On Generating a Dynamic Price Formation System with Rationality*
— Application to U.S. Fisheries —

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I. Introduction

There have been modest but growing studies on price formation systems over the past decade or so, both theoretically and empirically. By

* I would like to thank anonymous referees for helpful comments and suggestions but the usual caveat applies.
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price formation system, we mean a system where price variations are explained by quantity variations. In this and other contexts, applied economists are motivated to measure price flexibilities by demand shocks. For some goods such as fish, it is appropriate to estimate price flexibilities directly because the production process may be such that market supplies of related goods are determined largely in advance of prices or they are inelastic. In such cases, a complete price formation system makes both econometric and economic senses. By the pioneering work of Antonelli over a century ago, it is now widely recognized that such systems exist and can be manipulated analogous to Marshallian demand systems.

In his book, Hicks (1956) suggests “the dependence of prices on quantities can of course be worked out, ..., by solving the system of equations given by equality between demand and supply in the various markets” (Hicks, 1956, p. 149). Thus, we come to the part of the generalized theory of demand which corresponds to the inversion of the demand curve into the marginal valuation analysis.

Surveying the existing literature, an example of other studies than fisheries is found in the study of oligopolistic markets by Cheng (1985), in which some firms set prices by taking the quantities of other firms as given. Another is the set of studies of price formation of public goods for which quantities cannot adjust in the short run. Since quantities are predetermined by production at the hypothetical market level, price must adjust so that the available quantity is consumed. Similarly, it has been argued that such is likely the case for resources, food and fish.1) There

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1) Although statistical considerations have provided the main argument for price dependency, another consideration also supports the price formation approach: that
are many evidences compiled by Huang (1988) for U.S. composite foods, Barten and Bettendorf (1989) for fish, Eales and Unnevehr (1994) and Holt (2002) for U.S. meat, Brown et al. (1995) for fresh oranges. Another application of price formation systems can be found in the optimal taxation literature in which price formation functions are derived from the distance function in order to implement optimal Ramsey rules (See Deaton, 1980, 1981, 1986 and Stern, 1986; Park and Hwang, 2000). In addition, resource and environmental economists have widely used price formation functions as marginal willingness to pay functions in order to evaluate the welfare changes from changes in environmental quality and public services. Its application can be found in Palmquist (1988) and Rosen (1974) for the hedonic price models.

In many empirical studies, as Hicks pointed out, different demand specifications have generally different implications even though they are based on the same microeconomic theory of representative consumer behavior. Interests in price formation systems stem from the existence of goods for which the assumption of predetermined prices may not be viable and current supplies may be restricted by biological lags or production lags. Most of studies on fisheries have proposed price formation systems in a static framework, not dynamic one.

The purpose of this study is to overcome some of the deficiencies by incorporating dynamic structures into the static framework. This study takes a different approach from the existing studies by Barten (1993) and Brown et al. (1995) in three points. One is to extend a version of price formation system into a dynamic system, considering some form of quantities for perishable goods are usually subject to substantial measurement error and that retail prices are probably more accurate measures.
endogenous taste formation by past decision or future expectation. For example, Brown et al. (1995) applied the static marginal value functions to the U.S. orange producers assuming weekly supply is fixed. It seems a dubious application in that oranges are storable goods so that inventory adjustment is one of important factors for producers. Thus, it may not be argued that "the problem for the firm is essentially the same as that for the consumer." In their application, dynamic factors may be more important than in my application. Unfortunately, they did not extend it dynamically.

The second as the most important motive is to adapt the model to estimate the marginal values to consumers of commercial fisheries.\textsuperscript{2)} Since it is conceived of regulations as inducing movements along the marginal value curves, it is of growing importance to regional and national policy makers who are confronted with competing claims on diminishing fish stocks by commercial and recreational fisheries interests. Recent regulation in the United States are motivated by that large numbers of fish species are over fished. Especially the most valuable species, the grouper-snapper complex are under management jurisdiction of the National Marine Fisheries Council. In the empirical section, we will show how the U.S. fisheries data work well with the proposed price formation system here.

The third is to develop dynamically flexible price formation systems with full rationality. The words 'flexible' and 'rationality' should be

\textsuperscript{2)} When we are studying market demand, i.e. the demand from the whole group of consumers of the commodity, the 'quantity into price' approach becomes at least as important as 'price into quantity' approach. For we then very commonly begin with a given supply, and what we require to know is the price at which that given supply can be sold (See Hicks, 1950, p. 83).
emphasized in more details. The marginal valuation systems proposed here are more flexible in the sense that they have more degrees of freedom than existing demand systems. For consumer's rationality, we usually assume that the consumers are myopic in the sense that they adjust their behavior gradually. As a consequence, they may yield suboptimal behaviors. Instead, this study assumes that the consumers are farsighted and rational by incorporating taste changes immediately into the marginal valuation. In other words, they anticipate the expected future consequences of their current behaviors.

Thus, of the most concern is the issue of endogenous tastes by habit-formation related to past consumer decisions, or interdependent preferences, or prices and snob appeal. In the ordinary demand approach, they usually formulate habits or persistence in consumption patterns as taste changes. The concept of habit formation is then implied by change in the consumer's indifference map induced by past consumption (myopic view). However, this study extends the concept to include the case that consumers may be conscious of their effects in their behavior (rational view). For rationale of habit formation idea in the context, it may be argued that people get addicted not only to alcohol and cigarettes but also to music, work, and eating. In this sense, much more behavior than expected may be included into habit formation. Taking an example of aquatic resources, fish, it can thought of as habit forming. It is why consumers learn their utility of consuming one species from experience and generally know about only a few species since experimenting and getting information is costly. Thus, they may not treat experienced species and not-experienced ones as identical, even if the species are in fact almost the same in taste. Since tastes are themselves a product of
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experience, past consumption decisions will influence their current consumption through the lifetime marginal utility of consumption. This study will explain at least in part this idea.

A further advantage of this study is that, as Deaton and Muellbauer (1980) pointed out, the empirical static demand/inverse demand model may have a strong serial correlation in residuals which indicates that some important dynamic demand factors are omitted. Thus, a dynamic generalization of marginal valuation systems may be necessary in explaining market consumption behavior better. This attempt will have a new contribution to the existing literature of marginal valuation and demand models.

The layout of the article is as follows. Section II is concerned with generating the dynamic price formation system which incorporates the preferred income-equivalent scale and quantity effects on prices both in the short-run and long-run rationally as a first attempt. Section III deals with the data and stationarity issue in the U.S. fishery application as an empirical illustration and discusses some empirical results of a dynamic price formation system for U.S. fisheries. Section IV concludes.

II. A System of Dynamic Price Formation with Rationality

If we have a prior belief that quantities are predetermined or supply is inelastic in the short run and that prices adjust to clear the market, a price formation system is suggested as an appropriate consumer behavior. Several studies have found that demands for agricultural products and
environmental goods can often be well approximated by marginal valuations in the price formation systems (Barten and Bettendorf, 1989; Eales and Unnevehr, 1994). Related to this economic assumption, it seems that a consumer can determine the market price, which is hardly credible in general. It, however, assumes that there are traders between consumers and suppliers. In this case, the traders call the prices and consumers accept them. The resulting market behavior in the short run may be described as market prices set by consumers, not suppliers.

1. A Preliminary Exposition of Price Formation Systems

Taking the inverse of the ordinary demand system for the multi-good case, one can derive a system of price functions with arguments: all quantities and income.\(^3\) When discussing price function in one-good context, the slope of the inverted Marshallian demand function is simply the inverse of the slope of the ordinary Marshallian demand function. This is not generally true for price formation functions.

Towards empirical implementation to set a price formation system, consider the traditional consumer’s maximization problem that a representative consumer takes prices as given to maximize utility subject to a budget constraint. If \( q \) denotes a vector of quantities, \( p \) a vector of money prices, \( y \) total expenditure, and \( U \) concave utility function, then the basic choice problem may be

\[
\text{Max } U(q) \text{ s.t } (p/y)'q = 1
\]

\(^3\) To obtain a system of price functions from the ordinary demand system, conditions for global invertibility are required (See Gale and Nikaido, 1982; Cheng, 1985).
The necessary first-order conditions are

\[ U_q = \lambda \cdot (p/y) \quad (2) \]
\[ (p/y) q = 1 \quad (3) \]

where \( U_q = (\partial U/\partial q) \) is a derivative of \( U(q) \) with respect to \( q \) and \( \lambda \) is Lagrangean constant. It follows from (2) and (3) that a system of demands can be obtained. It is also possible to obtain another specification from (2) and (3) that represents a price formation system. Specifically, premultiplying \( q \) to (2)

\[ q \cdot U_q = q'(p/y) = \lambda \quad (4) \]

Similarly, it is evident from (4) that a normalized price vector in (2) may be written as

\[ p/y = \lambda^{-1} U_q = [q' U_q]^{-1} U_q \quad (5) \]

Let \( p/y = v \) be the normalized price vector and totally differentiate (5) to obtain

\[ \Delta v = \pi \cdot \Delta q \quad (6) \]

where \( \pi \) is a matrix of \( n \times n \) and defined as

\[ \pi = (I - v q') t U_{qq'} (I - q v') - [v - (I - v q') t U_{qq'} q] v' \quad (7) \]

and \( t = [q' U_q(q)]^{-1} \) and \( U_{qq'} \) is the second derivative of \( U \). It is a fundamental price formation system in the static context.
The above modelling explained, however, ignores the potential for dynamics to influence consumption decisions and long-run price formation behavior. Many recent studies report that dynamic specification is needed in demand systems to fit data and improve forecasting ability.

2. Generating Dynamic Price Formation Systems with Rationality

A dynamic version of consumer demand may be obtained by considering some form of endogenous taste formation by past decision or future expectation, and introducing a variable related to past behavior into the utility function. Since tastes are themselves a product of experience, the newly introduced variable would include the influence that the behavior in past period exerts on the present. Thus, it may be related to the habit strength, psychological or physical stock of goods.

A dynamically generalized model, however, should be a rational model fully consistent with a lifetime budget in the sense that the consumer is aware of the endogenous process in which future tastes are formed (See Pollak, 1978 and Blundell, 1988). The error correction type of demand model called as a naive habit formation model can be a dynamic demand model but is myopic without full rationality. If consumers insist on being myopic as in partial adjustment models, then it is less clear that the intertemporal utility function is the appropriate welfare criterion. Thus the range of application of those models would be much less.4)

In Ryder and Heal (1973), Boyce (1975), Stigler and Becker (1977), Klijn

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4) By the price-dependent error correction model, we mean that the role of price and quantity is exchanged in quantity-dependent error-correction model.
and Becker et al. (1994), one can find theoretical models that consumers
are rational or farsighted in the sense that they anticipate the expected
future consequences of their current actions. The main feature of these
models is that past consumption of some goods influences their current
consumption by affecting the marginal utility of current and future
consumptions.

To develop a fully rational model of dynamic price formation systems,
we relax the intertemporal separability as usually assumed. As Deaton
and Muellbauer (1980) and Becker et al. (1994) point out,\(^5\) the
intertemporal separability assumption seems somewhat restrictive because
it does not allow dynamics other than those arising from dynamic wealth
change and aggregate consumption effects. Relaxing this assumption to
follow Boyer (1983) and Becker et al. (1994), consider a model with two
goods \((q, z)\) and current-period utility in period \(t\) derived from the
consumption \(q_t\) at time \(t\) and \(q_{t-1}\) at time \(t-1\), namely that:\(^6\)

\[
U_t(q_t, z_t) = U(q_t, q_{t-1}, z_t)
\]

(8)

where \(z_t\) is consumption of a composite good in period \(t\) and \(U_t\) is the
one period or instantaneous utility function which is nonstationary
function because of habit formation.\(^7\)

\(^5\) They notice that some restriction on consumer demand may fail because of
intertemporal separability.

\(^6\) The \(q_t\) is dealt as a scalar, but it could be assumed as a consumption vector
without hurting the results drawn here. The presence of \(q_{t-1}\) may reflect
adjustment costs in consumption or habit persistence.

\(^7\) Note that \(U_t(\cdot, \cdot)\) is obtained from a stationary function \(U(\cdot, \cdot)\) having as arguments
both the current and the previous consumption.
Assuming that consumers be infinite-lived and maximize the sum of lifetime utility discounted at the rate $r$, the intertemporal consumer’s problem is given by

$$\text{Max. } \sum_{t=1}^{\infty} \beta^{t-1} U(q_t, q_{t-1}, z_t)$$

such that $q_{t=0} = q^0$ and

$$\sum_{t=1}^{\infty} \beta^{t-1} (z_t + p_t q_t) = A^0$$

where $A^0$ is the present value of wealth, $p_t$ is the price of $q_t$, $\beta = 1/(1+r)$ and $r$ is an interest rate which is assumed to equal the time preference rate.

The first order conditions of the optimization problem are derived as follows. Let $L$ be the Lagrangean associated with eq. (9),

$$L = \sum_{t=1}^{\infty} \beta^{t-1} U(q_t, q_{t-1}, z_t) + \lambda [A^0 - \sum_{t=1}^{\infty} \beta^{t-1} (z_t + p_t q_t)]$$

Then, the necessary conditions for optimization are, for each $t$

$$U_z(q_t, q_{t-1}, z_t) = \lambda$$

$$U_1(q_t, q_{t-1}, z_t) + \beta U_z(q_{t+1}, q_t, z_t) = \lambda p_t$$

where the subscripts of $U$ denote the partial derivatives with respect to its arguments in order.

Eq. (12) states that the marginal utility of other consumption in each
period equals the marginal utility of wealth, \( \lambda \). Eq. (13) implies that the marginal utility of current consumption in question (\( U_i \)) plus the discounted marginal effect on next period’s utility of today’s consumption (\( \beta U_2 \)) equal the marginal utility of wealth multiplied by prices.

By mean value theorem, the necessary conditions may be written in the form:\(^8\)

\[
U_{z1} \Delta q_t + U_{z2} \Delta q_{t-1} + U_{zz} \Delta z_t = 0
\]

\[
U_{11} \Delta q_t + U_{12} \Delta q_{t-1} + U_{zz} \Delta z_t \\
+ \beta(U_{21} \Delta q_t + U_{22} \Delta q_{t-1} + U_{zz} \Delta z_t) = \lambda \Delta p_t
\]

where the marginal utility of wealth (\( \lambda \)) is assumed to be constant under perfect foresight and \( U_i(x_1, \ldots x_n) = \partial^2 U / \partial x_i \partial x_j \). By arranging the condition (14) with respect to \( z_t \), one can obtain

\[
\Delta z_t = -\frac{U_{z1}}{U_{zz}} \Delta q_t - \frac{U_{z2}}{U_{zz}} \Delta q_{t-1}
\]

Substituting eq. (16) into eq. (15) yields

\[
\Delta p_t = \phi_1 \Delta q_t + \phi_0 \Delta q_{t-1} + \beta \phi_0 \Delta q_{t+1}
\]

where \( \phi \)'s are defined as

\[
\phi_1 = \frac{U_{11} U_{zz} - U_{1z}^2 + \beta (U_{21} U_{zz} - U_{2z}^2)}{\lambda U_{zz}}
\]

\(^8\) The price (\( p_i \)) can be normalized by the income (\( y_t \)) where \( y_t = z_t + p_i q_t \). In this case, the price of \( z_t \) will be \( 1/y_t \) and the price of \( q_t \) is \( v_t = p_t/y_t \).
\[ \phi_0 = \frac{U_{22}U_{zz} - U_{12}U_{2z}}{\lambda U_{zz}} \]

Equivalently, assuming that all the variables are scaled in natural logarithm, it could be written as

\[ \Delta \ln p_i = \phi_1 \Delta \ln q_i + \phi_0 \Delta \ln q_{i-1} + \beta \phi_0 \Delta \ln q_{i+1} \quad (17a) \]

Comparing to eq. (6), how can it be interpreted? It can be interpreted that when we predict changes in prices (marginal valuations), we need to know changes in quantities themselves in the previous, current and next periods. It looks a rather strong result which is obtained by tight linkages between its own consumption goods at a different time. Note that for one-good case, \( \phi_1 \) is negative by concavity of \( U \). Hence, it follows that an increase in quantity produces a decrease in price, other things being equal.

The effect of changes in past demand on marginal valuation is summarized in \( \phi_0 \). When \( \phi_0 \) is positive, the increase in past consumption increases marginal valuation. Since the increase in past consumption stimulates the current consumption in the case of habit forming goods, superimposing the current consumption unchanged (by the predetermined quantities assumption) should necessarily increase the marginal valuation (prices) at time \( t \). Thus, the positive sign of \( \phi_0 \) implies that past consumption and current consumption of \( q \) are \( q \)-habit complements and the negative \( \phi_0 \) implies that past and current consumptions of \( q \) are \( q \)-habit substitutes. Notice, however, that the notion of \( q \)-habit substitutes may give misleading interpretation because
it implies that the good in question is not habit forming. Instead, it will be interpreted that the past and future consumptions affect current marginal valuations for the good in question if consumers behavior rationally. Thus, negative $\phi_0$ means that total spending on that good in its lifetime budget is important rather than habits.

Extending to that $q$ is a vector of consumption bundle, we have more complex own and cross effects of changes in consumption on the marginal valuations of goods. In this case, $\phi_0$ is a matrix of coefficients which reflects the cross effects of past consumption of other goods and the own effects of past consumption of the goods themselves.

Note that the coefficients of past and future consumptions are not independent. This may give another restriction on the specification (17) in estimation. We shall see that point in empirical section. Given that specification, the long-run scale elasticity (income-equivalent effects in marginal values) and price flexibility (inverse demand elasticity) can be obtained when the long-run equilibrium is defined as a steady state, in which all equilibrium values grow at a constant rate. Note that in a steady state, we have that $\ln q_{it} = \ln q_{i,t-1} = \ln q_{i,t+1}$. Thus, the long-run scale elasticity and price flexibility can be defined.

### III. An Empirical Illustration of U.S. Fisheries

As a price formation system, demand models that specify prices as a function of quantities consumed seem reasonable in the setting of inelastic supply. One of such resources to fit would be fishery products.
To illustrate the dynamic modelling strategy outlined in the preceding section, we apply it to the U.S. fish demand data. They are particularly interesting to this analysis in which the species are over fished, including many of the most valuable species. For example, the 92 fish stocks are over fished in 2000 (NMFS, 2001). Thus, it has led to stock rebuilding plans that are overseen by regional councils. All the species in the data are under management as the most concern in the south Atlantic regions (SAFMC, 2003). With overfishing stocks and growing demand for fishery products, fishery managers are confronted with the difficult tasks of setting allowed harvest levels and allocating harvest among user groups. Without estimating demands or price functions of fishery products correctly, as Thurman et al. (2004) pointed out, the consumer and processor costs of harvest reductions and the future benefits of stock changed cannot be computed.

1. Data and Stationarity Issue

In order to apply a dynamic price formation system, the data in Thurman et al. (2004) are utilized, which consist of the monthly landings and monthly prices of commercial fishes over 1977 to 1992 in the Southeast region of the United States. The commercial fishes include six aggregate types of groupers, porgies, snappers, jacks, tilefishes, and sea basses. As presented in <Table 1>, the average expenditure shares within the group are 42.2, 5.2, 41.7, 3.8, 3.6, 3.4% for groupers, porgies, snappers, jacks, tilefishes, and sea basses, respectively. Those fish types are representative and show diminishing stocks recently. The sample shows wide ranges of the landed quantities of each type of fish from
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〈Table 1〉 The Summary Statistics of the Data

<table>
<thead>
<tr>
<th>Types of Fish</th>
<th>Average Share (Unit: %)</th>
<th>St. Dev. (1)</th>
<th>Average Price ($/10lbs)</th>
<th>St. Dev. (2)</th>
<th>Average Quantity (Unit: lbs)</th>
<th>St. Dev. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grouper</td>
<td>42.2</td>
<td>8.2</td>
<td>13.3</td>
<td>4.5</td>
<td>1,070,704</td>
<td>350,734</td>
</tr>
<tr>
<td>Porgy</td>
<td>5.2</td>
<td>3.0</td>
<td>4.5</td>
<td>1.3</td>
<td>381,963</td>
<td>234,741</td>
</tr>
<tr>
<td>Snapper</td>
<td>41.7</td>
<td>9.7</td>
<td>16.7</td>
<td>3.4</td>
<td>779,644</td>
<td>187,956</td>
</tr>
<tr>
<td>Jacks</td>
<td>3.8</td>
<td>2.6</td>
<td>2.8</td>
<td>1.5</td>
<td>477,913</td>
<td>340,017</td>
</tr>
<tr>
<td>Tilefish</td>
<td>3.6</td>
<td>2.5</td>
<td>10.0</td>
<td>3.7</td>
<td>124,627</td>
<td>93,925</td>
</tr>
<tr>
<td>SeaBass</td>
<td>3.4</td>
<td>3.5</td>
<td>9.0</td>
<td>3.3</td>
<td>123,029</td>
<td>113,809</td>
</tr>
</tbody>
</table>

Note: (1) The column St. Dev. (1) denotes the standard deviations of their shares. Similarly, the columns St. Dev. (2) and (3) indicate the standard deviations of their prices and quantities, respectively.

1,736 lbs to 2,594,571 lbs. It follows from average quantities that groupers and snappers are the dominant types of commercial fishes.

In all dynamic systems, stationarity is an important issue to yield reliable estimates of the parameters. If the levels of the variables are nonstationary, then the regression equations are subject to the spurious regression phenomenon. The test for the presence of unit roots takes the augmented Dickey–Fuller regressions:

\[ \Delta y_t = \alpha + \beta t + (\rho - 1)y_{t-1} + \sum_{i=1}^{m} d_i y_{t-i} + \varepsilon_t \]  \hspace{1cm} (18)

where \( y \) is the variable under consideration and \( m \) is the number of lags ensuring that errors are white noise.\(^9\) If \( \rho = 1 \) is rejected, then the series is stationary in levels. The result of unit root test is presented in 〈Table 2〉. The null hypothesis in this test is that the series is

\[^9\) Statistically, the lag length is chosen to minimize the Akaike Information Criterion.
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(2) Augmented Dickey-Fuller Test Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Augmented DF Statistic</th>
<th>AR Coefficient</th>
<th>Lag Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>log $v_1$</td>
<td>-3.5</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>log $v_2$</td>
<td>-7.7</td>
<td>0.51</td>
<td>1</td>
</tr>
<tr>
<td>log $v_3$</td>
<td>-3.6</td>
<td>0.16</td>
<td>1</td>
</tr>
<tr>
<td>log $v_4$</td>
<td>-5.3</td>
<td>0.33</td>
<td>1</td>
</tr>
<tr>
<td>log $v_5$</td>
<td>-3.0</td>
<td>0.12</td>
<td>2</td>
</tr>
<tr>
<td>log $v_6$</td>
<td>-7.0</td>
<td>0.39</td>
<td>1</td>
</tr>
<tr>
<td>log $q_1$</td>
<td>-4.6</td>
<td>0.22</td>
<td>1</td>
</tr>
<tr>
<td>log $q_2$</td>
<td>-6.3</td>
<td>0.34</td>
<td>1</td>
</tr>
<tr>
<td>log $q_3$</td>
<td>-7.5</td>
<td>0.61</td>
<td>1</td>
</tr>
<tr>
<td>log $q_4$</td>
<td>-7.2</td>
<td>0.44</td>
<td>1</td>
</tr>
<tr>
<td>log $q_5$</td>
<td>-3.8</td>
<td>0.12</td>
<td>1</td>
</tr>
<tr>
<td>log $q_6$</td>
<td>-6.8</td>
<td>0.32</td>
<td>1</td>
</tr>
<tr>
<td>log $Q$</td>
<td>-5.4</td>
<td>0.35</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: (1) The subscripts of the variables indicate item 1 for groupers, item 2 for porgies, item 3 for snappers, item 4 for jacks, item 5 for tilefishes, item 6 for seabasses, respectively.

(2) The critical value of ADF statistic is -2.6 and lag order for ADF test chosen by the Akaike Information Criterion. The variable log $Q$ is the divisia quantity index in which the change by $k$ implies all quantities increase by $k$ proportion. It is equivalent to income change by a proportional change in prices in ordinary demand systems.

non-stationary. It is tested on the basis of the \( t \)-statistic value of \( \rho \) in eq. (18). Using 5% significance level, the null is rejected for any variable.

To see the stability of the price formation system in the long-run, the Engel-Granger cointegration test is applied to the six fish prices and thus there are six cointegrating regressions to examine. We consider the regressions of the level variables and test for a unit root in their regressions residuals. The ADF statistic for each regression using a lag
length of 1, which was chosen by the minimum AIC technique, are -5.4, -7.7, -7.3, -7.1, -5.8, -8.0 for grouper, porgy, snapper, jacks, tilefish, sea bass, respectively. Checking the stationarity of the residuals of long-run price formation systems, the presence of cointegrating vectors is not rejected as expected.

2. Parameterization of A Dynamic Price Formation System with Rationality

As differential demand systems, the double logarithm or log linear demand systems are often used because of simplicity but clarification. However, their parameter matrix do not satisfy the usual demand conditions based on consumer’s rational choices. As a similar differential system but satisfying all usual demand conditions, one can propose to multiply eq. (6) by the quantities to arrive at a choice of constants satisfying demand conditions. It can be called as the Rotterdam price formation system corresponding to the Rotterdam demand system in the dual space.

Using the Slutsky equation, the price formation system can decompose change in prices into substitution effects and income-equivalent scale effects in primal space which is analogous to the usual decomposition into substitution and income effects. The difference comes from the way to compensate loss of utility. In the primal space, compensation is ensured by a proportional change in quantities (frequently using the quantity index) while compensation in the dual space is taken by a proportional change in prices or equivalently change in income.

Let the price formation equation for fish type $i$ be in the double
logarithm and apply the Slutsky equation in elasticity form:

\[ \Delta \log v_{it} = \sum_{j} e_{ij} \Delta \log q_{jt} = \sum_{j} \left( e_{ij}^c + e_{ij}^e \right) \Delta \log q_{jt} \]

\[ = \sum_{j} e_{ij}^c \Delta \log q_{jt} + e_{i} \sum_{j} w_{ij} \Delta \log q_{jt} \]  

(19)

where \( e_{ij}^c \), \( e_{ij}^e \) and \( e_{i} \) represent Hicksian and Marshall substitution effects and income-equivalent scale effects, respectively. Then the empirical price formation system for fish \( i \) can be read after multiplying its own budget share:

\[ w_{i} \Delta \log v_{it} = \sum_{j} \pi_{ij} \Delta \log q_{jt} + \pi_{i} \Delta \log Q_{t} + \eta_{it} \]  

(20)

where \( W_{i} \) is the budget share of fish \( i \) and \( \Delta \log Q_{t} \) is the Divisia (Stone) quantity index at time \( t \).10 The \( \eta_{i} \) denotes error terms, \( [\pi_{ij}] \) the substitution matrix, and \( [\pi_{i}] \) is a vector of income-equivalent scale effects that play a role of income effect in the primal space. Specifically,

\[ \pi_{ij} = w_{i} e_{ij}^c \]

\[ \pi_{i} = w_{i} e_{i} \]

The extension of this static parameterization into dynamic one seems straightforward. Since the coefficients of past consumption and future consumption bundles are included, a dynamic price formation system with rationality may be written empirically as

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10 This index can be derived by taylor expansion of the budget equation directly and defined as \( \sum w_{i} \Delta \log q_{i} \). Note also that the coefficients in the system satisfy the symmetry condition after multiplying the budget share \( w_{i} \) and are adjusted by \( w_{i} \Delta \log Q_{t} \) in estimation.
\[ w_i \Delta \log v_{i-t} = \sum_j \pi_{j} \Delta \log q_{j-t} + \pi_i \Delta \log Q, \]

\[ + \sum_j \phi_{j} \Delta \log q_{j-t-1} + \phi_i \Delta \log Q_{t-1} \]

\[ + \beta \left[ \sum_j \phi_{j} \Delta \log q_{j-t+1} + \phi_i \Delta \log Q_{t+1} \right] + \epsilon_i, \]

where \( \epsilon_i \) denotes error terms and \( \beta \) the discount factor. Eq. (21) will be the maintained framework of a dynamic price formation system with rationality in this study. Note that the coefficients of past consumption and future consumption are not independent. This may give another restriction on the specification (21) in estimation. All the restrictions including this are imposed on the price formation system.

Given eq. (21), the long-run income-equivalent scale elasticity and price flexibility can be obtained when the long-run equilibrium is defined as a steady state, in which all equilibrium values grow at a constant rate. Note that in a steady state, we have that \( \ln Q_t = \ln Q_{t-1} = \ln Q_{t+1} \) and \( \ln q_{i-t} = \ln q_{i-t-1} = \ln q_{i-t+1} \). Thus, the long-run price flexibility and scale elasticity can be defined by

\[ w_i f_{i} = \pi_i + (1 + \beta) \phi_i \]  \hspace{1cm} (22)

\[ w_i f_{i} = \pi_i + (1 + \beta) \phi_i \]  \hspace{1cm} (23)

if \( f_{i} \) denotes a long-run price flexibility and \( f_i \) indicates a long-run scale elasticity of price for fish \( i \).\(^{11)\)

\(^{11)\) Hereafter, \( e_i^{*} \) will be used as \( f_{i} \) for short and \( e_i \) as \( f_i \) for consistency with the existing literature.
3. Estimation of A Dynamic Price Formation System with Rationality

1) Estimation Method

To estimate the parameters of the price formation system, the specification should be modified in some respects. Firstly, to account for seasonality of demand in the monthly data, eq. (21) is augmented with eleven seasonal dummy variables, $D_{kt}$ ($k = 2, \ldots, 12$) whose associated coefficients must sum to zero to ensure adding up condition. Secondly, one equation is deleted in estimation and then recovered from the rest of the estimated equations due to the singularity of the full model’s residual covariance matrix by adding up condition. Barten (1993) shows that the estimates are not variant with respect to the deleted equation during estimation of maximum likelihood.

Given the form of (21), the instrumental variable method (IV) using instrumental variables for future consumption levels may be used since one period lags and leads of quantity variables are included in the system. In some empirical studies, actual future consumption is used for expected future consumption. However, this procedure raises a question about measurement error in the variables. Thus, the model is estimated to use predicted consumption, which is formed by regressing it to the second and more lagged values of quantities and other explanatory variables in the system.\(^{12}\)

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\(^{12}\) The instruments in estimation include the second and third lagged values of consumption plus the twelfth lag of consumption (quantities and scale). The reason for the inclusion of the twelfth lag of consumption is that because consumers have
The IV estimators yield consistent parameter estimates. However, this estimation technique yields inefficient estimates. Thus, the cross-equation correlation among errors is taken into account by applying the system estimation method, i.e., the three stage least square estimator. Once the two stage least square parameters have been calculated, the residuals of each equation are used to estimate the cross-equation variances and covariances. In the final stage of the estimation process, maximum likelihood parameter estimates are obtained.

Following Anderson and Blundell (1982), the restrictions suggested by economic theory are imposed on both the short-run and long-run structures of the system. As discussed in the section II. 2, eq. (21) has another restriction which was derived from the theory: a constant relationship between future and past quantity effects. The coefficients of future consumption are equal to $\beta$ (the discount rate) multiplied by the estimated coefficients of past consumption. It seems impossible to infer the discount rate reliably from the data. Thus, to make estimation tractable, we impose the discount factor $a \text{ priori}$ ranging from 0.70 to 0.95 which correspond to interest rates ranging from 5.3 percent to 42.9 percent. For this whole range, eq. (21) is estimated by 3SLS using predicted consumption.

2) Empirical Results

The first column in <Table 3> indicates the price formation equations for the specific fish types. The second and third columns shows the estimated price flexibilities ($f_i$) and income-equivalent scale elasticities information concerning the most recent seasonal consumption, one loses valuable information by discarding these variables as instruments.
### Table 3  The Estimated Dynamic Price Formation System

<table>
<thead>
<tr>
<th>Equation</th>
<th>Short-run Flexibilities</th>
<th>Values</th>
<th>t-values</th>
<th>Long-run Flexibilities</th>
<th>t-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grouper</td>
<td>$f_{s1}$</td>
<td>-0.078</td>
<td>(-5.42)</td>
<td>-0.154</td>
<td>(-4.82)</td>
</tr>
<tr>
<td></td>
<td>$f_{s2}$</td>
<td>0.022</td>
<td>(3.97)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s3}$</td>
<td>0.040</td>
<td>(3.37)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s4}$</td>
<td>0.008</td>
<td>(1.72)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s5}$</td>
<td>-0.001</td>
<td>(-0.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s6}$</td>
<td>0.010</td>
<td>(2.27)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s7}$</td>
<td>-1.002</td>
<td>(-37.72)</td>
<td>-1.039</td>
<td>(21.91)</td>
</tr>
<tr>
<td>Porgy</td>
<td>$f_{s1}$</td>
<td>0.176</td>
<td>(3.97)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s2}$</td>
<td>-0.113</td>
<td>(-3.24)</td>
<td>-0.140</td>
<td>(-1.77)</td>
</tr>
<tr>
<td></td>
<td>$f_{s3}$</td>
<td>0.022</td>
<td>(0.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s4}$</td>
<td>-0.013</td>
<td>(-0.72)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s5}$</td>
<td>-0.022</td>
<td>(-1.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s6}$</td>
<td>-0.049</td>
<td>(-2.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s7}$</td>
<td>-0.890</td>
<td>(-10.73)</td>
<td>-0.763</td>
<td>(-10.12)</td>
</tr>
<tr>
<td>Snapper</td>
<td>$f_{s1}$</td>
<td>0.040</td>
<td>(3.57)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s2}$</td>
<td>0.003</td>
<td>(0.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s3}$</td>
<td>-0.044</td>
<td>(-3.47)</td>
<td>-0.087</td>
<td>(-2.91)</td>
</tr>
<tr>
<td></td>
<td>$f_{s4}$</td>
<td>-0.002</td>
<td>(-0.42)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s5}$</td>
<td>0.005</td>
<td>(1.42)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s6}$</td>
<td>-0.002</td>
<td>(-0.51)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s7}$</td>
<td>-1.004</td>
<td>(-47.71)</td>
<td>-0.995</td>
<td>(-13.56)</td>
</tr>
<tr>
<td>Jack</td>
<td>$f_{s1}$</td>
<td>0.084</td>
<td>(1.72)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s2}$</td>
<td>-0.018</td>
<td>(-0.72)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s3}$</td>
<td>-0.018</td>
<td>(-0.42)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s4}$</td>
<td>-0.079</td>
<td>(-2.84)</td>
<td>-0.052</td>
<td>(-0.86)</td>
</tr>
<tr>
<td></td>
<td>$f_{s5}$</td>
<td>-0.006</td>
<td>(-0.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s6}$</td>
<td>0.036</td>
<td>(1.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s7}$</td>
<td>-1.032</td>
<td>(10.72)</td>
<td>-0.905</td>
<td>(-11.19)</td>
</tr>
<tr>
<td>Tilefish</td>
<td>$f_{s1}$</td>
<td>-0.012</td>
<td>(-0.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s2}$</td>
<td>-0.032</td>
<td>(-1.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s3}$</td>
<td>0.052</td>
<td>(1.42)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s4}$</td>
<td>-0.006</td>
<td>(-0.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s5}$</td>
<td>-0.006</td>
<td>(-0.19)</td>
<td>-0.023</td>
<td>(-0.44)</td>
</tr>
<tr>
<td></td>
<td>$f_{s6}$</td>
<td>0.001</td>
<td>(0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s7}$</td>
<td>-1.012</td>
<td>(-14.90)</td>
<td>-0.904</td>
<td>(-11.32)</td>
</tr>
<tr>
<td>SeaBass</td>
<td>$f_{s1}$</td>
<td>0.120</td>
<td>(2.27)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s2}$</td>
<td>-0.074</td>
<td>(-2.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s3}$</td>
<td>-0.024</td>
<td>(-0.51)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s4}$</td>
<td>0.040</td>
<td>(1.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s5}$</td>
<td>0.001</td>
<td>(0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{s6}$</td>
<td>-0.064</td>
<td>(-2.10)</td>
<td>-0.183</td>
<td>(-3.02)</td>
</tr>
<tr>
<td></td>
<td>$f_{s7}$</td>
<td>-1.099</td>
<td>(-10.82)</td>
<td>-1.139</td>
<td>(-11.81)</td>
</tr>
</tbody>
</table>
(\(f_i\)) using predicted future consumption for the leads of quantity variables and restricting \(\beta = 0.7\), an interest rate of 5.3 percent.\(^{13}\)

According to the table, the scale elasticities in the short-run are all close to minus one and are estimated with a high degree of precision, suggesting nearly homothetic preferences. The same is true for most of the long-run scale elasticities. Looking at price flexibilities in the short-run, the own-price flexibilities are all estimated negatively and 5 of 6 goods except tilefish are estimated with a high degree of precision. It is observed that these flexibilities are low and in a rather narrow range as found by Barten and Bettendorf (1989) and Holt and Bishop (2002). Typically, fish consumed at home is found to be elastic so that its marginal value is inelastic.

For the cross effects, 15 of 30 pairs are estimated as substitutes and 11 of 30 are statistically significant. By the homogeneity condition, the off-diagonal terms in the substitution matrix must average out to zero with each good being a substitute for itself and thus complementarity may dominate over substitutability or independence in the price formation system. The estimates in the long-run are quite similar to those in the short-run, though long-run own price flexibilities are greater except that for jacks which is not significant.\(^{14}\)

Another note should be taken to the effect of past consumption on the current price formation. \(<\text{Table 4}\>\) reports this effect with model fitness. The second column in the table shows the coefficients of determination \((R^2)\) as an indication of relative fit and the Durbin–Watson statistics.

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13) Note that \(\beta\) is just the discount factor, \((1+r)^{-1}\) where \(r\) is the interest rate.

14) The empirical results restricting \(\beta = 0.8\) are almost the same as restricting \(\beta = 0.7\) and thereby not reported here.
Table 4  Estimated Past Quantity Effects on The Current Prices

<table>
<thead>
<tr>
<th>Equation of</th>
<th>( R^2 )</th>
<th>( T )-Test for</th>
<th>Past Quantity</th>
<th>Own Past Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fishes</td>
<td>DW</td>
<td>Homotheticity</td>
<td>Parameters</td>
<td>Coefficients</td>
</tr>
<tr>
<td>Grouper</td>
<td>0.98/1.7</td>
<td>0.14</td>
<td>( Q_{1,t-1} )</td>
<td>-0.04 (3.30)</td>
</tr>
<tr>
<td>Porgy</td>
<td>0.87/1.8</td>
<td>1.32</td>
<td>( Q_{2,t-1} )</td>
<td>-0.02 (0.45)</td>
</tr>
<tr>
<td>Snapper</td>
<td>0.98/2.1</td>
<td>0.22</td>
<td>( Q_{3,t-1} )</td>
<td>-0.03 (1.99)</td>
</tr>
<tr>
<td>Jack</td>
<td>0.82/2.1</td>
<td>0.34</td>
<td>( Q_{4,t-1} )</td>
<td>0.02 (0.65)</td>
</tr>
<tr>
<td>Tilefish</td>
<td>0.85/2.1</td>
<td>0.27</td>
<td>( Q_{5,t-1} )</td>
<td>-0.01 (0.50)</td>
</tr>
<tr>
<td>Seabass</td>
<td>0.85/2.2</td>
<td>0.97</td>
<td>( Q_{6,t-1} )</td>
<td>-0.07 (2.58)</td>
</tr>
</tbody>
</table>

Note: The values in the parentheses indicate \( t \)-statistics.

which measure autocorrelation in the disturbances for each equation.\(^{15}\)

According to the table, the \( R^2 \)'s are relatively high due to the large variation in the data and none of the equations seem to have autocorrelation.\(^{16}\) Since homothetic preferences are likely suggested in Table 3, the test results for homotheticity are given in the third column of Table 4 and confirm homothetic preferences.

As a final note in this section, the signs of \( \ln v_{it}/\ln q_{it-1} \) are checked to see whether fishes follow consumption-habit formation. Since an increase in past consumption leads to an increase in current consumption in the case of habit forming goods, superimposing current consumption unchanged should increase the marginal valuation at time \( t \).

Thus, if the sign is positive, then commodity \( i \) is a \( q \)-habit complement.

---

15) The system weighted \( R^2 \) as a whole is the value of 0.9922, which is not shown in the table.
16) However, this does not imply no high-order of autocorrelations. Thus, we examine the test statistics for high-order of autocorrelations, Ljung-Box \( Q \)-statistics, which indicate no high-order autocorrelations after the correction in estimation.
which implies an addictive good. If the sign is negative, it is a $q$-habit substitute which implies a satiating good with the dynamic effect of fish stocks. The estimated results are shown in the last column of Table 4. According to the table, groupers, snappers and seabasses appear to be $q$-habit substitutes with the statistical significance. Thus, there seems no evidence that fish is a strong addictive good. However, past consumption is important on the current consumption decision as a dynamic factor of stock effects.

IV. Concluding Remarks

The article has considered two issues: (1) the usefulness of a price formation system for a market behavior, (2) a dynamic generalization of the price formation system by incorporating endogenous taste formation. Many studies report a dynamic generalization may be necessary in explaining market consumption behavior better. In this regard, this article will have a new contribution on the following two points: a rational dynamic generalization of the price formation systems by a new approach and an empirical illustration of price formation for natural resources such as fish dynamically and rationally.

In the same line of works by Barten (1993), Brown et al. (1995), Holt and Bishop (2002) considering a marginal valuation function as a proper model for the short-run market behavior, this article has generalized their works to the intertemporal optimization-based dynamic flexible system and applied it to aquatic resources. The underlying idea of the dynamic
price formation mechanism is that consumers are rational and farsighted, and thus consider past and future consumptions in addition to current consumption in order to decide current prices (marginal valuations) for the consumption demanded.

This study has focused on an empirical price formation model and its dynamic system with rationality. Such model satisfies all the theoretical restrictions based on consumers' rational choices, i.e., adding up, symmetry, and homogeneity. The empirical results for this model show that it fits the data well and gives plausible signs and magnitudes of income-equivalent scale elasticities and price flexibilities.

Finally, some future research may be suggested. A future application may be fit into land price formation, public and environmental services' price formation. Other future application may treat problems of aggregation across species and welfare analysis. These can be the topics of ongoing and future works. It is hoped that a dynamic generalization of the price formation system should contribute to the general methodology of applied economics.

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30. South Atlantic Fishery Management Council (SAFMC), South Atlantic Update, Fall Newsletter, Charleston, SC, 2003.

요 약

합리성을 가진 동태적 가격형성모형의 연구
— U.S. 수산자원에의 응용 —

박 환 재

본 논문은 최근 연구된 Barten (1993), Brown et al. (1995), Holt and Bishop (2002)의 가격형성체계(price formation system)를 확장 발전시키고 있다. 특히 기존 연구들과 달리 소비행위측면에서 완전합리성을 고려하여 가격이 형성될 수 있는 동태적 모형을 설정하고 있다. 본 연구의 동태적 모형의 기본 아이디어는 소비자가 완전히 합리적이어서 미래를 예측한다고 가정한다. 따라서 어떤 시장에서 중개자가 부르는 가격을 소비자가 수용할 것인지의 여부를 결정할 때 과거와 미래소비가 현재소비에 영향을 미칠 수 있다는 것이다. 본 연구의 실증분석에서는 이러한 모형을 미국 어류시장으로 고갈되어가고 있는 grouper-snapper complex에 적용하고 있다. 자원관리정책측면에서 볼 때 쿼터제를 실시할 경우 소비자의 한계가치가 어떻게 변하는지를 상파보는 것은 후생판도의 수단으로서 경제분석의 핵심이 될 것이다. 실증분석 결과는 본 연구가 제시하고 있는 동태적 가격형성체계(dynamic price formation system)가 통계적으로 적합함을 잘 보여 준다. 특히 그 측면 내에서 추정된 가격의 유동성이 적절한 부호와 범위를 보여주어서 만족할 만한 결과를 나타내고 있다. 미래에는 토지나 공공재, 환경제의 가격형성모형의 연구에도 응용될 수 있을 것이다.

주제어: 가격형성체계, 한계가치, 식관성, 단위근, 가격유동성

- 786 -
Abstracts

On Generating a Dynamic Price Formation System with Rationality
— Application to U.S. Fisheries —

Hoanjae Park

This article is basically an extension of Barten (1993), Brown et al. (1995), Holt and Bishop's (2002) price formation system. A new dynamic price formation system is attempted considering full rationality of the consumers' side. The underlying idea of the new dynamic price formation system is that consumers are rational and farsighted and thus consider past and future consumptions in addition to current consumption to accept the prices traders called. In an empirical application, the U.S. commercial fish demand data are particularly interesting to this analysis in which the species are over fished, including many of the most valuable species. Especially, the grouper-snapper complex are under management jurisdiction of the National Marine Fisheries Council. In the empirical section, it shows how to adapt the model to estimate the marginal values to consumers of commercial fisheries. Since it is conceived of regulations as inducing movements along the marginal value curves, it is of growing importance to regional and national policy makers who are confronted with competing claims on diminishing fish stocks by commercial fisheries interests. It performs well and shows the plausible signs and magnitudes of price flexibilities and interaction among species. It further contributes to the general methodology of applied economics.

Keywords: Price formation, Marginal Value, Habit formation, Unit Root, Price Flexibility