

A Maximum A Posterior Probability based Multiuser Detection Method in Space based Constellation Network

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Summary

In space based constellation network, users are allowed to enter or leave the network arbitrarily. Hence, the number, identities and transmitted data of active users vary with time and have considerable impacts on the receiver's performance. The so-called problem of multiuser detection means identifying the identity of each active user and detecting the data transmitted by each active user. Traditional methods assume that the number of active users is equal to the maximum number of users that the network can hold. The model of traditional methods are simple and the performance are suboptimal. In this paper a Maximum A Posteriori Probability (MAP) based multiuser detection method is proposed. The proposed method models the activity state of users as Markov chain and transforms multiuser detection into searching optimal path in grid map with BCJR algorithm. Simulation results indicate that the proposed method obtains 2.6dB and 1dB E_b/N_0 gains respectively when activity detection error rate and symbol error rate reach 10^{-3} , comparing with reference methods.

Keywords:

Multiuser detection; MAP; BCJR algorithm; Space based constellation network

1. Introduction

Space-based constellation network which contains geosynchronous orbiting satellites, medium orbiting satellites and low orbiting satellites, is self-organized wireless network without central nodes. In space-based constellation network, users are allowed to enter or leave the network arbitrarily which leading to the dynamic change of network topology [1-3]. Thus the number, identities and transmitted data of active users vary with time. Multiuser detection which aiming at identifying the identity of each active user and detecting data transmitted by each active user play an important role in mitigating multi-access interference and enhancing capacity of DS-CDMA system.

Over the past decades, a significant number of researchers have addressed various multiuser detection schemes. However, most past works [4-6] proceed under the assumption that the number of active users is constant, known at the receiver, and equal to the maximum number of users entitled to access the system. This assumption is often overly pessimistic, since many users might be inactive, and detection under the assumption of a number of users larger than the real one may lead to serious performance degradation. A two-stage decoder is proposed in [7], which

is a combination of two separate modules, namely, active user identifier and multiuser detector. The performance of the overall decoder is highly dependent on the performance of active user identifier. And the authors do not consider any model for the users' activity pattern, hence, the method can only be applied to static environments. A per-survivor processing (PSP) algorithm and two more algorithms built upon particle filtering methodology are suggested in [8]. Despite their extensive scope of transmission system model, the performance of the three methods is suboptimal. In this paper, a MAP-based method is developed, which uses BCJR algorithm to estimate the number, identities and transmitted data of active users. Simulation results show that the proposed method achieves a better performance comparing with the methods advocated in [8].

2. MAP based multiuser detection method

2.1 Activity state modeling

We consider a DS-CDMA space based constellation network with up to Q users. Each user is free to enter or leave the network in a random fashion with data transmitted in packets over an additive white Gaussian noise (AWGN) channel synchronously. Each user is assigned a spreading code of length M . For the k th symbol interval, the received signal can be modeled as

$$y_k(t) = x_k(t) + n(t), t \in [0, T] \quad (1)$$

where T denotes the symbol interval. $n(t)$ is the additive white Gaussian noise. $x_k(t)$ is a superposition of signals transmitted by active users whose number is unknown, which can be modeled as

$$x_k(t) = \sum_{q=1}^Q c_q(t) b_q(k), t \in [0, T] \quad (2)$$

where $b_q(k)$ is the symbol transmitted by the q th user at the k th symbol interval. If user q is active, $b_q(k)$ is assumed to be independent identically distributed (i.i.d.) random variable obeying Bernoulli distribution and taking "+1" or "-1" with equal probability where "+1" represents binary symbol "0" and "-1" represents binary symbol "1". If user q is inactive, $b_q(k)$ is assumed to be 0. $c_q(t)$ represents the spreading waveform of the q th user. It is of the form

$$c_q(t) = \sum_{m=0}^{M-1} \beta_m^q g(t - mT_c), t \in [0, T] \quad (3)$$

where $g(t)$ is a normalized chip waveform with chip duration T_c . $(\beta_0^q, \beta_1^q, \dots, \beta_{M-1}^q)$ is the spreading sequence of the q th user with a length of $M=T/T_c$, which comprising ± 1 .

Substituting (2) in (1) yields

$$y_k(t) = \sum_{q=1}^Q c_q(t) b_q(k) + n(t), t \in [0, T] \quad (4)$$

At the receiver, chip-matched filtering followed by chip rate sampling within the k th symbol interval transforms the received signal $y_k(t)$ into vector form $y_k \in \mathbb{R}^{M \times 1}$, which can be expressed as follows

$$\begin{aligned} y_k &= x_k + n_k \\ &= \sum_{q=1}^Q c_q b_q(k) + n_k \end{aligned} \quad (5)$$

where $x_k \in \mathbb{R}^{M \times 1}$ denotes the transmitted signal vector at the k th symbol interval, $n_k \in \mathbb{R}^{M \times 1}$ is the AWGN vector at the k th symbol interval with zero-mean and auto-covariance matrix $\sigma^2 I_M$. $c_q = [\beta_0^q, \beta_1^q, \dots, \beta_{M-1}^q]$ denotes the spreading code vector of the q th user. Rewrite (5) in compact form

$$\begin{aligned} y_k &= x_k + n_k \\ &= \begin{bmatrix} c_1 & c_2 & \dots & c_Q \end{bmatrix} \begin{bmatrix} b_1(k) \\ b_2(k) \\ \vdots \\ b_Q(k) \end{bmatrix} + n_k \\ &= CB_k + n_k \end{aligned} \quad (6)$$

where $C \in \{\pm 1\}^{M \times Q}$ is the spreading code matrix consisting of the spreading codes of Q users in the network. $B_k = [b_1(k), \dots, b_Q(k)] \in \{\pm 1, 0\}^{Q \times 1}$ is a ternary Q -dimensional vector constituted by ± 1 and 0, representing transmitted symbols of Q users at the k th symbol interval.

We define a random variable $u_q(k)$ to indicate the state of the q th user at the k th symbol interval, i.e.

$$u_q(k) = \begin{cases} 1, & \text{if user } q \text{ is active at the } k\text{th symbol interval} \\ 0, & \text{if user } q \text{ is inactive at the } k\text{th symbol interval} \end{cases} \quad (7)$$

$u_q(k)$ is completely determined by $b_q(k)$, i.e.

$$u_q(k) = \begin{cases} 1, & b_q(k) = \pm 1 \\ 0, & b_q(k) = 0 \end{cases} \quad (8)$$

Based on $u_q(k)$ a dynamic model for the activity or inactivity of user can be easily defined through the following probabilities:

$$\begin{aligned} p(u_q(0) = 0) &= 1 - \phi \\ p(u_q(0) = 1) &= \phi \\ p(u_q(k) = 1 | u_q(k-1) = 1) &= \mu \\ p(u_q(k) = 0 | u_q(k-1) = 1) &= 1 - \mu \\ p(u_q(k) = 1 | u_q(k-1) = 0) &= \eta \\ p(u_q(k) = 0 | u_q(k-1) = 0) &= 1 - \eta \end{aligned} \quad (9)$$

where ϕ is the a priori probability of a user being active at the beginning of transmission. μ is the probability of a user being active at the k th symbol interval, conditional on the fact that it was already active at the $k-1$ th symbol interval, and η is the probability of a user being active at the k th symbol interval given that it was inactive at the $k-1$ th symbol interval. The model described by (9) implies that $u_q(k)$ for a fixed q is a Markov chain. Due to the sequence of $u_q(k)$ being modeled as a Markov chain, the sequence of $b_q(k)$ can be also shown to be a Markov chain. In particular

$$\begin{aligned} p(b_q(0) = 0) &= 1 - \phi \\ p(b_q(0) = 1) &= \frac{\phi}{2} \\ p(b_q(0) = -1) &= \frac{\phi}{2} \\ p(b_q(k) = 1 | b_q(k-1) = 1) &= \frac{\mu}{2} \\ p(b_q(k) = -1 | b_q(k-1) = 1) &= \frac{\mu}{2} \\ p(b_q(k) = 0 | b_q(k-1) = 1) &= 1 - \mu \\ p(b_q(k) = 1 | b_q(k-1) = -1) &= \frac{\mu}{2} \\ p(b_q(k) = -1 | b_q(k-1) = -1) &= \frac{\mu}{2} \\ p(b_q(k) = 0 | b_q(k-1) = -1) &= 1 - \mu \\ p(b_q(k) = 1 | b_q(k-1) = 0) &= \frac{\eta}{2} \\ p(b_q(k) = -1 | b_q(k-1) = 0) &= \frac{\eta}{2} \\ p(b_q(k) = 0 | b_q(k-1) = 0) &= 1 - \eta \end{aligned} \quad (10)$$

2.2 Construction of grid map

Consider a packet with K symbols. The transmitted symbol of Q users can be written as

The probability density function $p(S', S, Y)$ can be expressed as Eq.(21)

$$p(S', S, Y) = p(S', S, Y_{t < k}, y_k, Y_{t > k}) \quad (21)$$

where $Y_{t < k}$ represents the portion of the received signal Y before symbol interval k , $Y_{t > k}$ represents the portion of the received signal Y after symbol interval k , and y_k represents the received signal at symbol interval k . According to Bayes rule

$$\begin{aligned} p(S', S, Y) &= p(S', S, Y_{t < k}, y_k) p(Y_{t > k} | S', S, Y_{t < k}, y_k) \\ &= p(S', Y_{t < k}) p(S, y_k | S', Y_{t < k}) p(Y_{t > k} | S', S, Y_{t < k}, y_k) \\ &= p(S', Y_{t < k}) p(S, y_k | S') p(Y_{t > k} | S) \end{aligned} \quad (22)$$

where the last equality follows from the fact that the received signal at k th symbol interval depends only on the state at k th column. Defining

$$\begin{aligned} \alpha_k(S') &\equiv p(S', Y_{t < k}) \\ \gamma_k(S', S) &\equiv p(S, y_k | S') \end{aligned} \quad (23)$$

$$\beta_{k+1}(S) \equiv p(Y_{t > k} | S)$$

Substituting Eq.(23) into Eq.(22), we obtain

$$p(S', S, Y) = \alpha_k(S') \gamma_k(S', S) \beta_{k+1}(S) \quad (24)$$

Perform forward recursion on $\alpha_{k+1}(S)$, We have

$$\begin{aligned} \alpha_{k+1}(S) &= p(S, Y_{t < k+1}) \\ &= \sum_{S' \in \delta_k} p(S', S, Y_{t < k+1}) \\ &= \sum_{S' \in \delta_k} p(S', S, Y_{t < k}, y_k) \\ &= \sum_{S' \in \delta_k} p(S', Y_{t < k}) p(S, y_k | S', Y_{t < k}) \\ &= \sum_{S' \in \delta_k} p(S', Y_{t < k}) p(S, y_k | S') \\ &= \sum_{S' \in \delta_k} \alpha_k(S') \gamma_k(S', S) \end{aligned} \quad (25)$$

where δ_k is the set of all states at the k th column.

Forward recursion begins from the first column with initial condition shown in Eq.(26) since the searching process starts from an arbitrary state.

$$\alpha_1(S) = 1/3^Q \quad (26)$$

Similarly, backward recursion of $\beta_k(S)$ can be performed as follow

$$\beta_k(S') = \sum_{S \in \delta_{k+1}} \gamma_k(S', S) \beta_{k+1}(S) \quad (27)$$

where δ_{k+1} is the set of all states at the $k+1$ th column.

Backward recursion begins from the last column with initial condition shown in Eq.(28) since the searching process ends to an arbitrary state.

$$\beta_{K+1}(S) = 1/3^Q \quad (28)$$

$\gamma_k(S', S), k=1, \dots, K$ can be expressed as follow

$$\begin{aligned} \gamma_k(S', S) &\equiv p(S, y_k | S') \\ &= \frac{p(S', S, y_k)}{p(S')} \\ &= \frac{p(S', S, y_k)}{p(S', S)} \times \frac{p(S, S')}{p(S')} \\ &= p(y_k | S', S) p(S | S') \\ &= p(y_k | e) p(S | S') \end{aligned} \quad (29)$$

where e is the input vector belonging to the branch connecting state S' and S . For an AWGN channel with noise variance σ^2 , we have

$$p(y_k | e) = (2\pi\sigma^2)^{-M/2} \exp\left(-\frac{\|y_k - Ce\|^2}{2\sigma^2}\right) \quad (30)$$

By taking into account that the symbol transmitted by different users are independent, $p(S|S')$ can be decomposed as

$$p(S | S') = \prod_{q=1}^Q p(s_q | s_q') \quad (31)$$

where s_q' and s_q are possible values of $b_q(k)$ and $b_q(k+1)$. Therefore, $p(s_q | s_q')$ has four possible values $\mu/2$, $1-\mu, \eta/2$, $1-\eta$ shown in (10). Thus, substituting (30) and (31) into (29) we obtain

$$\gamma_k(S', S) = (2\pi\sigma^2)^{-M/2} \exp\left(-\frac{\|y_k - Ce\|^2}{2\sigma^2}\right) \prod_{q=1}^Q p(s_q | s_q') \quad (32)$$

We simplify the calculations of (25), (27) and (32) by making use of the identity shown in (33) and (34) to replace the computationally more difficult operation $\ln(e^x + e^y)$ with a max function plus a lookup table for evaluating $\ln(1 + e^{-|x-y|})$.

$$\max^*(x, y) \equiv \ln(e^x + e^y) = \max(x, y) + \ln(1 + e^{-|x-y|}) \quad (33)$$

$$\begin{aligned} \max^*(x, y, z) &\equiv \ln(e^x + e^y + e^z) \\ &= \max^*(\max^*(x, y), z) \end{aligned} \quad (34)$$

To benefit from this simplification, we introduce the log-domain representation shown as follows

$$\begin{aligned} \gamma_k^*(S', S) &\equiv \ln(\gamma_k(S', S)) \\ &= -\frac{M}{2} \ln(2\pi\sigma^2) - \frac{\|y_k - Ce\|^2}{2\sigma^2} + \sum_{q=1}^Q \ln(p(s_q | s_q')), k=1, \dots, K \end{aligned} \quad (35)$$

$$\alpha_1^*(S) \equiv \ln(\alpha_1(S)) = (-Q) \ln 3 \quad (36)$$

$$\begin{aligned} \alpha_{k+1}^*(S) &\equiv \ln(\alpha_{k+1}(S)) \\ &= \ln\left(\sum_{S' \in \delta_k} \alpha_k(S') \gamma_k(S', S)\right) \end{aligned} \quad (37)$$

$$\begin{aligned} &= \ln\left(\sum_{S' \in \delta_k} e^{[\alpha_k^*(S') + \gamma_k^*(S', S)]}\right) \\ &= \max_{S' \in \delta_k}^* [\alpha_k^*(S') + \gamma_k^*(S', S)], k = 1, 2, \dots, K \\ \beta_{K+1}^*(S) &\equiv \ln(\beta_{K+1}(S)) = (-Q) \ln 3 \end{aligned} \quad (38)$$

$$\begin{aligned} \beta_k^*(S') &\equiv \ln(\beta_k(S')) \\ &= \ln\left(\sum_{S \in \delta_{k+1}} \gamma_k(S', S) \beta_{k+1}(S)\right) \\ &= \ln\left(\sum_{S \in \delta_{k+1}} e^{[\gamma_k^*(S', S) + \beta_{k+1}^*(S)]}\right) \\ &= \max_{S \in \delta_{k+1}}^* [\gamma_k^*(S', S) + \beta_{k+1}^*(S)], k = K, K-1, \dots, 1 \end{aligned} \quad (39)$$

Further, we now write the probability density function in Eq.(24) and the APP L-values in Eq.(20) as

$$p(S', S, Y) = e^{\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)} \quad (40)$$

$$\begin{aligned} L(b_q(k) = +1) &= \ln\left[\sum_{(S', S) \in \Sigma_k^{+1}} e^{\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)}\right] \\ &= \max_{(S', S) \in \Sigma_k^{+1}}^* (\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)) \end{aligned}$$

$$\begin{aligned} L(b_q(k) = 0) &= \ln\left[\sum_{(S', S) \in \Sigma_k^0} e^{\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)}\right] \\ &= \max_{(S', S) \in \Sigma_k^0}^* (\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)) \end{aligned}$$

$$\begin{aligned} L(b_q(k) = -1) &= \ln\left[\sum_{(S', S) \in \Sigma_k^{-1}} e^{\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)}\right] \\ &= \max_{(S', S) \in \Sigma_k^{-1}}^* (\alpha_k^*(S') + \gamma_k^*(S', S) + \beta_{k+1}^*(S)) \end{aligned} \quad (41)$$

3 Simulation and analysis

3.1 Scenario and parameters

We consider a synchronous DS-CDMA space based constellation network. All the parameters used in the simulation is summarized in Table1. The proposed method is compared with the methods advocated in [8] which are labeled as ‘‘SIS-OPT’’, ‘‘SIS-LF’’ and ‘‘PSP’’ respectively.

The performance of the methods is evaluated in terms of symbol error rate (SER) and activity detection error rate (ADER). All the results presented herein are referred to a user of interest, so that it is possible to compute the SER and ADER at every symbol interval. SER indicates that the estimated value of the symbol transmitted by user of interest differs from its real value Similarly, ADER indicates that the estimated state of user of interest differs

from its real state. It can be easily seen that an error in activity detection entails an error in symbol detection but not *vice versa*.

Table 1. Parameters

length of packet(K)	10 ³ bits
length of spreading codes(M)	64bits
ϕ	0.5
μ	0.9
η	0.1
simulation times	1000
modulation format	BPSK

3.2 Numerical results and discussion

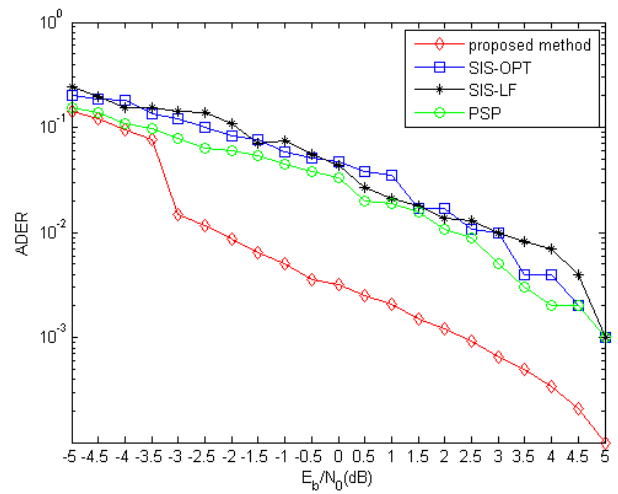


Fig. 2 ADER for several values of E_b/N_0

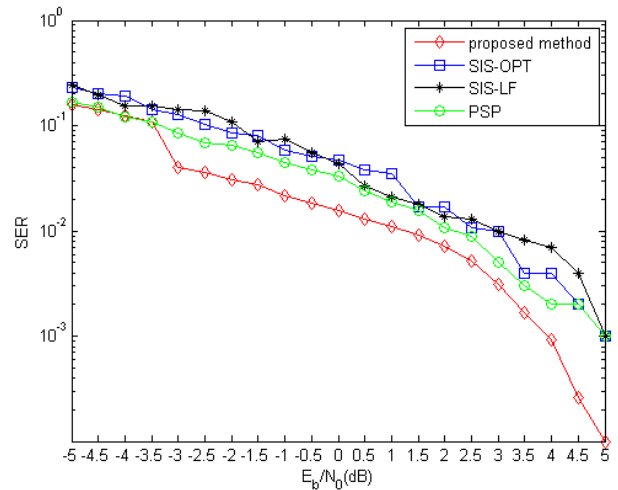


Fig. 3 SER for several values of E_b/N_0

Fig. 2 displays the ADER achieved by all the methods

against several values of E_b/N_0 . It can be seen that the ADER of the proposed method descends sharply when $E_b/N_0 = -3\text{dB}$, and the performance of the proposed method far outperforms the other three methods when $E_b/N_0 > -3\text{dB}$. Comparing with the other three methods, the proposed method achieves 2.6 dB E_b/N_0 gains when ADER reaches 10^{-3} .

Fig. 3 displays the SER achieved by all the methods against several values of E_b/N_0 . It can be observed that the proposed method achieves 1dB E_b/N_0 gains compared with the other three methods when SER reaches 10^{-3} . Moreover, it can be seen that Fig. 3 is very similar to Fig. 2. In fact, we have found that most symbol errors accounted when computing SER are due to the methods not correctly detecting the activity state of user of interest. Therefore, keeping ADER low is strictly necessary to attain a low SER.

4 Conclusions

In this paper, we tackle the problem of multiuser detection in DS-CDMA space based constellation network. We propose a MAP based method which transforming multiuser detection into searching optimal path in grid map with BCJR algorithm. Simulation results indicate that most symbol detection errors are due to the failure in detection of the user's activity state. The proposed method obtains 2.6dB and 1dB E_b/N_0 gains respectively when ADER and SER reach 10^{-3} , comparing with SIS-OPT, SIS-LF and PSP.

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