi and Augenti 2012	Progressive collapse is a chain reaction mechanism resulting in a pronounced disproportion in size between a relatively minor triggering event and resulting collapse, that is, between the initial amount of directly damaged elements and the final amount of failed elements.
Fallon et al. 2016	Progressive collapse of a structural system occurs when the local failure of a single primary load-bearing element or a small group of elements triggers a larger, more widespread collapse of adjoining portions of the structure.
Nazri et al. 2017	Progressive collapse is described as building collapse caused by the loss or failure of a structural load-bearing member because of load hazards. Localized failure facilitates load redistribution to the adjacent member, which then initiates partial or total progressive collapse of a building.
Pantidis and Gerasimidis 2017	Progressive collapse of structures is the phenomenon of an initial local failure mushrooming to the global level, resulting in the stiffness degradation of a relatively large part of the structure. Eventually, partial or total collapse of the structure occurs.
Xiao and Hedegaard 2018	Progressive collapse, a chain reaction or disproportionate propagation of failures following damage to a relatively small portion of a structure.
Adam et al. 2018	Progressive collapse is a collapse that begins with localised damage to a single or a few structural components and develops throughout the structural system, affecting other components.
Feng et al. 2020	the global disproportionate failure/collapse of a structure due to the initial local damage of a single or a few structural components triggered by some specific accidental events.
Kong et al. 2020	If a structure lacks sufficient robustness, local damage induced by accident events, such as blasts, impacts, fires, or a combination of such things could spread widely to the remaining parts of the structure, leading to a complete or disproportionate collapse.

Appendix B. List of definitions of robustness

Source	Definition of robustness
Eurocode 1 Part 1-7, 2006	The ability of a structure to withstand events like fire, explosions, impact or the consequences of human error without being damaged to an extent disproportionate to the original cause.
GSA	Ability of a structure or structural components to resist damage without premature and/or brittle failure due to events like explosions, impacts, fire or consequences of human error, due to its vigorous strength and toughness.
JCSS 2008	The robustness of a system is defined as the ratio between the direct risks and the total risks (total risks is equal to the sum of direct and indirect risks), for a specified time frame and considering all relevant exposure events and all relevant damage states for the constituents of the system.
IstructE 2002	the ability of an engineered structure or system that enables it to survive a potentially damaging incident or extreme event without disproportionate loss of function.
JSSC 2005	The state of being strong, tough and rigid.
Faber 2006	Robustness is broadly recognized to be a property which can not only be associated with the structure itself but must be considered as a product of several indicators: risk, redundancy, ductility, consequences of structural component and system failures, variability of loads and resistances, dependency of failure modes, performance of structural joints, occurrence probabilities of extraordinary loads and environmental exposures, strategies for structural monitoring and maintenance, emergency preparedness and evacuation plans and general structural coherence.
Starossek 2006	Insensitivity to local failure is referred to as robustness. Robustness is a property of the structure alone and independent of the cause and probability of initial local failure.
Maes et al. 2006	Robustness refers to the manner in which certain performance objectives or system properties are affected by hazardous or extreme conditions. It makes no sense to speak of a system being robust without first specifying both the feature and the perturbations of interest. Robustness is concerned with the "how and to what extent" these specified performance objectives are affected by the specified perturbations.
Agarwal and England 2008	Robustness is the ability of a structure to avoid disproportionate consequences in relation to the initial damage.
Biondini et al. 2008	Structural robustness can be viewed as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself.
Bontempi et al. 2007	The robustness of a structure, intended as its ability not to suffer disproportionate damages as a result of limited initial failure, is an intrinsic requirement, inherent to the structural system organization.
Vrouwenvelder 2008	The notion of robustness is that a structure should not be too sensitive to local damage, whatever the source of damage.
Starossek and Haberland 2010	Insensitivity of a structure to initial damage. A structure is robust if an initial damage does not lead to disproportionate collapse.
Brando et al. 2012	As the capacity of a structure to withstand damages without suffering disproportionate response to the triggering causes while maintaining an assigned level of performance.
Brett and Lu 2013	Ability of a structure in withstanding an abnormal event involving a localized failure with limited levels of consequences, or simply structural damages.
Formisano et al. 2015	Robustness is accomplished when the structure response is proportioned to the actions applied to it. These actions could appear in different ways, e.g. loads exceeding the design ones, accidental loads or damage to members.
Fallon et al. 2016	The concept of structural robustness is generally associated with the ability of a structural system to resist widespread collapse or failure as the result of an initial perturbation. This perturbation can be manifested as a change in the applied load (attributable to an extreme event), a change in the structure's capacity (because of damage), or both.
Li et al. 2019	Building robustness to progressive collapse can be generally defined as a measure of the ability of a system to remain functional in the event of the local failure of a single component or multiple connected components.