Application of High-Order Target Dynamics to Position and Force Control of a Manipulator

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고차수 동력학적 표적 모형을 이용한 로보트의 위치와 힘의 제어

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Key Words: Manipulator Control(매니퓰레이터 제어), Target Dynamics(표적모형), Contact Position(접촉위치), Contact Force(접촉력), Redundant Inputs(중복입력)

초 록

이 논문에서는 고차수 동력학적 표적모형을 이용하여 간단하고 우수한 성능을 지니는 위치와 힘의 추적제어법을 제안하였다. 이 표적모형은 위치오차와 힘오차의 상관관계를 자유운동을 위한 임피던스와 힘오차의 보상기로 모형화한다. 이들의 특성중, 적절한 보상기 설계에 의하여 위치임력과 힘임력 추적제어, 천이응답을 줄이기 위한 위치임력과 힘임력의 동시사용 및 매니퓰레이터의 접촉시에 부드러운 접촉을 위한 동특성의 연속성 유지 보상기 설계에 대하여 고찰하였다. 또한 접촉환경이 불규칙한 기하구조를 갖는 경우에, 위치교란중 힘의 추적제어를 모의실험으로 보였다.

1. INTRODUCTION

When a robot involves in a task of changing between free and contact motion, tracking contact position or force is a difficult control problem. Attempts were made to use the concept of impedance control(Hogan 1987). These tracking controllers try to feedback full dynamics to cancel the actual dynamics (Goldenberg 1988), or to modify the position input by minimizing force errors(Lasky and Hsia 1991) or by identifying the geometry and environmental properties(Anderson and Spong 1987, Lin 1991). During the operation of a manipulator, it is not easy to know the accurate position and mechanical properties, such
as inertia and stiffness of the contact environment. Consequently, application of these tracking controllers is restricted to the tasks for which the environment can be identified.

Instead of modifying the position input, force compensation is introduced to improve smooth transition between contact and free motions (Kazerooni 1987, Kazerooni and Kim 1989) and applied to force tracking (Payandeh and Goldenberg 1991). The former uses the modified position input for force control and lacks tracking capability. The latter shows a few cases of the compensator application, but lacks theoretical base and, consequently, limits its utilization.

In this research, a contact position or force tracking control method is suggested that guarantees stability and tracks position or force input in the presence of uncertainties of the contact environment. This control is based on target dynamics that relates position and force errors through free motion impedance and compensated force error. For tracking control, the compensator can be designed so that either of the position or force error, depending on control task, be zero at the steady states. It is also shown that the two inputs of position and force can cooperate to reduce one output of them. The controller can be, for smooth transition, designed so that the dynamics characteristics are similar between the free and contact motion. The suggested method does not require accurate knowledge of environmental geometry and mechanical properties for successful position or force tracking control.

2. TARGET DYNAMICS MODEL

Position or force tracking is achieved by combining inner and outer loop control. The inner-loop linearizes the system dynamics of the manipulator, and the outer-loop modifies the dynamics to follow the desired target dynamics. The control scheme based on the target dynamics is described.

Assume that an articulated manipulator, as shown in Fig. 1, is composed of n simply connected links, i.e., each joint which connects the links has one relative degree of freedom. A global coordinate system O-xyz is fixed on the ground. The position and orientation of the end-effector are \[ \mathbf{x} = [x, y, ...,]^T \in \mathbb{R}^{n+1} \] in the global coordinates. The joint coordinates are also used to express the relative translational or rotational motion as \[ \mathbf{q} = [q_1, q_2, ..., q_n]^T \in \mathbb{R}^{n+1} \] in vector form, and corresponding joint torque (including force for translational joint) \[ \mathbf{r} = [\tau_1, \tau_2, ..., \tau_n]^T \in \mathbb{R}^{n+1}. \] The symbol \( \mathbf{x}_0 \) is the environmental position before contact, and \( \mathbf{f}_{\text{ext}} \) is the force applied to the environment by the manipulator. For simplicity, it is assumed that the degree of the manipulator, \( n \), is same as the number of Cartesian coordinates used.

![Fig. 1 A robot configuration](attachment:image.png)
Manipulator dynamics can be derived, using the Lagrangian or variational principles, in the joint coordinates (Chern and Yae 1991) as,

$$\mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{r}_c + \mathbf{J}^T \mathbf{f}_{\text{ext}}$$

(1)

where $\mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times 1}$ is the gravitational, Coriolis, and centrifugal force; $\mathbf{r}_c \in \mathbb{R}^{n \times 1}$ is the control torque applied at joint actuators; $\mathbf{J}$ is the Jacobian of the Cartesian coordinates with respect to the joint coordinates; $\mathbf{f}_{\text{ext}} \in \mathbb{R}^{n \times 1}$ is the external force due to contact; and $\mathbf{M} \in \mathbb{R}^{n \times n}$ is the generalized mass matrix.

In designing the target dynamics, two things are considered. Firstly the target dynamics for control of a robot can accommodate the free motion and contact motion and smooth transition between the two. Since a robot interacts with the environment, it is important that the robot can swiftly move in free space following the desired trajectory, smoothly transit to contact with the environment, and exert the force of the desired magnitude. Second, the target dynamics also allows tracking of contact position or force. Considering these two things, we suggest a model of target dynamics that uses state error feedbacks and compensated force errors.

The simplest form of compensations for force tracking control is chosen in Cartesian formulation as,

$$\mathbf{G}(s)\mathbf{x}_e = \mathbf{H}(s)f_c$$

(2)

where $\mathbf{x}_e = \mathbf{x}_d - \mathbf{x}$, position error vector, $\mathbf{x}_d$ is desired trajectory $\in \mathbb{R}^{n \times 1}$, $\mathbf{x}$ is present Cartesian position and orientation vector $\in \mathbb{R}^{n \times 1}$, $\mathbf{f}_c = \mathbf{f}_d - \mathbf{f}_0$, the force error vector, $\mathbf{f}_d$ is the desired force or torque $\in \mathbb{R}^{n \times 1}$, $\mathbf{f}_0$ is the sensed force or torque ($\simeq \mathbf{f}_{\text{ext}}$). $\mathbf{G}(s) = (s^2 + K_p + K_v)$ impedance for free motion control $\in \mathbb{R}^{n \times n}$, $\mathbf{H}(s)$ - the force compensator $\in \mathbb{R}^{n \times n}$, and $s$ is the Laplace transformation. The constants $\mathbf{K}_p$ and $\mathbf{K}_v$ are diagonal matrices of derivative and proportional position feedback gains, respectively.

For simplicity, $\mathbf{H}(s)$ and $\mathbf{G}(s)$ are diagonal matrices which decouple the control dynamics. When both the sensed force and the desired force are zero as in free motion, the right hand side of Eq. (2) vanishes, and it becomes a free motion controller as $\mathbf{G}(s) \mathbf{x}_e = \mathbf{0}$. It is noted that only position and velocity feedbacks of the contact point are used in free motion control.

When the end-effector of the robot begins to contact a surface, or when the desired force is activated with or without an actual contact, the force error is nonzero, and the nonzero force activate the right hand side of Eq. (2). Equation (2), then, becomes the contact controller. The target dynamics, designed appropriately, simultaneously satisfies both the specifications of contact and free motion control.

The joint torque is derived from the control algorithm. Feedback and feedforward of states (position and velocity) and force are used in the control. The joint driving torques based on the target dynamics Eq. (2) are obtained as,

$$\mathbf{r}_c(t) = \dot{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}}) + \ddot{\mathbf{M}}(\mathbf{q}) \mathbf{u}(t) - \mathbf{J}^T \mathbf{f}_c$$

(3)

where

$$\mathbf{u}(t) = \mathbf{J}^T [\ddot{\mathbf{x}}_d + \mathbf{K}_p \dot{\mathbf{x}}_e + \dot{\mathbf{K}}_v \mathbf{x}_e - \mathbf{H}^s (\mathbf{f}_s - \mathbf{f}_0) - \mathbf{J} \dot{\mathbf{q}}]$$

(4)

and the symbol $*$ is a time convolution, and $\dot{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}})$ and $\ddot{\mathbf{M}}(\mathbf{q})$ are the estimates of nonlinear force $\mathbf{v}$ and inertia $\mathbf{M}$, respectively. In practice, it is possible that the sensed force $\mathbf{f}_s$ may contain errors, and the estimates, $\dot{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}})$ and $\ddot{\mathbf{M}}(\mathbf{q})$, which results from on-line computation of the manipulator model dynamics, may have estimation errors. However, we assume, for
simplicity in developing the control algorithm, that the sensing and the estimations are so accurate that the measurement and estimation errors are negligible.

3. STABILITY

The compensator has to be designed so that it guarantees the control stability of the end effector in contact with the environment. Introducing the mechanical model of the environment, the stability condition is presented considering the control system as a bounded-input and bounded-output system. The sensitivity functions, the transfer functions between the input disturbance and output errors, are introduced to analyze the stability and tracking performance.

Environmental Model

Environmental properties are also modelled to show the stability and tracking performance. When the end effector contacts the environment, the reaction force will be induced by the deformation and friction of the environment. It is assumed that the environment can be modelled as a simple passive mechanical system that consists of inertia, damping, and stiffness as,

\[ f_e = E(s)(x - x_o) + f_o - f_u \]  (5)

where

\[ E(s) = M(s)^+ + C(s) + K(s) \]  (6)

and the symbol \( f_o \) is force disturbance from the static load. The coefficients matrices: inertia \( M(s) \), damping \( C(s) \), and stiffness \( K(s) \) of the environment are diagonal matrices. These matrices as well as the environmental geometry \( x_o \) may vary as functions of contact positions.

Sensitivity Functions

A simplified linear control loop can be drawn as in Fig. 2 by substituting Eq. (5) to Eq. (2). The closed loop can be viewed as a system with two-input \((x_d, f_o)\) and two-output \((x, f_e)\) with position disturbance \(x_o\) and force disturbance \(f_o\). The switching symbol (Kazerooni 1987) in Fig. 2 is used in order to represent the contact between the end-effector and environment.

![Fig. 2 Closed loop of target dynamics](image)

Viewing the control loop in Fig. 2 as a multi-input and multi-output system, each component of sensitivity functions can be derived. The force error \( f_e = f_d - f_u \) is written as,

\[ f_e = -E(s)(x - x_o) + f_d - f_u = E(s)x_o - E(s)(x_d - x_o) + f_d - f_u \]  (7)

From Eq. (2) and (7), the output errors are derived in terms of the sensitivity functions as,

\[
\begin{bmatrix}
    x_e \\
    f_e
\end{bmatrix} =
\begin{bmatrix}
    S_{xx}(s) & S_{xf}(s) \\
    S_{hx}(s) & S_{hf}(s)
\end{bmatrix}
\begin{bmatrix}
    x_d - x_o \\
    f_d - f_u
\end{bmatrix}
\]  (8)

where the sensitivity functions are defined as,
\[
\begin{bmatrix}
S_x(s) & S_d(s) \\
S_n(s) & S_f(s)
\end{bmatrix}
= 
\left[ I + E G H \right]^{-1}
\begin{bmatrix}
E G H - G H \ I
E \ I
\end{bmatrix}
\]

(9)

The Laplace transform parameter, \( s \), is omitted for brevity. In Eq. (8), \( S_x(s), S_d(s), S_n(s) \) and \( S_f(s) \) denotes the sensitivity functions of position–position, position–force, force–position, and force–force, respectively. Equations (8) shows that the output forces and positions are coupled, and the environmental effects are mixed in the sensitivity functions.

**Stability Condition**

The local stability in contact motion applies when the environment has a fixed mechanical property. Stability in contact dynamics requires that the two position and force outputs are bounded (Doyle et al. 1992). The system is called bounded–input–bounded–output (BIBO) stable if the two outputs are bounded to the bounded inputs.

It is shown that the controlled system has BIBO stability when the position inputs and environmental geometry are bounded for less than 2 order in Laplace transform parameter, and the characteristic roots of the closed–loop system have stable zeros (Lee 1993).

\[
\text{det} \left[ I + E(s)G(s)H(s) \right] = 0
\]

(10)

The stability condition, Eq. (10), shows that stability of the controller is affected by the mechanical property of the contact environment. It implies that the the compensator must be designed in conjunction of the environment.

### 4. PERFORMANCE

The merits of the present controller are discussed. It is shown that the controller can track either position or force input with the appropriate design of the compensator. The two inputs are used to control one output, and the coordination of the two inputs can reduce the output error. The introduction of the high–order compensator enables to design the controller that satisfies the free motion and contact motion specification.

**Position and Force Tracking**

The position and force tracking conditions are imposed on the position error \( x_e \) and the force error \( f_e \), respectively. The following theorem is used to show the tracking performance.

**Theorem 3.1**

Assume that the feedback system is stable.

a) If \( x_o, f_o, x_d \) and \( f_d \) are step inputs, then \( x_e \to 0 \) as \( t \to \infty \) if and only if the stable sensitivity functions, \( S_x(s) \) and \( S_d(s) \), have at least one zero at the origin.

b) If \( x_o, f_o, x_d \) and \( f_d \) are step inputs, then \( f_e \to 0 \) as \( t \to \infty \) if and only if the stable sensitivity functions, \( S_n(s) \) and \( S_f(s) \), have at least one zeros at the origin.

The theorem implies that under simultaneous step inputs and disturbance, the controller can track either position or force, exclusively.

To show the relation between the sensitivity functions and the compensators, it is assumed that we can freely choose the shape of the proper compensator, \( H(s) \), i.e., its order and coefficients. In the present research, a diagonal form of compensator in arbitrary order is chosen. The compensator can be factorized as,
\[ \text{H}(s) = D(s)^{-1} N(s) \]  \hspace{1cm} (11) \\
where,

\[ D(s) = I s^k + A_1 s^{k-1} + \ldots + A_k \]  \hspace{1cm} (12) \\
\[ N(s) = B_0 s^m + B_1 s^{m-1} + \ldots + B_m \]  \hspace{1cm} (13) \\

and where the superscripts \( m \) and \( k \) are integers, and \( A_1, A_2, \ldots, A_k \) and \( B_0, B_1, \ldots, B_m \) are constant diagonal matrices. They are determined according to the designer’s specification and goal. Each diagonal element in the compensator \( H(s) \) is a rational coprime polynomial, i.e., a polynomial with no common factor. It is assumed that each element of the compensator has relative degree, defined as \( k \geq m \), greater than or equal to zero.

The next step is how to place zeros at the origin for the sensitivity functions. The following fact makes it possible to design tracking controllers.

**Fact 3.1**

a) If the compensator, \( H(s) \), has zeros at the origin, the sensitivity functions, \( S_{\alpha}(s) \) and \( S_{\beta}(s) \), simultaneously have at least the same number of zeroes at the origin.

b) If the compensator, \( H(s) \), has poles at the origin, the sensitivity functions, \( S_{\alpha}(s) \) and \( S_{\beta}(s) \), simultaneously have the same number of zeros at the origin.

Proof is shown in Lee (1993).

The position tracking controller rejects the effect of the variation of the payload and tracks the input position. This position tracking controller is simpler than other adaptive algorithms, such as model reference adaptive control (Dubowsky and Desforges 1979), adaptive control using an autoregressive model (Koivo and Guo 1983), resolved motion adaptive control (Wu and Paul 1982, Lee and Lee 1984).

The force tracking controller rejects the disturbance from the uncertainties of the environmental geometry and tracks the desired force input. Moreover, the geometry and mechanical property of the environment, or the position input do not affect the tracking.

**Cooperation of the Two Inputs**

In robotics control, steady state is only achieved when the end-effector of the robot operates solely in one static configuration. During the operation of the robot, the configuration of the robot changes, and the robot remains in the transient state. The robot task requires small tracking error in the transient state. In general this transient response can be improved by increasing the bandwidth of the error response. In addition to increasing bandwidth, the target dynamics has another option to reduce the tracking errors. Since the target dynamics uses redundant inputs, they can coordinate to reduce the errors. If force tracking control, for example, is aimed, the force input is set as a desired value. The position input is then a supplementary, which can be used to reduce the position disturbance.

In position tracking the force feedback can be used to reduce force disturbance. When the stability condition Eq. (10) is satisfied, the \( L_2 \)-norm of the position error is bounded and written from Eq. (8) as,

\[
\| x_0 \|_2 \leq \| S_{\alpha}(s) \|_\infty \| (x_0 - x_e) \|_2 \]

\[ + \| S_{\alpha}(s) \|_\infty \| f_d - f_e \|_2 \]  \hspace{1cm} (14)

The position error is a sum of the errors due to position disturbance \( x_e \) and force disturbance \( f_e \). The first term in the right-hand side in Eq.
(14) shows that the attenuation of the position disturbance is achieved only by minimizing the norm of position-position sensitivity $\| S_{\alpha}(s) \|_\infty$, since the position input $x_0$ is fixed. In position tracking control, the force input is redundant. This force input can be used to reduce the effect of force disturbance. By sensing and estimation, the force disturbance is used in the feedback as $f_d = f_e + \varepsilon$ so that the error, $\| f_d - f_e \|_2 = \| \varepsilon \|_2$ approaches zero as the estimation error $\varepsilon$ goes to zero. When the norm $\| f_d - f_e \|_2$ approaches zero, the norm of the position tracking error Eq. (14) also decreases.

The same argument can be applied to force control. The position feedback can coordinate to reduce the force tracking error. Taking $\mathcal{L}_2$-norm on the force error in Eq. (14), it is bounded from Eq. (8) as,

$$
\| f_e \|_2 \leq \| S_{\alpha}(s) \|_\infty \| E(x_d - x_0) \|_2 + \| S_{\alpha}(s) \|_\infty \| f_d - f_e \|_2
$$

Equation (15) shows the contribution of the disturbances to the force error. For the given force input, the last term in Eq. (15) shows that the effect of the force disturbance $f_e$ can be reduced only by suppressing the force-force sensitivity function $\| S_{\alpha}(s) \|_\infty$. The more significant error source in force tracking control is the uncertainty of the contact geometry, which results in the position disturbance. In force control, the position input is supplementary and can be used to reduce the effect of the position disturbance. If the geometry is approximately known by sensing or estimations, the knowledge can be used as the position input $x_0$ so that it minimizes the $\mathcal{L}_2$-norm, $\| E(x_d - x_0) \|_2$.

It is noted that the position disturbance $x_0$ is coupled with the environmental property $E(s)$. When the environment is very stiff, the coupled effect $\| E(x_d - x_0) \|_2$ can be large, although the position disturbance itself $\| x_d - x_0 \|_2$ is small. That can cause significant force error. Therefore, it is importance to know the contact geometry as accurately as possible. It is also noted that the force-force sensitivity function $\| S_{\alpha}(s) \|_\infty$ is a common factor in both terms in Eq. (15). It implies that the minimization of the sensitivity function $S_{\alpha}(s)$ alone can reduce both the effects of position and force disturbances.

**Smooth Transition**

Smoothness in transition can be discussed by the change of the location of the characteristic roots (Lin 1991). The dynamics behavior maintains its continuity during the transition, if the characteristic roots of contact motion are close to those of free motion. The characteristic roots for free motion are chosen for stable motion of the end-effector in free space. To show the smooth transition, it will suffice that the characteristic roots of the contact motion can be placed at arbitrary location. The characteristic roots of the contact motion can be placed close to those of free motion.

The location of the roots can be determined by the characteristic equation Eq. (10). For simplicity, the matrix form of the characteristic equation is used, and the order of the denominator, $k = m \geq 2$. Using Eqs. (6), (12), and (13), the characteristic equation Eq. (10) can be rearranged as,

$$
(I + M_B s^k + \sum_{k=0}^{m} A_k K_k + A_k + K_k B_k + C_k B_k) s^k
$$

where $A_k$ and $K_k$ are the system matrices of the characteristic roots.
\[ (A_k K_0 + A_b K_v + K_b B_k + C_b B_b) s \]
\[ + (A_k K_v + K_b B_b) = \sum_{i=0}^{k} Q_i s^i \] (16)

where the coefficients \( Q_i \) denote the ones of the characteristic equation that have roots at the desired location. Equation (16) has a sufficient number of coefficients to be matched to the arbitrary characteristic matrix equation. The matching of the matrix equation implies the matching of eigenvalues and eigenvectors. As an example, a position tracking control is considered. The position tracking condition requires that the coefficient of the numerator be \( B_k = 0 \). Choose arbitrary coefficients for \( B_0 \) and \( B_i, i=3, 4, ..., k-1 \). Multiply the constant, \( (I+M_b B_0) \), to the given characteristic equation. The two equations can be recursively matched in the backward direction of the Eq. (16) to determine the coefficients of the denominator. \( A_k = 0, \) \( k-1, ..., 1 \). The remaining coefficients \( B_1 \) and \( B_2 \) can be used to match the last two coefficients.

When the environment has a different mechanical property, say \( M_b = 0 \), the procedure can be applied with slight modification.

In conclusion, an arbitrary characteristic equation can be matched with a sufficient high-order compensator. By placing the characteristic roots of contact motion close to the ones of the free motion controller, the similar response, consequently smooth transition, is obtained. This is one aspect different to impedance control (Hogan 1987). In impedance control, the closed-loop poles cannot be arbitrarily placed at the desired locations for both contact dynamics and free motion dynamics (Lin 1991).

5. SIMULATION

The effectiveness of the present controller is confirmed by numerical simulation. A force controller for a two-degree-of-freedom robot is tested on the task that the end-effector applies constant force while it moves on a surface with a slope as uncertainties.

The task is to move the end-effector from free space to the surface, to move to the prescribed position exerting the constant force to the environment, and then to turn back into the free space. To show that the tracking capability and disturbance rejection, there is a cosine slope as geometric uncertainties. The controller is designed so that force tracking control is applied to the direction normal to the nominal surface, and position control is applied in the direction horizontal to the nominal surface.

The planar two-link manipulator and its environment are shown in Fig. 3. Each link is 2 meters long, with centroid located at its center. The contact surface has properties, such as \( M_b = 0 \), and \( K_b = 10^7 \text{N/m} \). The robot is supposed to contact during the movement between \( y = 2 \text{m} \) and \( y = 3 \text{m} \). The slope is located at \( y = 2.5 \text{m} \) with the depth, 0.01m, as,

\[ z = -0.005 \left[ 1 + \sin(2 \pi (y-2.5)) \right] \] (17)

![Fig. 3 Two-link planar robot](image)

The free motion impedance has roots at \( s = -20 \), and the contact controller has close-loop poles at \( s = -80 \). The target dynamics are designed as,
6. CONCLUSION

A simple and high performance position or force tracking control based on a higher order target dynamics model is proposed.

The controller is able to achieve position or force tracking in uncertain geometry and mechanical properties of the contact environment. For tracking control, the compensator is designed so that either of the position or force error, depending on control task, be zero at the steady states. It is also shown that the controller simultaneously uses the position and force inputs to improve transient responses. The two inputs are used to control one output, and the cooperation of the two inputs can reduce the output error. Finally, the compensator can be, for smooth transition, designed so that the controlled behavior of the end-effector maintains its dynamics continuity during the contact. The introduction of the high-order compensator enables to design the controller that satisfies the free motion and contact motion specification. The characteristic roots of the contact motion are placed close to those of free motion.

The simulation result shows the validity of the presented force control method.

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