Discrete Optimum Design of Space Truss Structures Using Genetic Algorithms

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Abstract

The objective of this study is the development of discrete optimum design algorithms which is based on the genetic algorithms. The developed algorithms were implemented in a computer program. For the optimum design, the objective function is the weight of space trusses structures and the constraints are stresses and displacements. This study solves the problem by introducing the genetic algorithms. The genetic algorithms consists of genetic process and evolutionary process. The genetic process selects the next design points based on the survivability of the current design points. The evolutionary process evaluates the survivability of the design points selected from the genetic process. The efficiency and validity of the developed discrete optimum design algorithms was verified by applying the algorithms to optimum design examples.

Keywords: genetic algorithms, discrete optimization, space truss structures, evolutionary process

1. INTRODUCTION

Genetic algorithms (GAs) are computational modeling based on the evolution of the nature, optimum algorithms of global parallel search research by Holland (Holland, 1975). GAs are based on Darwinian’s evolution, survival of fittest. GAs are stochastic optimization techniques called evolutionary algorithms, which implements reproduction, crossover and mutation, simulates the natural evolutionary process of living things with the node of design space. It is part of artificial intelligence (AI) that make the properties of nature computer algorithms, find the optimization solution adapted evolutionary process. In point of optimization techniques, GAs are useful method in structure optimization as it can find the global optimization solution in spite of nonlinear, process the discrete variables and search the design large space (Mitsuo, 1997). This study shows the organization of discrete optimization design using genetic algorithms and develops the programs which perform geometric nonlinear analysis and discrete design of space truss structures at the same time with constrained of stresses and displacements. And Numerical examples for a space truss structures of 25-bar and dome are presented to explain the applications of the proposed programs in this paper.

2. FOUNDATIONS OF GENETIC ALGORITHMS

GAs, using binary bit string by chromosome to express real value of design variables, are able to mutate and crossover as genotype of chromosome make form arrayed with composed of binary string, 0 and 1. Finally, these strings are individual making population and the node of design space (Rajeev, 1992). This study generate the population composed of individual, that is, strings that the number expressed by encoding the probability solution which made discrete information of design variables, data base with binary bit. Thus, based on schema theorem, the more increase the number of generation, the higher probability which character having higher the fittest can survive. The survival individual make higher the fittest as reproduction and crossover on stochastic survival probability. To implement preceding process repeatedly, the individual having low fitness got weeded out. Since the individual having high fitness generate the individual having higher fitness with crossover, it can be expressed the fittest individual (Goldberg, 1987).

2-1 Reproduction

The reproduction is the process that each individual is selected with new individual of next generation on fitness theorem. This process presents the natural theory, that is, the higher fitness make the more offspring (Mitsuo, 1997). There are three individual selection methods; roulette-wheel selection, tournament selection, proportional selection. This study chooses roulette-wheel selection.

2-2 Crossover

One of the most important operators in a GAs is crossover. Crossover is a means for two strings (parents) to produce two offspring by mixing and matching their desirable qualities through a random process.
2-3 Mutation

Another important operator used in GAs is called mutation (Goldberg, 1987), which mimics the phenomenon of natural mutation. When mutation is applied to a string it sweeps down the list of bits and changes the bit from 0 to 1 or from 1 to 0, if a probability test is passed.

3. APPLICATION PROCEDURE OF GENETIC ALGORITHMS

The Fig. 1 shows the application method of GAs. In this study, GAs is composed of three processes; initial process, evolutionary process and genetic process. In initial process, GAs have to make chromosomes same as the number of individuals with developing random variables. That chromosome is transmitted to genetic process, changed into values used to real design variables through decoding and implements structure analysis with using attained design variables. The evolution process analyze the result of structure analysis, compute the fitness of chromosome and transmit the fitness to genetic process. The genetic process generate the string of next generation, is retransmitted to evolution process by three genetic operators, reproduction, crossover and mutation. This preceding method is the basic search method of GAs, which iterate evolution and genetic process and find optimum solution.

![Flowchart for optimum design using GAs](image)

The Fig 2 ~ Fig 8 shows the main module of program developed in this study.

Fig 2. Start view
Fig 3. Main view
Fig 4. Evolutionary process
Fig 5. Genetic process
Fig 6. Analysis process
4. FITNESS EXPRESSION OF OPTIMIZATION

Fitness is a quality value which is a measure of the reproductive efficiency of living creatures according to the principle of survival of the fittest. In GA’s fitness is used to allocate reproductive trials and thus is some measure of goodness to be maximized. This means that strings with higher fitness value will have higher probability of being selected as parent. Therefore we have to transform the objective function minimization problem to the fitness function maximization problem (Jenkins, 1991). This study use the reciprocal of object function by the fitness in implementation of optimization design. To avoid the object function expressed by the value less than 1.0, Eq.1 present the ratio of maximum object function to the fitness.

\[
\text{Fitness} = \frac{\text{Object}_{\text{Max}}}{\text{Object}_{\text{Sum}}} \quad (\text{Eq. 1})
\]

\[
\text{Object}_{\text{Max}} = \sum_{i=1}^{\text{NEL}} A_{\text{max}} \times L_i
\]

\[
\text{Object}_{\text{Sum}} = \sum_{i=1}^{\text{NEL}} A_i \times L_i
\]

where, \( A_{\text{max}} \) : maximum value of cross-section database in use

4.1 Application of constraints

The common method to apply constraints to optimization is using the penalty function, when the design variables have the constraints in GAs. The constrained problem stated by Eq. 2 is transformed into an unconstrained one by the addition to the object function of a penalty term which takes care of the effect of the constraints. An unconstrained minimization may then be performed on the transformed function which generally is written as

\[
\begin{align*}
\text{Minimize} & \quad F(X) \\
\text{Subject to} & \quad G_i(X) \leq 0, \quad i = 1, 2, \ldots, m \\
\text{Equv}(X) & = F(X) + P(X) \\
P(X) & = 0 \quad \text{if } X \text{ is feasible} \\
P(X) & > 0 \quad \text{otherwise}
\end{align*}
\quad (\text{Eq. 2})
\]

Where,
X : represents a chromosome
Equv(X) : evaluation function
F(X) : objective function
P(X) : penalty function
\( G_i(X) \) : constraints
m : the number of constraints

The penalty function used in this study is composed of Eq. 3. This is taken from Kavilie and Moe (Kavilie & Moe, 1971)

\[
P(X) = \alpha \sum_{i=1}^{m} \frac{1}{G_i(X)} \quad (\text{Eq. 3})
\]

where , \( P(X) \) : penalty function,

m : a Number of constrain condition

5. DESIGN EXAMPLES

5.1 Object function

The optimization process is formulated in Eq. 4.

\[
\begin{align*}
\text{Minimize} & \quad W(X) \\
\text{Subject to} & \quad G_i(X) \geq 0 \\
W(X) & = \rho \sum_{i=1}^{\text{NEL}} V_i, \quad V_i = A_i \times L_i \\
\end{align*}
\quad (\text{Eq. 4})
\]

where,
W : object function(tf), X : design variable
G : constraints,
\( V_i \) : volume of design member(\( cm^3 \))
\( A_i \) : cross-section area(\( cm^2 \))
\( L_i \) : member length (cm)
\( \rho \) : the weight per unit volume of steel (\( tf / cm^3 \))
**Example 1** The configuration and dimension of the 25-bar space truss are shown in Fig. 9. The details of the considering load data and the grouping of members are given in Table 1 and 2. The assumed data are: $E=703 \text{ tf/} \text{cm}^2$, $\rho = 2.768 \times 10^{-6} \text{ tf/cm}^3$. The displacements at joint 1 and 2 in the directions of X and Y are restricted to be less than $\pm 0.89 \text{cm}$. The stresses are limited to $\pm 2.8 \text{ tf/cm}^2$.

### Table 1. Loading conditions for 25bar space truss

<table>
<thead>
<tr>
<th>Loading condition</th>
<th>node</th>
<th>Load(tf)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>0.45</td>
<td>-4.54</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-4.54</td>
</tr>
<tr>
<td>3</td>
<td>0.23</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.27</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 2. Member Grouping Details

<table>
<thead>
<tr>
<th>Member Group</th>
<th>Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2,3,4,5</td>
</tr>
<tr>
<td>3</td>
<td>6,7,8,9</td>
</tr>
<tr>
<td>4</td>
<td>10,11</td>
</tr>
<tr>
<td>5</td>
<td>12,13</td>
</tr>
<tr>
<td>6</td>
<td>14,15,16,17</td>
</tr>
<tr>
<td>7</td>
<td>18,19,20,21</td>
</tr>
<tr>
<td>8</td>
<td>22,23,24,25</td>
</tr>
</tbody>
</table>

The following control parameters are used for the GAs in the design of the 25bar space truss; a crossover probability of 0.85, a mutation probability of 0.01, a population size of 60, a string length per variable of 6, total length of chromosome is 48. The set of available sections used for this problem is given in Reference (Rajeev, 1992). Comparisons of results are presented in Table 3. It is shown that the optimum designs obtained are in good agreement.

### Table 3. Summary of discrete optimized designs of 25bar space truss

<table>
<thead>
<tr>
<th>Member Group</th>
<th>This Paper</th>
<th>Rajeev(1992)</th>
<th>Zhu(1986)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6452</td>
<td>0.6452</td>
<td>0.6452</td>
</tr>
<tr>
<td>2</td>
<td>11.6136</td>
<td>11.6136</td>
<td>12.2588</td>
</tr>
<tr>
<td>3</td>
<td>14.8396</td>
<td>14.8396</td>
<td>16.7752</td>
</tr>
<tr>
<td>4</td>
<td>1.2904</td>
<td>1.2904</td>
<td>0.6452</td>
</tr>
<tr>
<td>5</td>
<td>0.6452</td>
<td>0.6452</td>
<td>0.6452</td>
</tr>
<tr>
<td>6</td>
<td>5.1616</td>
<td>5.1616</td>
<td>5.1616</td>
</tr>
<tr>
<td>7</td>
<td>11.6136</td>
<td>11.6136</td>
<td>13.5492</td>
</tr>
<tr>
<td>8</td>
<td>19.356</td>
<td>19.356</td>
<td>16.7752</td>
</tr>
<tr>
<td>Weight(t)</td>
<td>0.2476</td>
<td>0.2476</td>
<td>0.2553</td>
</tr>
</tbody>
</table>

Table 4 shows result used the set of commercially available fabricated steel pipe sizes. The size is based on KS material database.

### Table 4. Discrete optimized designs of 25bar space truss

<table>
<thead>
<tr>
<th>Member Group</th>
<th>This Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ø-21.7×1.9</td>
</tr>
<tr>
<td>2</td>
<td>Ø-76.3×2.8</td>
</tr>
<tr>
<td>3</td>
<td>Ø-216.3×5.8</td>
</tr>
<tr>
<td>4</td>
<td>Ø-21.7×1.9</td>
</tr>
<tr>
<td>5</td>
<td>Ø-101.6×4.2</td>
</tr>
<tr>
<td>6</td>
<td>Ø-48.6×3.2</td>
</tr>
<tr>
<td>7</td>
<td>Ø-42.7×2.4</td>
</tr>
<tr>
<td>8</td>
<td>Ø-139.8×4.0</td>
</tr>
<tr>
<td>Weight(t)</td>
<td>0.2574</td>
</tr>
</tbody>
</table>

The generation and objective function history of the optimization process is shown in Fig. 10, Fig. 11.
[Example 2] We take the case of space truss dome and all joints of the dome are located on the surface of a sphere as shown in Fig 12. The material properties of the dome are; \( E=2140 \text{ tf/cm}^2 \), \( \rho=7.85\times10^{-6} \text{ tf/cm}^3 \). The stresses are limited to \( \sigma_t=1.325 \text{ tf/cm}^2 \) (tensile), to \( \sigma_c=-1.06 \text{ tf/cm}^2 \) (compression). Node1 was constrained to a vertical displacement limit of 2.6cm.

Fig. 12 shows a paraboloid layout for the dome with a diameter of 30m, which is assumed to be loaded by four vertical load systems;
1. -30.591tonf acting only at joint number 1.
2. -3.059tonf acting at all the joints 1 to 13.
3. -15.295tonf acting at joint number 1 and -10.197tonf at the joint 4 and 5.

The following control parameters are used for the GAs in the design of space truss dome; a crossover probability of 0.85, a mutation probability of 0.01, a population size of 60, a string length per variable of 6. Comparisons of results are presented in Table 5. From the results, it is found that GAs search technique is very effective for discrete optimal design of space truss structures and has high robustness.

Table 5 Summary of discrete optimized designs of Space truss dome

<table>
<thead>
<tr>
<th>Member Group</th>
<th>This paper</th>
<th>Pauli(1973)</th>
<th>MIDAS-GENw</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ø-267.4x6.6</td>
<td>43.99</td>
<td>Ø-216.3x7.0</td>
</tr>
<tr>
<td>2</td>
<td>Ø-165.2x5.0</td>
<td>14.37</td>
<td>Ø-114.3x5.6</td>
</tr>
<tr>
<td>3</td>
<td>Ø-139.8x4.0</td>
<td>24.24</td>
<td>Ø-165.2x4.5</td>
</tr>
<tr>
<td>4</td>
<td>Ø-165.2x5.0</td>
<td>29.25</td>
<td>Ø-165.2x4.5</td>
</tr>
<tr>
<td>5</td>
<td>Ø- 76.3x3.2</td>
<td>4.99</td>
<td>Ø- 76.3x2.8</td>
</tr>
<tr>
<td>6</td>
<td>Ø-101.6x4.2</td>
<td>27.69</td>
<td>Ø-190.7x4.5</td>
</tr>
<tr>
<td>7</td>
<td>Ø- 48.6x3.2</td>
<td>17.11</td>
<td>Ø-190.7x4.5</td>
</tr>
<tr>
<td>8</td>
<td>Ø- 76.3x2.8</td>
<td>13.25</td>
<td>Ø-190.7x4.5</td>
</tr>
</tbody>
</table>

| Weight (tf) | 6.749     | 6.675     | 8.287     |

The generation and objective function history of the optimization process is shown in Fig. 13, Fig. 14.
6. CONCLUSION

The following observations are made on the discrete optimum design using the proposed genetic algorithms based on the solutions of the problems discussed in this paper.

This study is developed a useful discrete optimum design program using GAs. It is found that the developed program gives more accurate results than it by traditional methods and commercial program (MIDAS-GENw) in optimum design of space truss structures.

This study’s Algorithm optimum search technique is effective to find the good optimum solution as well as has validity.

REFERENCES

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Fig 14. Process of objective function