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PRIME KNOTS WITH ARC INDEX 12 UP TO 16 CROSSINGS

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ABSTRACT. We obtain the list of prime knots with arc index 12 up to 16 crossings and their minimal grid diagrams. This is a continuation of the works [5] and [8] in which Cromwell matrices were generated to obtain minimal grid diagrams of all prime knots up to arc index 11. We provide minimal grid diagrams of the prime alternating knots with arc index 12. They are the 10 crossing prime alternating knots. The full list of 19,513 prime knots of arc index 12 up to 16 crossings and their minimal grid diagrams can be found in the arXiv [6].

1. Introduction

An *arc presentation* of a knot is an embedding of a knot into the union of finitely many vertical half planes whose common boundary is the z-axis so that each of the half planes intersect the knot in a single properly embedded curve. These curves are called the *arcs* of the arc presentation. The minimal number of arcs among all arc presentations of a given knot is called the *arc index* [1,2].

A grid diagram is a knot diagram whose projection is a closed curve which is composed of finitely many horizontal line segments and the same number of vertical line segments such that vertical line segments always cross over horizontal line segments. An $n \times n$ Cromwell matrix is an $n \times n$ square matrix whose entries are either 0 or 1 such that there are exactly two 1's in each row and each column. By joining the 1's in a Cromwell matrix by horizontal line segments and vertical line segments, we obtain a grid diagram. See Figure 1.

	0	0	1	0	1
	0	1	0	1	0
┍╴│╶┘│││	1	0	1	0	0
	0	1	0	0	1
	1	0	0	1	0

FIGURE 1. A grid diagram and its Cromwell matrix

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On the other hand, a grid diagram can be converted easily to an arc presentation with the number of arcs equal to the number of vertical line segments as illustrated in Figure 2. Conversely, an arc presentation can be easily converted to a grid diagram with the number of vertical line segments equal to the number of arcs.

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Arc index	5	6	7	8	9	10	11	12	13	14	15	Total
Crossings												
3	1											1
4		1										1
5			2									2
6				3								3
7					7							7
8			1	2		18						21
9				2	6		41					49
10				1	9	32		123				165
11					4	46	135		367			552
12					2	48	211	627		1288		2176
13						49	399	1412	3250		4878	9988
14						17	477	3180				
15					1	22	441	6216				
16						7	345	7955				
17						1	192	10283	1			
18							75	8584				
19							12	6063				
20							3	3540				
21							3	1284				
22								761				
23								124				
24							1	132				
25								39				
26								3				
27												
28								1				
Total	1	1	3	8	29	240	2335	50327				

TABLE 1. Prime knots up to arc index 12 or crossing number 13

For this work, we've generated 12×12 Cromwell matrices so that they cover all prime knots with arc index 12. Using 'Knotscape' [3, 4] we identified the knots corresponding to these matrices. Then we removed those corresponding to the unknot, composite knots, links, and knots with arc index less than 12. The remaining knots have arc index 12. Finally we removed duplicates.



FIGURE 2. Construction of an arc presentation from a grid diagram

Table 1 shows the number of prime knots of given arc index and minimal crossing number. Minimal grid diagrams of the 282 knots counted for the arc index 5 to 10 appear in [5] and those of the 2,335 knots counted for the arc index 11 appear in [7,8]. In the columns of the arc index 11 and 12, the knots counted by the thirteen italicized numbers may have smaller crossing numbers but not smaller than 17. The five numbers in boldface are newly determined by this work.

2. Minimal grid diagrams of the prime knots with arc index 12

In grid diagrams, we use the convention that the vertical lines cross over the horizontal ones at every crossing. To save pages, we provide minimal grid diagrams only for the prime alternating knots with arc index 12. They are the 123 prime alternating 10 crossing knots. The full list of 19,513 prime knots of arc index 12 up to 16 crossings and their minimal grid diagrams can be found in the arXiv [6]. The labels of the grid diagrams are the DT names [9].



 $10a_{11}$





 $10a_{17}$



 $10a_{10}$

























 $10a_{23}$



 $10a_{51}$





 $10a_{57}$



 $10a_{50}$





 $10a_{52}$













 $10a_{65}$









 $10a_{63}$





 $10a_{91}$





 $10a_{97}$



 $10a_{90}$























 $10a_{105}$





 $10a_{103}$



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References

 P. R. Cromwell, Embedding knots and links in an open book. I. Basic properties, Topology Appl. 64 (1995), no. 1, 37–58. https://doi.org/10.1016/0166-8641(94)00087-J

- [2] P. R. Cromwell and I. J. Nutt, Embedding knots and links in an open book. II. Bounds on arc index, Math. Proc. Cambridge Philos. Soc. 119 (1996), no. 2, 309–319. https: //doi.org/10.1017/S0305004100074181
- [3] J. Hoste and M. Thistlethwaite, Knotscape, http://www.math.utk.edu/~morwen/ knotscape.html
- [4] J. Hoste, M. Thistlethwaite, and J. Weeks, *The first 1,701,936 knots*, Math. Intelligencer 20 (1998), no. 4, 33–48. https://doi.org/10.1007/BF03025227
- [5] G. T. Jin, H. Kim, G. Lee, J. H. Gong, H. Kim, H. Kim, and S. A. Oh, *Prime knots with arc index up to 10*, in Intelligence of low dimensional topology 2006, 65–74, Ser. Knots Everything, 40, World Sci. Publ., Hackensack, NJ, 2007. https://doi.org/10. 1142/9789812770967_0009
- [6] G. T. Jin, H. Kim, S. Lee, and H. J. Myung, Prime knots with arc index 12 up to 16 crossings, arXiv:2007.05711
- [7] G. T. Jin and W. K. Park, Prime knots with arc index up to 11 and an upper bound of arc index for non-alternating knots, J. Knot Theory Ramifications 19 (2010), no. 12, 1655–1672. https://doi.org/10.1142/S0218216510008595
- [8] _____, A tabulation of prime knots up to arc index 11, J. Knot Theory Ramifications 20 (2011), no. 11, 1537–1635. https://doi.org/10.1142/S021821651100942X
- [9] C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, http://www. indiana.edu/~knotinfo, November 25, 2020.

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